Missing Discussions: Institutional Constraints in the Islamic Political Tradition

Online Appendix

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A Welfare Inclusive of Revolt Costs

In this section, we argue that all the main results in the main text go through if the costs of revolt are included in the payoff of majority citizens, in addition to the policy payoff.

In the equilibrium of the coordination game, a citizen i revolts if and only if his direct cost of revolt c_i is below the equilibrium threshold c^* and the revolt succeeds if and only if the $\bar{c} < \bar{c}^*$. Moreover, in the limit as $\rho \to 0$: $\lim_{\rho\to 0} c^*(\rho) = \lim_{\rho\to 0} \bar{c}^*(\rho)$ (Boleslavsky, Shadmehr and Sonin, 2021). If $\bar{c} < \bar{c}^*$, then almost all citizens (members of the Majority) revolt; if $\bar{c} > \bar{c}^*$, then almost no citizen revolts. Thus, when revolt is attempted, the expected cost of revolt is:

$$\Pr(\bar{c} < \bar{c}^*) \cdot \mathbb{E}[\bar{c} \mid \bar{c} < \bar{c}^*]$$

Given $\bar{c} \sim U[0, 1]$, this is equal to:

$$\Pr(\bar{c} < \bar{c}^*) \cdot \mathbb{E}[\bar{c} \mid \bar{c} < \bar{c}^*] = \frac{(\bar{c}^*)^2}{2}$$

Under the cost threshold c^*, there is a revolt with probability $\beta \in [0, 1]$. Note that $\beta = \bar{c}^*$ whenever $\bar{c}^* > 0$ and $\beta = 0$ whenever $\bar{c}^* \leq 0$. Thus, $\Pr(\bar{c} < \bar{c}^*) \cdot \mathbb{E}[\bar{c}|\bar{c} < \bar{c}^*] = \beta^2/2$. This, in turn, implies that to account for the expected costs of revolt in the citizens' expected payoffs, we can simply subtract $\beta^2/2$ whenever a revolt is attempted by a strictly positive measure of citizens. Because when the revolt succeeds, the citizen payoff increases by 2, this means that to account for the expected costs of revolt in the citizen's payoffs, we can simply

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substitute β with $\beta_c(\beta) = \beta - \beta^2/4$ when calculating the value of the expected payoffs: $Pr(revolt\ attempted) \cdot (2\beta - \beta^2/2) = 2Pr(revolt\ attempted) \cdot (\beta - \beta^2/4)$. Because $\beta_c(0) = 0$, $\beta_c(1) = 1/2$, and $\beta_c(\beta)$ is strictly increasing in β, all our main results go through if we add the direct costs of revolt into the citizens' payoff, and then compare them under different institutional arrangements. For completeness, we derive these results below.

Recall that, in the equilibrium without institutional constraints, there is a revolt only if $\hat{s} = 0$ and the ruler takes action 1. In particular,

- When $\beta(1, M, \gamma) > 1 - \delta_0$, the threat of revolt disciplines the minority-congruent ruler, and the minority-congruent ruler takes action $a = 0$ when $\hat{s} = 0$. Consequently, there are no revolts, and there are no costs of revolt. In this case, payoffs inclusive of revolt costs are equal to policy payoffs. The majority citizens' expected payoff inclusive of revolt costs is:

$$1 - q(1 - p)$$

- When $\beta(1, M, \gamma) < 1 - \delta_0$, the minority-congruent ruler takes action $a = 1$ when $\hat{s} = 0$. When that happens, a revolt is attempted by a strictly positive measure of citizens. Therefore, a revolt is attempted by a strictly positive measure of citizens with probability

$$\Pr(t = b) \cdot \Pr(s = 0) \cdot \Pr(\hat{s} = 0 \mid s = 0) = q \cdot \frac{1}{2} \cdot p = \frac{qp}{2},$$

  in which case the expected costs of revolt is $\beta^2/2$.

  Thus, to calculate the expected payoff of majority citizens inclusive of revolt costs, one needs to subtract $(qp/2)(\beta^2/2) = qp\beta^2/4$ from the policy payoff. Recalling that $\beta_c(\beta) = \beta - \beta^2/4$, the majority citizens' expected payoff inclusive of revolt costs is:

$$1 - q\,(1 - p\beta) - \frac{qp\beta^2}{4} = 1 - q\,(1 - p\beta_c)\,.$$

Proposition 2 in the main text is therefore modified as follows.

Proposition 1. *In equilibrium,*

$$\sigma(\hat{s}, 1) = \sigma(\hat{s} = \emptyset, 0) = 1 \quad and \quad \sigma(\hat{s} = s, 0) = \begin{cases} 0 & ;\ \beta(1, M, \gamma) > 1 - \delta_0 \\ 1 & ;\ \beta(1, M, \gamma) < 1 - \delta_0 \end{cases}$$

There is a revolt only if $\hat{s} = 0$ and the ruler takes action 1. This revolt succeeds with probability $\beta(1, M, \gamma)$. Moreover, the expected payoff for a majority citizen, inclusive of policy payoffs and revolt costs, is

$$\begin{cases} 1 - q(1 - p) & ;\ \beta(1, M, \gamma) > 1 - \delta_0 \\ 1 - q(1 - p\beta_c(1, M, \gamma)) & ;\ \beta(1, M, \gamma) < 1 - \delta_0. \end{cases}$$

Following the same steps, Proposition 3 in the main text is modified as follows.

Proposition 2. *Recall that A is the aggregate government action, and $Pr_{(t_1,t_2)}(A)$ is the probability of A conditional on rulers' types (t_1, t_2). In equilibrium,*

$$Pr_{(t_1,t_2)}(A = s) = 1, \quad \text{if } (t_1, t_2) \neq (b, b).$$

Otherwise,

$$Pr_{(b,b)}(A = 1|\hat{s}, s = 1) = Pr_{(b,b)}(A = 1|\hat{s} = \emptyset, s = 0) = 1$$

and

$$Pr_{(b,b)}(A = 1|\hat{s} = s, s = 0) = \begin{cases} 1 & ; \beta(1, M, \gamma) < 1 - \delta_0 \\ 0 & ; otherwise. \end{cases}$$

There is a revolt only if $\hat{s} = 0$ and both rulers take action 1. This revolt succeeds with probability $\beta(1, M, \gamma)$. Moreover, the expected payoff for a majority citizen, inclusive of policy payoffs and revolt costs, is

$$\begin{cases} 1 - q^2(1 - p) - \mu & ; \beta(1, M, \gamma) > 1 - \delta_0 \\ 1 - q^2(1 - p\beta_c(1, M, \gamma)) - \mu & ; \beta(1, M, \gamma) < 1 - \delta_0. \end{cases}$$

Corollary 1 in the main text is modified as follows.

Corollary 1. *The value of institutional constraints is:*

$$\begin{cases} (1 - p)(q - q^2) - \mu & ; \beta(1, M, \gamma) > 1 - \delta_0 \\ (1 - p\beta_c(1, M, \gamma))(q - q^2) - \mu & ; \beta(1, M, \gamma) < 1 - \delta_0. \end{cases}$$

Proposition 4 in the main text is modified as follows.

Proposition 3. *There is threshold $p^*(M, \gamma, q, \mu)$ such that a majority citizen's expected payoff, inclusive of policy payoffs and revolt costs, is higher without institutional constraints if and only if the scope of the divine law $p > p^*$, where*

$$p^*(M, \gamma, q, \mu) = \begin{cases} 1 - \frac{\mu}{q(1-q)} & ; \beta(1, M, \gamma) > 1 - \delta_0 \\ \\ \frac{1}{\beta_c(1,M,\gamma)} \left(1 - \frac{\mu}{q(1-q)}\right) & ; \beta(1, M, \gamma) < 1 - \delta_0. \end{cases}$$

Moreover,

1. *If $p^*(M, \gamma, q, \mu) > 0$, then $p^*(M, \gamma, q, \mu)$ is decreasing in M and γ; strictly so if and only if $\beta(1, M, \gamma) < 1 - \delta_0$.*
2. *$p^*(M, \gamma, q, \mu = 0) \geq 1$. For $\mu > 0$, $p^*(M, \gamma, q, \mu)$ has an inverted U-shape in q, with*

$$lim_{q \to 0^+} p^*(M, \gamma, q, \mu) = lim_{q \to 1^-} p^*(M, \gamma, q, \mu) = -\infty.$$

Proposition 5 in the main text is modified as follows.

Proposition 4. *There is a cost threshold such that the majority citizen's expected payoff, inclusive of policy payoffs and revolt costs, is higher without institutional constraints if and only if* $\mu > \mu^*$*, where*

$$\mu^*(\beta, p, q) = \begin{cases} (1-p)(q-q^2) & ; \beta > 1-\delta_0 \\ \\ (1-p(\beta-\beta^2/4))(q-q^2) & ; \beta < 1-\delta_0, \end{cases}$$

where $\beta = \beta(1, M, \gamma)$*. Moreover,*

1. μ^* *is strictly decreasing in* p*, and weakly decreasing in* $\beta(1, M, \gamma)$ *(and hence in* M *and* γ*); strictly so when* $\beta < 1 - \delta_0$*.*
2. *Suppose* $\delta_0 < T/M$*, so that there is sufficient conflict of interest that the threat of revolt does not deter the minority-congruent ruler (*$\beta < 1-\delta_0$*). Then,*

$$\frac{\partial^2 \mu^*(\beta, p, q)}{\partial p \partial \beta} = -(q-q^2)\left(1 - \frac{\beta}{2}\right) < 0.$$

Finally, Proposition 6 in the main text is modified as follows.

Proposition 5. *Suppose* $\gamma \sim U[0,1]$*. Let* $Q = Pr_\gamma(\mu \leq \mu^*(\gamma))$ *be the probability that institutional constraints improve the majority citizen's expected payoff, inclusive of policy payoffs and revolt costs. Suppose* $\delta_0 < T/M$*, so that there is sufficient conflict of interest that the threat of revolt does not deter the minority-congruent (*$\beta < 1-\delta_0$*). Let* $\mu' = \mu/(q-q^2)$*. Then,*

1. Q *is decreasing in* μ'*,* p *and in* M*; strictly so when* $\mu' \in (1-p\left((1-T/M) - \frac{1}{4}(1-T/M)^2\right), 1)$*.*
2. $|Q(\mu_2') - Q(\mu_1')|$ *is strictly decreasing in* p *and* M *for all* μ_1', μ_2'*, with* $\mu_1', \mu_2' \in (1 - p\left((1-T/M) - \frac{1}{4}(1-T/M)^2\right), 1)$*.*

Proof. Using Proposition 4,

$$\begin{aligned} Q &= Pr_\gamma(\mu \leq \mu^*(\gamma) \mid \beta < 1-\delta_0) \\ &= Pr_\gamma\left(\mu \leq (1-p(\beta - \frac{\beta^2}{4}))(q-q^2)\right) \end{aligned}$$

Using the fact that $\beta = \beta(1, M, \gamma)$, and substituting Proposition 1 in the main text, we have: $\beta = H\left((1-\frac{T}{M})\gamma\right)$. Because $H = U[0,1]$, $\beta = (1-\frac{T}{M})\gamma$. Substituting, we have:

$$\begin{aligned} Q &= Pr_\gamma\left(\mu \leq (1-p(\gamma(1-\frac{T}{M}) - \frac{\gamma^2(1-\frac{T}{M})^2}{4}))(q-q^2)\right) \\ &= Pr_\gamma\left(\mu' \leq 1-p(\gamma(1-\frac{T}{M}) - \frac{\gamma^2(1-\frac{T}{M})^2}{4})\right) \\ &= Pr_\gamma\left(\gamma(1-\frac{T}{M}) - \frac{\gamma^2}{4}(1-\frac{T}{M})^2 \leq \frac{1-\mu'}{p}\right) \end{aligned}$$

For any $\gamma \in [0,1]$ and $M \in (T,1]$, let:

$$\zeta(\gamma, M) \equiv \gamma(1-\frac{T}{M}) - \frac{\gamma^2}{4}(1-\frac{T}{M})^2$$

Then,

$$Q = Pr_\gamma\left(\zeta(\gamma, M) \leq \frac{1-\mu'}{p}\right) \tag{1}$$

We continue with a few observations that will play a crucial role in the following arguments.

- $\zeta(\gamma, M)$ is strictly increasing in γ and M, because:

$$\frac{\partial\zeta}{\partial\gamma} = 1 - \frac{T}{M} - \frac{\gamma}{2}(1-\frac{T}{M})^2 \qquad > 0$$
$$\frac{\partial\zeta}{\partial M} = \gamma\frac{T}{M^2} - \frac{\gamma^2}{2}(1-\frac{T}{M})\frac{T}{M^2} \qquad > 0$$

- $\zeta(\gamma, M)$ is strictly concave in γ, because:

$$\frac{\partial^2\zeta}{\partial\gamma^2} = -\frac{1}{2}(1-\frac{T}{M})^2 \qquad < 0$$

- $\zeta(\gamma, M)$ is supermodular in γ and M, because:

$$\frac{\partial^2\zeta}{\partial\gamma\partial M} = \frac{T}{M^2} - \gamma(1-\frac{T}{M})\frac{T}{M^2} \qquad > 0$$

  As a result, $\zeta(\gamma, M)$ satisfies strict increasing differences in (γ, M). That is, for any $\gamma_1 < \gamma_2$ and $M_1 < M_2$,

$$\zeta(\gamma_2, M_1) - \zeta(\gamma_1, M_1) < \zeta(\gamma_2, M_2) - \zeta(\gamma_1, M_2).$$

Because $\zeta(\gamma, M)$ is strictly increasing in γ, and since $\gamma \sim U[0,1]$, Equation (1) implies:

$$Q = \begin{cases} 0 & ;\ \frac{1-\mu'}{p} < \zeta(0, M) \\ \gamma^* \text{ s.t. } \zeta(\gamma^*, M) = \frac{1-\mu'}{p} & ;\ \zeta(0, M) \leq \frac{1-\mu'}{p} \leq \zeta(1, M) \\ 1 & ;\ \frac{1-\mu'}{p} > \zeta(1, M) \end{cases}$$

Substituting $\zeta(0, M) = 0$ and $\zeta(1, M) = (1-\frac{T}{M}) - \frac{1}{4}(1-\frac{T}{M})^2$,

$$Q = \begin{cases} 0 & ;\ \frac{1-\mu'}{p} < 0 \\ \gamma^* \text{ s.t. } \zeta(\gamma^*, M) = \frac{1-\mu'}{p} & ;\ 0 \leq \frac{1-\mu'}{p} \leq (1-\frac{T}{M}) - \frac{1}{4}(1-\frac{T}{M})^2 \\ 1 & ;\ \frac{1-\mu'}{p} > (1-\frac{T}{M}) - \frac{1}{4}(1-\frac{T}{M})^2 \end{cases}$$

Rearranging,

$$Q = \begin{cases} 1 & ; \ \mu' < 1 - p\left((1-\frac{T}{M}) - \frac{1}{4}(1-\frac{T}{M})^2\right) \\ \gamma^* \text{ s.t. } \zeta(\gamma^*, M) = \frac{1-\mu'}{p} & ; \ \mu' \in \left[1 - p\left((1-\frac{T}{M}) - \frac{1}{4}(1-\frac{T}{M})^2\right), 1\right] \\ 0 & ; \ \mu' > 1 \end{cases}$$

The fact that Q is decreasing in μ' and p, strictly so when $\mu' \in (1-p\left((1-\frac{T}{M}) - \frac{1}{4}(1-\frac{T}{M})^2\right), 1)$, follows from $\zeta(\gamma, M)$ being strictly increasing in γ. Moreover, the fact that Q is decreasing in M, strictly so when $\mu' \in (1-p\left((1-\frac{T}{M}) - \frac{1}{4}(1-\frac{T}{M})^2\right), 1)$, follows from $\zeta(\gamma, M)$ being strictly increasing in γ and strictly increasing in M.

Next, we show that $|Q(\mu_2') - Q(\mu_1')|$ is strictly decreasing in p for all μ_1', μ_2', with $\mu_1', \mu_2' \in (1-p\left((1-T/M) - \frac{1}{4}(1-T/M)^2\right), 1)$. Without loss of generality, take $\mu_1' < \mu_2'$ and $p_1 < p_2$ such that $\mu_1', \mu_2' \in (1-p_1\left((1-T/M) - \frac{1}{4}(1-T/M)^2\right), 1)$.

Let $Q(\mu_1' \mid p_1)$ and $Q(\mu_2' \mid p_1)$ denote the relevant probabilities under p_1. Note that $Q(\mu_1' \mid p_1) = \gamma_{11}$ and $Q(\mu_2' \mid p_1) = \gamma_{21}$, where:

$$\zeta(\gamma_{11}, M) = \frac{1-\mu_1'}{p_1} \qquad \zeta(\gamma_{21}, M) = \frac{1-\mu_2'}{p_1}$$

Similarly, $Q(\mu_1' \mid p_2) = \gamma_{12}$ and $Q(\mu_2' \mid p_2) = \gamma_{22}$, where:

$$\zeta(\gamma_{12}, M) = \frac{1-\mu_1'}{p_2} \qquad \zeta(\gamma_{22}, M) = \frac{1-\mu_2'}{p_2}$$

Now,

$$\zeta(\gamma_{11}, M) - \zeta(\gamma_{21}, M) = \frac{\mu_2' - \mu_1'}{p_1} > \frac{\mu_2' - \mu_1'}{p_2} = \zeta(\gamma_{12}, M) - \zeta(\gamma_{22}, M)$$

Therefore,

$$\zeta(\gamma_{11}, M) - \zeta(\gamma_{21}, M) > \zeta(\gamma_{12}, M) - \zeta(\gamma_{22}, M) \tag{2}$$

Because Q is strictly decreasing in μ' in the range considered, $\gamma_{11} > \gamma_{21}$ and $\gamma_{12} > \gamma_{22}$. Because Q is strictly decreasing in p in the range considered, $\gamma_{11} > \gamma_{12}$ and $\gamma_{21} > \gamma_{22}$. Finally, recall that $\zeta(\gamma, M)$ is strictly concave in γ. For Equation (2) to hold, therefore, one must have: $\gamma_{11} - \gamma_{21} > \gamma_{12} - \gamma_{22}$. Therefore,

$$\begin{aligned} |Q(\mu_2' \mid p_1) - Q(\mu_1' \mid p_1)| &= |\gamma_{21} - \gamma_{11}| \\ &= \gamma_{11} - \gamma_{21} \\ &> \gamma_{12} - \gamma_{22} \\ &= |\gamma_{22} - \gamma_{12}| \\ &= |Q(\mu_2' \mid p_2) - Q(\mu_1' \mid p_2)| \end{aligned}$$

and the result follows.

Finally, we show that $|Q(\mu_2') - Q(\mu_1')|$ is strictly decreasing in M for all μ_1', μ_2', with $\mu_1', \mu_2' \in (1 - p\left((1 - T/M) - \frac{1}{4}(1 - T/M)^2\right), 1)$. Without loss of generality, take $\mu_1' < \mu_2'$ and $M_1 < M_2$ such that $\mu_1', \mu_2' \in (1 - p\left((1 - T/M_1) - \frac{1}{4}(1 - T/M_1)^2\right), 1)$.

Let $Q(\mu_1' \mid M_1)$ and $Q(\mu_2' \mid M_1)$ denote the relevant probabilities under M_1. Note that $Q(\mu_1' \mid M_1) = \gamma_{11}$ and $Q(\mu_2' \mid M_1) = \gamma_{21}$, where:

$$\zeta(\gamma_{11}, M_1) = \frac{1 - \mu_1'}{p} \qquad \zeta(\gamma_{21}, M_1) = \frac{1 - \mu_2'}{p}$$

Similarly, $Q(\mu_1' \mid M_2) = \gamma_{12}$ and $Q(\mu_2' \mid M_2) = \gamma_{22}$, where:

$$\zeta(\gamma_{12}, M_2) = \frac{1 - \mu_1'}{p} \qquad \zeta(\gamma_{22}, M_2) = \frac{1 - \mu_2'}{p}$$

Now,

$$\zeta(\gamma_{11}, M_1) - \zeta(\gamma_{21}, M_1) = \frac{\mu_2' - \mu_1'}{p} = \zeta(\gamma_{12}, M_2) - \zeta(\gamma_{22}, M_2) \tag{3}$$

Because Q is strictly decreasing in μ' in the range considered, $\gamma_{11} > \gamma_{21}$. Because $\zeta(\gamma, M)$ satisfies strictly increasing differences in (γ, M), $\zeta(\gamma_{11}, M_1) - \zeta(\gamma_{21}, M_1) < \zeta(\gamma_{11}, M_2) - \zeta(\gamma_{21}, M_2)$. This, along with Equation (3), implies:

$$\zeta(\gamma_{11}, M_2) - \zeta(\gamma_{21}, M_2) > \zeta(\gamma_{12}, M_2) - \zeta(\gamma_{22}, M_2) \tag{4}$$

Because Q is strictly decreasing in μ' in the range considered, $\gamma_{12} > \gamma_{22}$. Because Q is strictly decreasing in M in the range considered, $\gamma_{11} > \gamma_{12}$ and $\gamma_{21} > \gamma_{22}$. Finally, recall that $\zeta(\gamma, M)$ is strictly concave in γ. For Equation (4) to hold, therefore, one must have: $\gamma_{11} - \gamma_{21} > \gamma_{12} - \gamma_{22}$. Therefore,

$$\begin{aligned} |Q(\mu_2' \mid M_1) - Q(\mu_1' \mid M_1)| &= |\gamma_{21} - \gamma_{11}| \\ &= \gamma_{11} - \gamma_{21} \\ &> \gamma_{12} - \gamma_{22} \\ &= |\gamma_{22} - \gamma_{12}| \\ &= |Q(\mu_2' \mid M_2) - Q(\mu_1' \mid M_2)| \end{aligned}$$

and the result follows. □

B An Alternative Model of Institutional Constraints

In this section, we present an alternative model with institutional constraints and provide a characterization. Throughout this section, we maintain our assumption that $0 = \delta_1 < \delta_0 < 1$ in the main text. The difference is that we consider $y(a_1, a_2) = \max\{a_1, a_2\}$. That is, in the setup considered here, if one of the rulers choose the minority-congruent policy $a_i = 1$, the aggregate policy is $A = 1$. A majority-congruent ruler, therefore, does not have the blocking power by himself. However, since citizens observe (a_1, a_2), they can still receive information from the majority-congruent ruler's proposed policy and base their revolt decisions on this information. In this sense, the institutional arrangement has a learning benefit for the citizens.

B.1 Formal Definition of Equilibrium

The majority-congruent ruler $j \in \{1, 2\}$ (i.e., ruler j of type $t_j = g$) always chooses $a_j = s$ by assumption.

The strategy of the minority-congruent ruler 1 (i.e., ruler 1 of type $t_1 = b$) in state s, when public signal is $\hat{s}$ and ruler 2's type is $t_2 \in \{b, g\}$ is:

$$\sigma_1(\widehat{s}, s, t_2) \equiv \Pr(a_1 = 1 | s, \hat{s}, t_2) \quad \in [0, 1]$$

The strategy of minority-congruent ruler 2 (i.e., ruler 2 of type $t_2 = b$) in state s, given the public signal is $\hat{s}$ and ruler 1's action a_1 is:[1]

$$\sigma_2(\widehat{s}, s, a_1) \equiv \Pr(a_2 = 1 | s, \hat{s}, a_1) \quad \in [0, 1]$$

The posterior beliefs of citizens that the aggregate policy is incongruent, given information $(\hat{s}, a_1, a_2)$, is denoted by:

$$q(\hat{s}, a_1, a_2) \equiv \Pr\left(\max\{a_1, a_2\} \neq s | \hat{s}, a_1, a_2\right) \quad \in [0, 1]$$

The strategy of a citizen i when with posterior beliefs q' and the cost of revolt is c_i is denoted by:

$$\varphi(q', c_i) \equiv \Pr(r_i = 1 | q', c_i) \quad \in [0, 1]$$

The Perfect Bayesian Nash Equilibrium of the game is a quadruple $(\sigma_1^*, \sigma_2^*, \varphi^*, q^*)$ such that the following are satisfied.

1. $\varphi^*(q', c_i)$ maximizes the payoff of the citizens in majority for any $q' = q^*(\hat{s}, a_1, a_2)$.
2. $q^*(\hat{s}, a_1, a_2)$ is given by Bayes' Rule.
3. Given φ^* and σ_2^*, σ_1^* maximizes the payoff of the minority-congruent ruler 1. Similarly, given φ^* and σ_1^*, σ_2^* maximizes the payoff of the minority-congruent ruler 2.

[1] As discussed in the main text, ruler 2's strategy may also condition on t_1. However, because t_1 is not payoff-relevant for ruler 2, the dependence can be dropped.

We consider the symmetric cutoff strategy equilibrium with cutoffs greater than one as $\rho \to 0$. Once again, there are multiple equilibria in this model. As an equilibrium selection device, we impose the following assumption on the minority-congruent ruler.

Assumption 1. *When a minority-congruent ruler j is indifferent between the two actions, he chooses $a_j = 1$ with probability 1.*

Assumption 1 is a mild restriction on the minority-congruent ruler's behavior: it applies only when the ruler is indifferent between the two actions. It can be microfounded by assuming that the minority-congruent ruler j obtains some infinitesimal material payoff from taking action $a_j = 1$.

B.2 Equilibrium Characterization

B.2.1 Citizens' Actions

As we will show later, the members of minority never take part in a revolution in equilibrium. Therefore, the only citizens who potentially participate in a revolution are majority citizens, whose size is M. As discussed in Proposition 1 in the main text, in a symmetric cutoff strategy equilibrium as $\rho \to 0$, a successful revolution occurs with probability:

$$\beta(q', M, \gamma) = H\left((1 - \frac{T}{M}) \cdot \gamma \cdot (2q' - 1)\right)$$

B.2.2 Beliefs Following Proposed Policy

When $\hat{s} \in \{0, 1\}$, $q^*(\hat{s}, a_1, a_2) = |\hat{s} - \max\{a_1, a_2\}| \in \{0, 1\}$. When $\hat{s} = \emptyset$, the posterior beliefs are given by:

$$\begin{aligned}
q^*(\emptyset, 0, 0) &\equiv \Pr(\max\{a_1, a_2\} \neq s | a_1 = a_2 = 0, \hat{s} = \emptyset) \\
&= \Pr(s = 1 | a_1 = a_2 = 0, \hat{s} = \emptyset) \\
&= \frac{\Pr(s = 1, a_1 = a_2 = 0, \hat{s} = \emptyset)}{\Pr(s = 1, a_1 = a_2 = 0, \hat{s} = \emptyset) + \Pr(s = 0, a_1 = a_2 = 0, \hat{s} = \emptyset)} \\
&= \frac{\frac{1}{2} q^2 (1 - \sigma_1^*(\emptyset, 1, b))(1 - \sigma_2^*(\emptyset, 1, 0))}{\frac{1}{2} q^2 (1 - \sigma_1^*(\emptyset, 1, b))(1 - \sigma_2^*(\emptyset, 1, 0)) + \frac{1}{2}\left(q^2 (1 - \sigma_1^*(\emptyset, 0, b))(1 - \sigma_2^*(\emptyset, 0, 0)) + q(1-q)(1 - \sigma_1^*(\emptyset, 0, g)) + (1-q)q(1 - \sigma_2^*(\emptyset, 0, 0)) + (1-q)^2\right)} \\
&= \frac{q^2 (1 - \sigma_1^*(\emptyset, 1, b))(1 - \sigma_2^*(\emptyset, 1, 0))}{q^2 (1 - \sigma_1^*(\emptyset, 1, b))(1 - \sigma_2^*(\emptyset, 1, 0)) + \left(q^2 (1 - \sigma_1^*(\emptyset, 0, b))(1 - \sigma_2^*(\emptyset, 0, 0)) + q(1-q)(1 - \sigma_1^*(\emptyset, 0, g)) + (1-q)q(1 - \sigma_2^*(\emptyset, 0, 0)) + (1-q)^2\right)}
\end{aligned}$$

$$\begin{aligned}
q^*(\emptyset, 0, 1) &\equiv \Pr(\max\{a_1, a_2\} \neq s | a_1 = 0, a_2 = 1, \hat{s} = \emptyset) \\
&= \Pr(s = 0 | a_1 = 0, a_2 = 1, \hat{s} = \emptyset) \\
&= \frac{\Pr(s = 0, a_1 = 0, a_2 = 1, \hat{s} = \emptyset)}{\Pr(s = 0, a_1 = 0, a_2 = 1, \hat{s} = \emptyset) + \Pr(s = 1, a_1 = 0, a_2 = 1, \hat{s} = \emptyset)} \\
&= \frac{\frac{1}{2} q^2 (1 - \sigma_1^*(\emptyset, 0, b))\sigma_2^*(\emptyset, 0, 0) + (1-q)q(1 - \sigma_2^*(\emptyset, 0, 0))}{\frac{1}{2} q^2 (1 - \sigma_1^*(\emptyset, 0, b))\sigma_2^*(\emptyset, 0, 0) + (1-q)q(1 - \sigma_2^*(\emptyset, 0, 0)) + \frac{1}{2}\left(q^2 (1 - \sigma_1^*(\emptyset, 1, b))\sigma_2^*(\emptyset, 1, 0) + q(1-q)(1 - \sigma_1^*(\emptyset, 1, g))\right)}
\end{aligned}$$

$$\begin{aligned}
q^*(\emptyset, 1, 0) &\equiv \Pr(\max\{a_1, a_2\} \neq s | a_1 = 1, a_2 = 0, \hat{s} = \emptyset) \\
&= \Pr(s = 0 | a_1 = 1, a_2 = 0, \hat{s} = \emptyset) \\
&= \frac{\Pr(s = 0, a_1 = 1, a_2 = 0, \hat{s} = \emptyset)}{\Pr(s = 0, a_1 = 1, a_2 = 0, \hat{s} = \emptyset) + \Pr(s = 1, a_1 = 1, a_2 = 0, \hat{s} = \emptyset)} \\
&= \frac{\frac{1}{2} q^2 \sigma_1^*(\emptyset, 0, b)(1 - \sigma_2^*(\emptyset, 0, 1)) + q(1-q)\sigma_1^*(\emptyset, 0, g)}{\frac{1}{2} q^2 \sigma_1^*(\emptyset, 0, b)(1 - \sigma_2^*(\emptyset, 0, 1)) + q(1-q)\sigma_1^*(\emptyset, 0, g) + \frac{1}{2}\left(q^2 \sigma_1^*(\emptyset, 1, b)(1 - \sigma_2^*(\emptyset, 1, 1)) + (1-q)q(1 - \sigma_2^*(\emptyset, 1, 1))\right)}
\end{aligned}$$

$$
\begin{aligned}
q^*(\emptyset,1,1) &\equiv \Pr(\max\{a_1,a_2\} \neq s | a_1 = a_2 = 1, \hat{s} = \emptyset) \\
&= \Pr(s = 0 | a_1 = a_2 = 1, \hat{s} = \emptyset) \\
&= \frac{\Pr(s = 0, a_1 = a_2 = 1, \hat{s} = \emptyset)}{\Pr(s = 0, a_1 = a_2 = 1, \hat{s} = \emptyset) + \Pr(s = 1, a_1 = a_2 = 1, \hat{s} = \emptyset)} \\
&= \frac{\frac{1}{2}q^2\sigma_1^*(\emptyset,0,b)\sigma_2^*(\emptyset,0,1)}{\frac{1}{2}q^2\sigma_1^*(\emptyset,0,b)\sigma_2^*(\emptyset,0,1) + \frac{1}{2}\left(q^2\sigma_1^*(\emptyset,1,b)\sigma_2^*(\emptyset,1,1) + q(1-q)\sigma_1^*(\emptyset,1,g) + (1-q)q\sigma_2^*(\emptyset,1,1) + (1-q)^2\right)}
\end{aligned}
$$

B.2.3 Rulers' Actions

When the Issue is Preordained We begin by pinning down the strategies of minority-congruent ruler 2 at every history.

1. Consider the case $\hat{s} = s = 0$ and $a_1 = 0$. In this case, $\max\{a_1, a_2\} = a_2$ and $q^*(0, 0, a_2) = a_2$ for any $a_2 \in \{0, 1\}$.

   The majority members never revolt against $a_2 = 0$, and since $M > 1/2$, there is never a revolt against $a_2 = 0$. In contrast, the minority members never revolt against $a_2 = 1$, and therefore the probability of a successful revolt against $a_2 = 1$ is $\beta(1, M, \gamma)$. Thus, ruler 2's policy when $(\hat{s}, s, a_1) = (0, 0, 0)$ is:

   $$\sigma_2^*(0,0,0) \in \arg\max_{\sigma\in[0,1]} \sigma \cdot (1 - \beta(1, M, \gamma)) + (1 - \sigma) \cdot \delta_0$$

   Therefore, ruler 2's PBE strategy is:

   $$\sigma_2^*(0,0,0) = \begin{cases} 0 & ; \delta_0 > 1 - \beta(1, M, \gamma) \\ 1 & ; \delta_0 < 1 - \beta(1, M, \gamma) \end{cases}$$

2. Consider the case $\hat{s} = s = 0$ and $a_1 = 1$. In this case, $\max\{a_1, a_2\} = 1$ regardless of a_2, and $q^*(0, 1, a_2) = 1$ for any $a_2 \in \{0, 1\}$. Ruler 2 is indifferent between the two actions, and by Assumption 1, $\sigma_2^*(0, 0, 1) = 1$.

3. Consider the case $\hat{s} = s = 1$ and $a_1 = 0$. In this case, $\max\{a_1, a_2\} = a_2$ and $q^*(1, 0, a_2) = 1 - a_2$ for any $a_2 \in \{0, 1\}$.

   Because $\delta_1 = 0$, ruler 2 receives a payoff of 0 if he chooses $a_2 = 0$. If he chooses $a_2 = 1$, the citizens will not revolt, and ruler 2 will receive a payoff of 1. Therefore, $\sigma_2^*(1, 1, 0) = 1$.

4. Consider the case $\hat{s} = s = 1$ and $a_1 = 1$. In this case, $\max\{a_1, a_2\} = 1$ regardless of a_2, and $q^*(1, 1, a_2) = 0$ for any $a_2 \in \{0, 1\}$. Ruler 2 is indifferent between the two actions, and by Assumption 1, $\sigma_2^*(1, 1, 1) = 1$.

Next, we pin down the strategy of minority-congruent ruler 1 in every history.

1. Consider the case $\hat{s} = s = 0$ and $t_2 = g$. In this case, $a_2 = 0$, and $\max\{a_1, a_2\} = a_1 \in \{0, 1\}$. Moreover, $q^*(0, a_1, a_2) = a_1$ for any $a_1 \in \{0, 1\}$.

   The majority members never revolt against $a_1 = 0$, and since $M > 1/2$, there is never a revolt against $a_1 = 0$. The minority members never revolt against $a_1 = 1$, and therefore the probability of a successful revolt against $a_1 = 1$ is $\beta(1, M, \gamma)$. Thus, ruler 1's policy when $(\hat{s}, s, t_2) = (0, 0, g)$ is:

$$\sigma_1^*(0,0,g) \in \arg\max_{\sigma \in [0,1]} \sigma \cdot (1 - \beta(1, M, \gamma)) + (1 - \sigma) \cdot \delta_0$$

Therefore, ruler 1's PBE strategy is:

$$\sigma_1^*(0,0,g) = \begin{cases} 0 & ; \delta_0 > 1 - \beta(1, M, \gamma) \\ 1 & ; \delta_0 < 1 - \beta(1, M, \gamma) \end{cases}$$

2. Consider the case $\hat{s} = s = 0$ and $t_2 = b$. In this case,

$$a_2 = \begin{cases} 0 & ; \delta_0 > 1 - \beta(1, M, \gamma) \\ 1 & ; \delta_0 < 1 - \beta(1, M, \gamma) \end{cases}$$

   - If $\delta_0 > 1 - \beta(1, M, \gamma)$, ruler 1's optimal strategy is:

   $$\sigma_1^*(0,0,b) \in \arg\max_{\sigma \in [0,1]} \sigma \cdot (1 - \beta(1, M, \gamma)) + (1 - \sigma) \cdot \delta_0$$

   which is maximized when $\sigma_1^*(0,0,b) = 0$.

   - If $\delta_0 < 1 - \beta(1, M, \gamma)$, $\max\{a_1, a_2\} = 1$ for any $a_1 \in \{0,1\}$ in any PBE. Ruler 1 is indifferent between the two actions. By Assumption 1, $\sigma_1^*(0,0,b) = 1$.

3. Consider the case $\hat{s} = s = 1$ and $t_2 = g$. In this case, $a_2 = 1$, and $\max\{a_1, a_2\} = 1$ for any $a_1 \in \{0,1\}$ in any PBE. Ruler 1 is indifferent between the two actions. By Assumption 1, $\sigma_1^*(1,1,g) = 1$.

4. Consider the case $\hat{s} = s = 1$ and $t_2 = b$. Since $\sigma_2^*(1,1,0) = \sigma_2^*(1,1,1) = 1$, $a_2 = 1$ with probability one. Then, $\max\{a_1, a_2\} = 1$ for any $a_1 \in \{0,1\}$ in any PBE. Ruler 1 is indifferent between the two actions, and by Assumption 1, $\sigma_1^*(1,1,b) = 1$.

Note that $\sigma_1^*(1,1) = \sigma_2^*(1,1,1) = 1$ in any PBE. That is, when $\hat{s} = s = 1$, the aggregate policy is $A = 1$ with probability one.

If $\delta_0 < 1 - \beta(1, M, \gamma)$, $\sigma_1^*(0,0,b) = \sigma_1^*(0,0,g) = \sigma_2^*(0,0,0) = 1$. That is, when $\hat{s} = s = 0$, the aggregate policy taken by two rulers, when at least one of them is minority-congruent, is $A = 1$ with probability one. This is accompanied by a revolt with probability $\beta(1, M, \gamma)$.

If $\delta_0 > 1 - \beta(1, M, \gamma)$, $\sigma_1^*(0,0,b) = \sigma_1^*(0,0,g) = \sigma_2^*(0,0,0) = 0$. That is, when $\hat{s} = s = 0$, the aggregate policy is $A = 0$ with probability one.

When the Issue is Non-Preordained We begin this analysis with two observations, which will considerably simplify the following arguments.

Remark 1. *In any PBE, $\sigma_2^*(\emptyset, 1, 0) = 1$. This is because when $s = 1$ and $a_1 = 0$, choosing $a_2 = 0$ yields a payoff of 0 to ruler 2 (recall that $\delta_1 = 0$). On the other hand, choosing $a_2 = 1$ yields a strictly positive payoff because the probability of revolt is strictly less than one.*

Remark 2. *In any PBE, $q^*(\emptyset, 0, 0) = 0$. This follows from Remark 1 and the Equation defining $q^*(\emptyset, 0, 0)$ in Section B.2.2. In words, the citizens know that when $s = 1$, ruler 2 follows up $a_1 = 0$ with $a_2 = 1$. Therefore, whenever $a_1 = 0$ is followed up with $a_2 = 0$, the citizens deduce that the state is $s = 0$.*

By Remark 2, the majority citizens do not revolt upon observing $(\hat{s}, a_1, a_2) = (\emptyset, 0, 0)$. Since $M > 1/2$, the minority members do not revolt either, and there are no revolts. In any other $(\hat{s}, a_1, a_2) = (\emptyset, a_1, a_2)$, the aggregate action is $A = 1$. The minority citizens never attempt revolt against this action, and thus the only citizens possibly attempting revolt are the majority citizens. The probability of revolt is given by $\beta(q', M, \gamma)$.

Given these observations, the equilibrium strategy of minority-congruent ruler 2 the remaining histories is characterized by the following equations.

$$\sigma_2^*(\emptyset, 0, 0) \in \arg\max_{\sigma \in [0,1]} \sigma \cdot (1 - \beta(q^*(\emptyset, 0, 1), M, \gamma)) + (1 - \sigma) \cdot \delta_0 \tag{5}$$

$$\sigma_2^*(\emptyset, 0, 1) \in \arg\max_{\sigma \in [0,1]} \sigma \cdot (1 - \beta(q^*(\emptyset, 1, 1), M, \gamma)) + (1 - \sigma) \cdot (1 - \beta(q^*(\emptyset, 1, 0), M, \gamma)) \tag{6}$$

$$\sigma_2^*(\emptyset, 1, 1) \in \arg\max_{\sigma \in [0,1]} \sigma \cdot (1 - \beta(q^*(\emptyset, 1, 1), M, \gamma)) + (1 - \sigma) \cdot (1 - \beta(q^*(\emptyset, 1, 0), M, \gamma)) \tag{7}$$

We continue with two observations.

- In any PBE, $1 - \beta(q^*(\emptyset, 1, 1), M) \geq 1 - \beta(q^*(\emptyset, 1, 0), M)$. To see this, suppose not: suppose $1 - \beta(q^*(\emptyset, 1, 1), M) < 1 - \beta(q^*(\emptyset, 1, 0), M)$. Then, by (6), $\sigma_2^*(\emptyset, 0, 1) = 0$. Then, by the equation defining $q^*(\emptyset, 1, 1)$ in Section B.2.2, $q^*(\emptyset, 1, 1) = 0$. But then, $\beta(q^*(\emptyset, 1, 1), M) = 0$, a contradiction.
- The observation above, along with Assumption 1, implies that $\sigma_2^*(\emptyset, 0, 1) = \sigma_2^*(\emptyset, 1, 1) = 1$ in any PBE.

The only part of ruler 2's PBE strategy we have not pinned down so far is $\sigma_2^*(\emptyset, 0, 0)$.

We now proceed with ruler 1. For the equilibrium strategy of minority-congruent ruler 1, consider four possible histories.

1. Consider the case when $\hat{s} = \emptyset$, $s = 0$ and $t_2 = g$. Ruler 2 chooses $a_2 = 0$ with probability one, and the aggregate action is $A = \max\{a_1, a_2\} = a_1$.

   If ruler 1 chooses $a_1 = 1$, there is a revolt with probability $\beta(q^*(\emptyset, 1, 0), M, \gamma)$. If ruler 1 chooses $a_1 = 0$, there is a revolt with probability $\beta(q^*(\emptyset, 0, 0), M, \gamma) = 0$. Therefore, minority-congruent ruler 1's optimal strategy when $(\hat{s}, s, t_2) = (\emptyset, 0, g)$ is:

$$\sigma_1^*(\emptyset, 0, g) \in \arg\max_{\sigma \in [0,1]} \sigma \cdot (1 - \beta(q^*(\emptyset, 1, 0), M, \gamma)) + (1 - \sigma) \cdot \delta_0 \tag{8}$$

2. Consider the case when $\hat{s} = \emptyset$, $s = 0$ and $t_2 = b$.

   If ruler 1 chooses $a_1 = 1$, ruler 2 will follow with $a_2 = 1$ with probability one, because we established that $\sigma_2^*(\emptyset, 0, 1) = 1$. The aggregate action will be $A = 1$ and there will be a revolt with probability $\beta(q^*(\emptyset, 1, 1), M, \gamma)$.

If ruler 1 chooses $a_1 = 0$, ruler 2 will follow with a_2 with probability $\sigma_2^*(\emptyset, 0, 0)$. The aggregate action will be a_2, and ruler 1's payoff will be:

$$\sigma_2^*(\emptyset, 0, 0) \cdot (1 - \beta(q^*(\emptyset, 0, 1), M, \gamma)) + (1 - \sigma_2^*(\emptyset, 0, 0)) \cdot \delta_0$$

which, by (5), equals: $\max\{1 - \beta(q^*(\emptyset, 0, 1), M, \gamma), \delta_0\}$.

Therefore, minority-congruent ruler 1's optimal strategy when $(\hat{s}, s, t_2) = (\emptyset, 0, b)$ is:

$$\sigma_1^*(\emptyset, 0, b) \in \arg\max_{\sigma \in [0,1]} \sigma \cdot (1 - \beta(q^*(\emptyset, 1, 1), M, \gamma)) + (1 - \sigma) \cdot \max\{1 - \beta(q^*(\emptyset, 0, 1), M, \gamma), \delta_0\} \tag{9}$$

3. Consider the case when $\hat{s} = \emptyset$, $s = 1$ and $t_2 = g$. Ruler 2 chooses $a_2 = 1$ with probability one, and the aggregate action will be $A = 1$ with probability one.

   If ruler 1 chooses $a_1 = 1$, there will be a revolt with probability $\beta(q^*(\emptyset, 1, 1), M, \gamma)$. If ruler 1 chooses $a_1 = 0$, there will be a revolt with probability $\beta(q^*(\emptyset, 0, 1), M, \gamma)$. Therefore, minority-congruent ruler 1's optimal strategy when $(\hat{s}, s, t_2) = (\emptyset, 1, g)$ is:

$$\sigma_1^*(\emptyset, 1, g) \in \arg\max_{\sigma \in [0,1]} \sigma \cdot (1 - \beta(q^*(\emptyset, 1, 1), M, \gamma)) + (1 - \sigma) \cdot (1 - \beta(q^*(\emptyset, 0, 1), M, \gamma)) \tag{10}$$

4. Consider the case when $\hat{s} = \emptyset$, $s = 1$ and $t_2 = b$. We have already established that $\sigma_2^*(\emptyset, 1, 0) = \sigma_2^*(\emptyset, 1, 1) = 1$ in any PBE. Thus, ruler 2 chooses $a_2 = 1$ with probability one, and the aggregate action will be $A = 1$ with probability one.

   If ruler 1 takes $a_1 = 1$, there will be a revolt with probability $\beta(q^*(\emptyset, 1, 1), M, \gamma)$. If ruler 1 chooses $a_1 = 0$, there will be a revolt with probability $\beta(q^*(\emptyset, 0, 1), M, \gamma)$. Therefore, minority-congruent ruler 1's optimal strategy when $(\hat{s}, s, t_2) = (\emptyset, 1, b)$ is:

$$\sigma_1^*(\emptyset, 1, b) \in \arg\max_{\sigma \in [0,1]} \sigma \cdot (1 - \beta(q^*(\emptyset, 1, 1), M, \gamma)) + (1 - \sigma) \cdot (1 - \beta(q^*(\emptyset, 0, 1), M, \gamma)) \tag{11}$$

Once again, we continue with two observations.

- In any PBE, $1 - \beta(q^*(\emptyset, 1, 1), M, \gamma) \geq 1 - \beta(q^*(\emptyset, 0, 1), M, \gamma)$. To see this, suppose not: suppose $1 - \beta(q^*(\emptyset, 1, 1), M, \gamma) < 1 - \beta(q^*(\emptyset, 0, 1), M, \gamma)$. Then, by (9), $\sigma_1^*(\emptyset, 0, b) = 0$. Then, by the equation defining $q^*(\emptyset, 1, 1)$ in Section B.2.2, $q^*(\emptyset, 1, 1) = 0$. But then, $\beta(q^*(\emptyset, 1, 1), M, \gamma) = 0$, a contradiction.
- The observation above, along with Assumption 1, implies that $\sigma_1^*(\emptyset, 1, g) = \sigma_1^*(\emptyset, 1, b) = 1$ in any PBE.

So far we have argued that $\sigma_2^*(\emptyset, 0, 1) = \sigma_2^*(\emptyset, 1, 1) = \sigma_1^*(\emptyset, 1, g) = \sigma_1^*(\emptyset, 1, b) = 1$. Substituting these into the equation defining $q^*(\emptyset, 1, 1)$ in Section B.2.2,

$$q^*(\emptyset,1,1) = \frac{q^2\sigma_1^*(\emptyset,0,b)\sigma_2^*(\emptyset,0,1)}{q^2\sigma_1^*(\emptyset,0,b)\sigma_2^*(\emptyset,0,1) + (q^2\sigma_1^*(\emptyset,1,b)\sigma_2^*(\emptyset,1,1) + q(1-q)\sigma_1^*(\emptyset,1,g) + (1-q)q\sigma_2^*(\emptyset,1,1) + (1-q)^2)}$$
$$= \frac{q^2\sigma_1^*(\emptyset,0,b)}{q^2\sigma_1^*(\emptyset,0,b) + q^2 + q(1-q) + (1-q)q + (1-q)^2} \leq \frac{1}{2}$$

Therefore, $\beta(q^*(\emptyset,1,1),M,\gamma)=0$. This, along with Assumption 1 and $\delta_0<1$, implies that $\sigma_1^*(\emptyset,0,b)=1$ in any PBE.

The only part of ruler 1's PBE strategy we have not pinned down so far is $\sigma_1^*(\emptyset,0,g)$. The rest of the analysis considers two separate cases.

- Suppose $\delta_0 < 1-\beta(1,M,\gamma)$. By (5), $\sigma_2^*(\emptyset,0,0)=1$. By (8), $\sigma_1^*(\emptyset,0,g)=1$. This completes the characterization of equilibrium strategies.

  Note that under these strategies, $q^*(\emptyset,1,0)=q^*(\emptyset,0,1)=1$. Therefore, whenever $(\hat{s},s)=(\emptyset,0)$ and $t_1\neq t_2$, there is a mismatch in the actions, and there is a successful revolution with probability $\beta(1,M,\gamma)$.

- Suppose $\delta_0 > 1-\beta(1,M,\gamma)$.

  Our first claim is that $\sigma_2^*(\emptyset,0,0)=0$. To see why, suppose not: $\sigma_2^*(\emptyset,0,0)>0$. Given the strategies pinned down so far and the equation defining $q^*(\emptyset,0,1)$ in Section B.2.2, $q^*(\emptyset,0,1)=1$. But then, by (5), $\sigma_2^*(\emptyset,0,0)=0$, a contradiction. On the other hand, when $\sigma_2^*(\emptyset,0,0)=0$, the history $(\emptyset,0,1)$ is never reached on equilibrium path. The Bayes' rule does not apply to $q^*(\emptyset,0,1)$. Then, any choice of $q^*(\emptyset,0,1)$ high enough so that $1-\beta(q^*(\emptyset,0,1),M,\gamma)\leq\delta_0$ is consistent with $\sigma_2^*(\emptyset,0,0)=0$ as an equilibrium strategy.

  Next, we similarly claim that $\sigma_1^*(\emptyset,0,g)=0$. Suppose not: $\sigma_1^*(\emptyset,0,g)>0$. Given the strategies pinned down so far and the equation defining $q^*(\emptyset,1,0)$ in Section B.2.2, $q^*(\emptyset,1,0)=1$. But then, by (8), $\sigma_1^*(\emptyset,0,g)=0$, a contradiction. On the other hand, when $\sigma_1^*(\emptyset,0,g)=0$, the history $(\emptyset,1,0)$ is never reached on equilibrium path. The Bayes' rule does not apply to $q^*(\emptyset,1,0)$. Then, any choice of $q^*(\emptyset,1,0)$ high enough so that $1-\beta(q^*(\emptyset,1,0),M,\gamma)\leq\delta_0$ is consistent with $\sigma_1^*(\emptyset,0,g)=0$ as an equilibrium strategy.

  We conclude that $\sigma_2^*(\emptyset,0,0)=\sigma_1^*(\emptyset,0,g)=0$ in any PBE. This completes the characterization of equilibrium strategies.

  Note that under these strategies, whenever $(\hat{s},s)=(\emptyset,0)$ and $t_1\neq t_2$, the aggregate action is $A=0$ and there are no revolts.

Our findings imply the following result.

Proposition 6. *Recall that A is the aggregate government action, and $Pr_{(t_1,t_2)}(A)$ be the equilibrium probability of A conditional on rulers' types (t_1,t_2).*

- *When $\beta(1,M,\gamma)>1-\delta_0$, the equilibrium outcomes are identical to those of the model* *in the main text. That is, in equilibrium,*

$$Pr_{(t_1,t_2)}(A = s) = 1, \quad \textit{if } (t_1, t_2) \neq (b, b).$$

Otherwise,

$$Pr_{(b,b)}(A = s|\hat{s} = s) = Pr_{(b,b)}(A = 1|\hat{s} = \emptyset) = 1$$

There are no revolts in equilibrium.

- *When* $\beta(1, M, \gamma) < 1 - \delta_0$,

$$Pr_{(t_1,t_2)}(A = 1) = 1, \quad \textit{if } (t_1, t_2) \neq (g, g).$$

There is a revolt when $\hat{s} = 0$ *and at least one ruler takes action* 1*, and when* $\hat{s} = \emptyset$ *and the rulers' actions do not match each other. These revolts succeed with probability* $\beta(1, M, \gamma)$.

The expected policy payoff for a majority citizen is

$$\begin{cases} 1 - q^2(1-p) - \mu & ; \beta(1, M, \gamma) > 1 - \delta_0 \\ (1-q)^2 + (2q(1-q) + pq^2)\, \beta(1, M, \gamma) - \mu & ; \beta(1, M, \gamma) < 1 - \delta_0. \end{cases}$$

Corollary 1 of the main text is then modified as follows:

Corollary 2. *The value of institutional constraints is:*

$$\begin{cases} (1-p)(q - q^2) - \mu & ; \beta(1, M, \gamma) > 1 - \delta_0 \\ ((2-p)\beta(1, M, \gamma) - 1)\,(q - q^2) - \mu & ; \beta(1, M, \gamma) < 1 - \delta_0. \end{cases}$$

Proposition 4 of the main text is modified as follows.

Proposition 7. *There is threshold* $p^*(M, \gamma, q, \mu)$ *such that a majority citizen's policy payoff is higher without institutional constraints if and only if the scope of the divine law* $p > p^*$*, where*

$$p^*(M, \gamma, q, \mu) = \begin{cases} 1 - \frac{\mu}{q(1-q)} & ; \beta(1, M, \gamma) > 1 - \delta_0 \\ \\ 2 - \frac{1}{\beta(1,M,\gamma)} \left(1 + \frac{\mu}{q(1-q)}\right) & ; \beta(1, M, \gamma) < 1 - \delta_0. \end{cases}$$

Moreover,

1. $p^*(M, \gamma, q, \mu)$ *is increasing in* M *and* γ*; strictly so if and only if* $\beta(1, M, \gamma) < 1 - \delta_0$.
2. *For* $\mu > 0$, $p^*(M, \gamma, q, \mu)$ *has an inverted U-shape in* q*, with*

$$lim_{q \to 0^+} p^*(M, \gamma, q, \mu) = lim_{q \to 1^-} p^*(M, \gamma, q, \mu) = -\infty.$$

As in the model in the main text, a higher scope of the law makes it more likely for a society to adopt institutional constraints. The difference is regarding the comparative statics with respect to M and γ. In this model, a more homogeneous society and a society with higher solidarity is *more* likely to adopt institutional constraints.

Why are the comparative statics going in the opposite direction? In the model with $y(a_1, a_2) = \min\{a_1, a_2\}$, the main advantage of institutional constraints is that a good ruler can *block* the bad ruler: he can just impose $a_j = 0$ on the aggregate action. Therefore, when institutional constraints are imposed, society needs to resort to revolt *less* than it would without institutional constraints. However, in the model with $y(a_1, a_2) = \max\{a_1, a_2\}$, the bad ruler *cannot* block the good ruler: even when the good ruler takes $a_j = 0$, the aggregate action is dictated by the other ruler's choice. In this model, the main advantage of institutional constraints is that the good ruler can *inform* the citizens by taking a different action than the bad ruler. The citizens can learn the state better with institutional constraints, yet, it still needs to revolt against an incongruent policy. In this model, therefore, when institutional constraints are imposed, the society resorts to revolt *more* than it would without institutional constraints. Because higher M and higher γ facilitate revolt, they favor the adoption of institutional constraints.

Proposition 5 of the main text is modified as follows.

Proposition 8. *There is a cost threshold such that the majority citizen's policy payoff is higher without institutional constraints if and only if* $\mu > \mu^*$, *where*

$$\mu^*(\beta, p, q) = \begin{cases} (1-p)(q-q^2) & ; \beta > 1-\delta_0 \\ \\ ((2-p)\beta - 1)(q-q^2) & ; \beta < 1-\delta_0, \end{cases}$$

where $\beta = \beta(1, M, \gamma)$. *Moreover,*

1. μ^* *is strictly decreasing in* p. μ^* *is weakly increasing in* $\beta(1, M, \gamma)$ *(and hence in* M *and* γ*), strictly so when* $\beta < 1 - \delta_0$.
2. *Suppose* $\delta_0 < T/M$, *so that there is sufficient conflict of interest that the threat of revolt does not deter the minority-congruent ruler (*$\beta < 1 - \delta_0$*). Then,*

$$\frac{\partial^2 \mu^*(\beta, p, q)}{\partial p \partial \beta} = -(q - q^2) < 0.$$

As in the model in the main text, higher scope of the law p improves the majority's ability to control the ruler, thereby reducing the marginal value of institutional constraints, and hence the cost threshold below which they are adopted. Recall that societal homogeneity M or solidarity γ improve the majority's ability to revolt. Contrary to the model in the main text, in this model, institutional constraints provide information about incongruent policies, leading the majority towards revolting more. Therefore, societal homogeneity and solidarity increase the marginal value of institutional constraints, and hence they increase the cost threshold below which they are adopted. Indeed, if M and γ are low enough so that

$\beta < 1 - \delta_0$ and $(2-p)\beta < 1$, it follows that $\mu^*(\beta, p, q) < 0$, and institutional constraints are never adopted. That is, in societies where homogeneity and solidarity are extremely low, it is never worth adopting institutional constraints. Intuitively, in this model, institutional constraints provide information to citizens and citizens use this information to revolt against incongruent policies. When the threat of revolt does not discipline the ruler and it is not likely to overturn incongruent policies, such information has no value, and it is not worth bringing in a second ruler for the sole purpose of providing information.

Note, however, that even though the comparative statics with respect to M and γ change, the second part of Proposition 5 remains intact. Recall that μ^* is decreasing in p, and it decreases faster when β is higher. Therefore, this model maintains the idea that homogeneity M and solidarity γ complements the scope of the law p. Intuitively, higher scope of the law is useful insofar as it is accompanied by a revolt. On the other hand, μ^* is increasing in β, and it increases slower when p is higher. Therefore, in this model, the scope of the law p substitutes homogeneity M and solidarity γ. Intuitively, the information provided through institutional constraints is more useful when revolt capabilities are higher. Yet, a higher scope of the law renders this information (and therefore the revolt capability) less useful by providing information regardless of institutions.

Regarding the inertia of institutional constraints, Proposition 6 of the main text is modified as follows. As in Proposition 6 in the main text, we focus on the case where institutional constraints may be adopted or not. This means restricting attention to the $(2-p)\beta > 1$ case; otherwise, institutional constraints are never adopted.

Proposition 9. *Suppose $\gamma \sim U[0,1]$. Let $Q = Pr_\gamma(\mu \leq \mu^*(\gamma))$ be the probability that institutional constraints improve the majority citizen's policy payoff. Suppose $\delta_0 < T/M$, so that there is sufficient conflict of interest that the threat of revolt does not deter the minority-congruent ($\beta < 1 - \delta_0$). Moreover, suppose $(2-p)(1 - \frac{T}{M}) > 1$, so that the institutional constraints are sometimes adopted ($(2-p)\beta > 1$ for high enough γ). Then,*

$$Q(\mu'; M, p) = \begin{cases} 1 - \frac{1+\mu'}{(2-p)(1-T/M)} & ; \mu' \leq (2-p)(1-T/M) - 1 \\ 0 & ; \mu' > (2-p)(1-T/M) - 1, \end{cases}$$

where $\mu' = \mu/(q - q^2)$. Moreover,

1. *Q is decreasing in p and increasing in M; strictly so when $\mu' \leq (2-p)(1-T/M) - 1$.*
2. *$|Q(\mu'_2) - Q(\mu'_1)|$ is strictly increasing in p and strictly decreasing in M for all $\mu'_2 > \mu'_1$, with $\mu'_2 \leq (2-p)(1-T/M) - 1$.*

Proof. Using Proposition 8,

$$\begin{aligned} Q &= Pr_\gamma(\mu \leq \mu^*(\gamma) \mid \beta < 1 - \delta_0) \\ &= Pr_\gamma\left(\mu \leq ((2-p)\beta - 1)(q - q^2)\right) \end{aligned}$$

Using the fact that $\beta = \beta(1, M, \gamma)$, and substituting Proposition 1 in the main text, we have:

$\beta = H\left((1-\frac{T}{M})\gamma\right)$. Because $H = U[0,1]$, $\beta = (1-\frac{T}{M})\gamma$. Substituting, we have:

$$\begin{aligned}
Q &= Pr_\gamma\left(\mu \leq \left((2-p)(1-\frac{T}{M})\gamma - 1\right)(q-q^2)\right) \\
&= Pr_\gamma\left(\mu' \leq \left((2-p)(1-\frac{T}{M})\gamma - 1\right)\right) \\
&= Pr_\gamma\left((2-p)(1-\frac{T}{M})\gamma \geq 1+\mu'\right) \\
&= Pr_\gamma\left(\gamma \geq \frac{1+\mu'}{(2-p)(1-\frac{T}{M})}\right)
\end{aligned}$$

Recall that $\gamma \sim U[0,1]$. Under the restriction $(2-p)(1-\frac{T}{M}) > 1$, $\frac{1}{(2-p)(1-\frac{T}{M})} < 1$, which means Q is strictly positive for $\mu' = 0$. Moreover, as long as $\mu' \leq (2-p)(1-T/M) - 1$, $\frac{1+\mu'}{(2-p)(1-\frac{T}{M})} \leq 1$, which means Q is positive. Indeed, when $\mu' \leq (2-p)(1-T/M) - 1$,

$$Q = Pr_\gamma\left(\gamma \geq \frac{1+\mu'}{(2-p)(1-\frac{T}{M})}\right) = 1 - \frac{1+\mu'}{(2-p)(1-\frac{T}{M})}$$

On the other hand, when $\mu' > (2-p)(1-T/M) - 1$, $\frac{1+\mu'}{(2-p)(1-\frac{T}{M})} > 1$, which means $Q = 0$.

The first part of Proposition 9 is evident from these formulas. Regarding the second part, as $Q(\mu')$ is decreasing in μ', with $\mu'_1 < \mu'_2$:

$$|Q(\mu'_2) - Q(\mu'_1)| = Q(\mu'_1) - Q(\mu'_2)$$

Moreover, since $\mu'_1 < \mu'_2 \leq (2-p)(1-T/M) - 1$, $Q(\mu'_1) = 1 - \frac{1+\mu'_1}{(2-p)(1-\frac{T}{M})}$ and $Q(\mu'_2) = 1 - \frac{1+\mu'_2}{(2-p)(1-\frac{T}{M})}$. Therefore,

$$\begin{aligned}
Q(\mu'_1) - Q(\mu'_2) &= \left(1 - \frac{1+\mu'_1}{(2-p)(1-\frac{T}{M})}\right) - \left(1 - \frac{1+\mu'_2}{(2-p)(1-\frac{T}{M})}\right) \\
&= \frac{\mu'_2 - \mu'_1}{(2-p)(1-\frac{T}{M})}
\end{aligned}$$

which is strictly increasing in p and strictly decreasing in M. □

Proposition 9 provides new insights into the effect of changes in the costs of institutions. For a given μ', societies with sufficiently high solidarity levels adopt institutional constraints. Consider a reduction in the costs of institutional constraints from μ'_2 to μ'_1, e.g., due to peacetime. Then, societies with even lower levels of γ tend to adopt institutional constraints. As part 2 of the Proposition shows, this change tends to be larger when p is larger. This is because the scope of the law substitutes solidarity in this model: when the scope of the law p is larger, the capacity of revolt obtained through γ needs to change a lot for a society to adopt institutional constraints. Therefore, the cutoff of solidarity above which institutional constraints are adopted varies strongly with μ'. Consequently, societies with high scope of law are more responsive to a decrease in μ'.

We conclude our discussion by presenting the analogue of Proposition in the main text. Given that institutional constraints make revolt more likely by providing information, the following result is not surprising.

Proposition 10. *Suppose that $p^* \in (0,1)$ and that $\delta_0 < T/M$, so that there is sufficient conflict of interest and the threat of revolt does not deter the minority-congruent ruler ($\beta < 1 - \delta_0$). Focusing on the scope of the law p as the only source of variation, the equilibrium probabilities of revolt attempts and successful revolt are both higher in societies with institutional constraints. Formally,*

$$\mathbb{E}[\frac{pq}{2} \mid p > p^*] < \mathbb{E}[q(1-q)+\frac{pq^2}{2} \mid p < p^*] \quad \textit{and} \quad \mathbb{E}[\frac{pq\beta}{2} \mid p > p^*] < \mathbb{E}[\left(2q(1-q) + \frac{pq^2}{2}\right)\beta \mid p < p^*],$$

for a given q and $\beta = \beta(1, M, \gamma)$.

C An Extended Model of Institutional Constraints

In this section, we present an extended version of the model with institutional constraints in Section 2.1 of the main text and provide a characterization of the equilibrium. The extended model is different from our main model in two ways. First, we do not require that the rulers observe each others' types. Second, we allow for $\delta_1 > 0$, but we still maintain the assumption that $\delta_1 < \delta_0 < 1$. That is, throughout this section, we will maintain the following assumption.

Assumption 2. $\delta_1 < \delta_0 < 1$, *i.e., the minority-congruent ruler always prefers to propose* $a = 1$, *and his incentives to propose* $a = 0$ *are stronger in state* $s = 0$.

Note that under Assumption 2, the PBE with one ruler discussed in the main text (Proposition 2) applies verbatim. This is because $\delta_1 < \delta_0$ ensures $\sigma(\emptyset, 1) \geq \sigma(\emptyset, 0)$ in any PBE. Then, Bayesian updating implies that following $\hat{s} = \emptyset$ and $a = 0$, the belief that the ruler's action does not match the state satisfies $q'(a) \leq \frac{1}{2}$. As a result, there are no revolts following $\hat{s} = \emptyset$. Throughout the reminder of this section, we analyze the game with two rulers.

Timing The timing of the game is as follows.

1. The nature determines the realizations of rulers' types, the state of the world s, signal $\hat{s}$, the common value of costs $\bar{c}$, and idiosyncratic elements of costs ϵ_i's.
2. Each ruler observes his own type, the state s, and $\hat{s}$. Each citizen i observes $\hat{s}$ and her private cost c_i.
3. Ruler 1 proposes action a_1, which ruler 2 and the citizens observe.
4. Ruler 2 proposes action a_2, which the citizens observe.
5. The aggregate policy is $A = \min\{a_1, a_2\}$. Citizens simultaneously decide whether or not to revolt against the aggregate policy A.
6. Success of revolution r is determined, payoffs are received, and the game ends.

We consider the Perfect Bayesian Nash Equilibrium of this game. The existence of two rulers who do not observe each others' types can generate multiple equilibria, in which case we use *forward induction* criterion of Govindan and Wilson (2009) to select an outcome. This criterion implies the Intuitive Criterion of Cho and Kreps (1987) for simple signaling games with one sender. The formal definition of forward induction criterion is provided below in Section C.2.

C.1 Formal Definition of Equilibrium

The majority-congruent ruler $j \in \{1, 2\}$ (i.e., ruler j of type $t_j = g$) always chooses $a_j = s$ by assumption. The strategy of the minority-congruent ruler 1 (i.e., ruler 1 of type $t_1 = b$) in state s when public signal is $\hat{s}$ is:

$$\sigma_1(\widehat{s}, s) \equiv \Pr(a_1 = 1 | s, \hat{s}) \quad \in [0, 1]$$

The strategy of minority-congruent ruler 2 (i.e., ruler 2 of type $t_2 = b$) in state s, given public signal $\hat{s}$ and ruler 1's action a_1 is:

$$\sigma_2(\widehat{s}, s, a_1) \equiv \Pr(a_2 = 1 | s, \hat{s}, a_1) \quad \in [0, 1]$$

The posterior beliefs of citizens that the aggregate policy is incongruent, given information $(\hat{s}, a_1, a_2)$, is denoted by:

$$q(\hat{s}, a_1, a_2) \equiv \Pr\left(\min\{a_1, a_2\} \neq s | \hat{s}, a_1, a_2\right) \quad \in [0, 1]$$

Let $r_i \in \{0, 1\}$ denote the revolting decision of citizen i, with $r_i = 1$ corresponding to revolting. The strategy of a majority citizen i when posterior beliefs are q' and the cost of revolt is c_i is denoted by:

$$\varphi(q', c_i) \equiv \Pr(r_i = 1 | q', c_i) \quad \in [0, 1]$$

As we will see later, in this version of the model, the minority citizens sometimes participate in revolt against $A = 0$ when they believe a sufficient number of majority citizens participate as well. The strategy of a minority citizen i when the aggregate action is $A = 0$, the posterior beliefs are q', and the cost of revolt is c_i, is denoted by:

$$\phi(q', c_i) \equiv \Pr(r_i = 1 | q', c_i) \quad \in [0, 1]$$

The Perfect Bayesian Nash Equilibrium of the game is a tuple $(\sigma_1^*, \sigma_2^*, \varphi^*, \phi^*, q^*)$ such that the following are satisfied.

1. $\varphi^*(q', c_i)$ maximizes the payoff of the citizens in majority for any $q' = q^*(\hat{s}, a_1, a_2)$.
2. $\phi^*(q', c_i)$ maximizes the payoff of the citizens in minority for any $q' = q^*(\hat{s}, a_1, a_2)$ when $A = 0$.
3. $q^*(\hat{s}, a_1, a_2)$ is given by Bayes' Rule.
4. Given φ^*, ϕ^* and σ_2^*, σ_1^* maximizes the payoff of the minority-congruent ruler 1. Similarly, given φ^*, ϕ^* and σ_1^*, σ_2^* maximizes the payoff of the minority-congruent ruler 2.

C.2 Forward Induction

The following definitions are adapted from Govindan and Wilson (2009).

A **terminal node** of the game with two rulers is:

$$(s, \hat{s}, a_1, a_2, r) \qquad \in \{(0, 0), (1, 1), (0, \emptyset), (1, \emptyset)\} \times \{0, 1\} \times \{0, 1\} \times \{0, 1\}$$

As we will demonstrate later, any subgame following $\hat{s} = s$ has a unique PBE. While refining the equilibrium, we will focus on the subgame following $\hat{s} = \emptyset$.

We begin by a formal definition of an outcome.

Definition 1. *The* **outcome** *of a Perfect Bayesian Nash Equilibrium is the induced probability distribution over the terminal nodes.*

Definition 2. *Consider an outcome. A pure strategy of a player is* **relevant** *for that outcome if:*

1. *There is a Perfect Bayesian Nash Equilibrium with that outcome, and,*
2. *The pure strategy is optimal under the beliefs in the said equilibrium.*

In words, given an outcome, the relevant strategies are reasonable deviations that a player may consider.

Definition 3. *Consider an outcome. An information set is* **relevant** *for that outcome if it is reached with strictly positive probability by some relevant strategy for that outcome.*

In words, relevant information sets are those that can be reached via reasonable deviations by some players.

Definition 4. *An outcome satisfies* **forward induction** *if it results from a Perfect Bayesian Nash Equilibrium in which at every information set that is relevant for that outcome the support of the beliefs are confined to profiles of Nature's strategies and other players' strategies that are relevant for that outcome.*

In words, an outcome satisfies forward induction if, in any relevant information set, the players believe that information set is reached via a reasonable deviation.

C.3 Equilibrium Characterization

C.3.1 Citizens' Actions

Suppose $A = 1$. In this case, the minority citizens never participate in the revolt, and the measure of citizens who may contemplate a revolt is M. As discussed in Proposition 1 of the main text, in a symmetric cutoff strategy equilibrium as $\rho \to 0$, a successful revolution occurs with probability:

$$\beta(q', M, \gamma) = H\left((1 - \frac{T}{M}) \cdot \gamma \cdot (2q' - 1)\right)$$

In contrast, when $A = 0$, the minority citizens always prefer to participate in the revolt, if they believe a sufficient number of majority citizens also revolt. In this case, the measure of citizens who may contemplate a revolt is 1. In any equilibrium, let the probability of a successful revolt be given by:

$$\bar{\beta}(q') \qquad \in [0, 1]$$

For our purposes, a closed-form equation characterizing $\bar{\beta}(q')$ is unnecessary. This is because we will show that in PBE that survives forward induction, there are no revolts against

$A = 0$.[2] However, we will maintain the assumption that $\bar{\beta}(q')$ is continuous in q', and:

$$\bar{\beta}(q') = 0 \qquad \text{for any } q' \leq \frac{1}{2} \tag{12}$$

Equation (12) holds because, in any equilibrium, majority citizens do not participate in a revolt against $A = 0$ when $q' \leq \frac{1}{2}$. Foreseeing this, minority members do not participate either, and hence there are no revolts.

C.3.2 Beliefs Following Proposed Policy

When the issue is predordained ($\hat{s} \in \{0, 1\}$), $q^*(\hat{s}, a_1, a_2) = |\hat{s} - \min\{a_1, a_2\}| \in \{0, 1\}$.

When the issue is non-preordained ($\hat{s} = \emptyset$), the posterior beliefs are given by:

$$\begin{aligned}
q^*(\emptyset,0,0) &\equiv \Pr(\min\{a_1,a_2\} \neq s | a_1 = a_2 = 0, \hat{s} = \emptyset) \\
&= \Pr(s = 1 | a_1 = a_2 = 0, \hat{s} = \emptyset) \\
&= \frac{\Pr(s = 1, a_1 = a_2 = 0, \hat{s} = \emptyset)}{\Pr(s = 1, a_1 = a_2 = 0, \hat{s} = \emptyset) + \Pr(s = 0, a_1 = a_2 = 0, \hat{s} = \emptyset)} \\
&= \frac{\frac{1}{2}q^2(1-\sigma_1^*(\emptyset,1))(1-\sigma_2^*(\emptyset,1,0))}{\frac{1}{2}q^2(1-\sigma_1^*(\emptyset,1))(1-\sigma_2^*(\emptyset,1,0)) + \frac{1}{2}\left(q^2(1-\sigma_1^*(\emptyset,0))(1-\sigma_2^*(\emptyset,0,0)) + q(1-q)(1-\sigma_1^*(\emptyset,0)) + (1-q)q(1-\sigma_2^*(\emptyset,0,0)) + (1-q)^2\right)} \\
&= \frac{q^2(1-\sigma_1^*(\emptyset,1))(1-\sigma_2^*(\emptyset,1,0))}{q^2(1-\sigma_1^*(\emptyset,1))(1-\sigma_2^*(\emptyset,1,0)) + \left(q^2(1-\sigma_1^*(\emptyset,0))(1-\sigma_2^*(\emptyset,0,0)) + q(1-q)(1-\sigma_1^*(\emptyset,0)) + (1-q)q(1-\sigma_2^*(\emptyset,0,0)) + (1-q)^2\right)}
\end{aligned}$$

$$\begin{aligned}
q^*(\emptyset,0,1) &\equiv \Pr(\min\{a_1,a_2\} \neq s | a_1 = 0, a_2 = 1, \hat{s} = \emptyset) \\
&= \Pr(s = 1 | a_1 = 0, a_2 = 1, \hat{s} = \emptyset) \\
&= \frac{\Pr(s = 1, a_1 = 0, a_2 = 1, \hat{s} = \emptyset)}{\Pr(s = 1, a_1 = 0, a_2 = 1, \hat{s} = \emptyset) + \Pr(s = 0, a_1 = 0, a_2 = 1, \hat{s} = \emptyset)} \\
&= \frac{\frac{1}{2}q^2(1-\sigma_1^*(\emptyset,1))\sigma_2^*(\emptyset,1,0) + q(1-q)(1-\sigma_1^*(\emptyset,1))}{\frac{1}{2}q^2(1-\sigma_1^*(\emptyset,1))\sigma_2^*(\emptyset,1,0) + q(1-q)(1-\sigma_1^*(\emptyset,1)) + \frac{1}{2}\left(q^2(1-\sigma_1^*(\emptyset,0))\sigma_2^*(\emptyset,0,0) + (1-q)q\sigma_2^*(\emptyset,0,0)\right)}
\end{aligned}$$

$$\begin{aligned}
q^*(\emptyset,1,0) &\equiv \Pr(\min\{a_1,a_2\} \neq s | a_1 = 1, a_2 = 0, \hat{s} = \emptyset) \\
&= \Pr(s = 1 | a_1 = 1, a_2 = 0, \hat{s} = \emptyset) \\
&= \frac{\Pr(s = 1, a_1 = 1, a_2 = 0, \hat{s} = \emptyset)}{\Pr(s = 1, a_1 = 1, a_2 = 0, \hat{s} = \emptyset) + \Pr(s = 0, a_1 = 1, a_2 = 0, \hat{s} = \emptyset)} \\
&= \frac{\frac{1}{2}q^2\sigma_1^*(\emptyset,1)(1-\sigma_2^*(\emptyset,1,1)) + (1-q)q(1-\sigma_2^*(\emptyset,1,1))}{\frac{1}{2}q^2\sigma_1^*(\emptyset,1)(1-\sigma_2^*(\emptyset,1,1)) + q(1-q)(1-\sigma_2^*(\emptyset,1,1)) + \frac{1}{2}\left(q^2\sigma_1^*(\emptyset,0)(1-\sigma_2^*(\emptyset,0,1)) + q(1-q)\sigma_1^*(\emptyset,0)\right)}
\end{aligned}$$

[2] It is, however, possible to characterize the value of $\bar{\beta}(q')$. In any symmetric cutoff strategy equilibrium as $\rho \to 0$, the minority citizens use a cutoff c^m such that $r_i = 1$ if and only if $c_i \leq c^m$. Similarly, majority citizens use a cutoff c^M such that $r_i = 1$ if and only if $c_i \leq c^M$. The revolution is successful as long as $\bar{c} \leq \bar{c}^*$ for some c^*. The three cutoff values, c^m, c^M and $\bar{c}^*$ satisfy:

$$\begin{aligned}
\gamma \cdot \Pr\left(\bar{c} \leq \bar{c}^* | c_i = c^m\right) &= c^m \\
\gamma \cdot (2q'-1) \cdot \Pr\left(\bar{c} \leq \bar{c}^* | c_i = c^M\right) &= c^M \\
(1-M) \cdot \Pr(c_i \leq c^m | \bar{c} = \bar{c}^*) + M \cdot \Pr(c_i \leq c^M | \bar{c} = \bar{c}^*) &= T
\end{aligned}$$

The probability of a successful revolt is $\bar{\beta}(q') = H(\bar{c}^*)$.

$$
\begin{aligned}
q^*(\emptyset,1,1) &\equiv \Pr(\min\{a_1,a_2\}\neq s|a_1=a_2=1,\hat{s}=\emptyset)\\
&= \Pr(s=0|a_1=a_2=1,\hat{s}=\emptyset)\\
&= \frac{\Pr(s=0,a_1=a_2=1,\hat{s}=\emptyset)}{\Pr(s=0,a_1=a_2=1,\hat{s}=\emptyset)+\Pr(s=1,a_1=a_2=1,\hat{s}=\emptyset)}\\
&= \frac{\frac{1}{2}q^2\sigma_1^*(\emptyset,0)\sigma_2^*(\emptyset,0,1)}{\frac{1}{2}q^2\sigma_1^*(\emptyset,0)\sigma_2^*(\emptyset,0,1)+\frac{1}{2}\left(q^2\sigma_1^*(\emptyset,1)\sigma_2^*(\emptyset,1,1)+q(1-q)\sigma_1^*(\emptyset,1)+(1-q)q\sigma_2^*(\emptyset,1,1)+(1-q)^2\right)}
\end{aligned}
$$

C.3.3 Rulers' Actions

When the Issue is Preordained We proceed in the fashion of backward induction, first pinning down the strategies of minority-congruent ruler 2 at every history.

1. Consider the case $\hat{s}=s=a_1=0$. In this case, $\min\{a_1,a_2\}=0$ regardless of a_2, and $q^*(0,0,a_2)=0$ for any $a_2\in\{0,1\}$. We conclude that any $\sigma_2^*(0,0,0)\in[0,1]$ can be a part of a PBE.

   Note that because $q^*(0,0,a_2)=0$, the majority citizens never participate in revolt, and consequently, there are no revolts. Therefore, the payoff of minority-congruent ruler 2 is δ_0 in any PBE.

2. Now, consider the case $\hat{s}=s=0$ and $a_1=1$. In this case, $\min\{a_1,a_2\}=a_2$ and $q^*(0,1,a_2)=a_2$ for any $a_2\in\{0,1\}$. When $a_2=0$, majority citizens do not participate in the revolt and there are no revolts. When $a_2=1$, only majority citizens participate in the revolt, which is successful with probability $\beta(1,M,\gamma)$. Thus, ruler 2's optimal strategy when $(\hat{s},s,a_1)=(0,0,1)$ is:

   $$\sigma_2^*(0,0,1)\in\arg\max_{\sigma\in[0,1]}\sigma\cdot(1-\beta(1,M,\gamma))+(1-\sigma)\cdot\delta_0$$

   Therefore, ruler 2's PBE strategy is:

   $$\sigma_2^*(0,0,1)=\begin{cases}0 & ;\delta_0>1-\beta(1,M,\gamma)\\ 1 & ;\delta_0<1-\beta(1,M,\gamma)\end{cases}$$

3. Now, consider the case $\hat{s}=s=1$ and $a_1=0$. In this case, $\min\{a_1,a_2\}=0$ regardless of a_2, and $q^*(1,0,a_2)=1$ for any $a_2\in\{0,1\}$. We conclude that any $\sigma_2^*(1,1,0)\in[0,1]$ can be a part of a PBE.

   Note that because $q^*(1,0,a_2)=1$, majority citizens participate in a revolt against $A=0$. Foreseeing this, minority citizens also participate. Therefore, all citizens contemplate participating in a revolt, which is successful with probability $\bar{\beta}(1)$. The payoff of minority-congruent ruler 2 is $\delta_0\cdot\left(1-\bar{\beta}(1)\right)$ in any PBE.

4. Finally, consider the case $\hat{s}=s=a_1=1$. In this case, $\min\{a_1,a_2\}=a_2$ and $q^*(1,1,a_2)=1-a_2$ for any $a_2\in\{0,1\}$. When $a_2=0$, all citizens contemplate participating in a revolt, which is successful with probability $\bar{\beta}(1)$. When $a_2=1$, none of the citizens revolt. Thus, ruler 2's optimal strategy when $(\hat{s},s,a_1)=(1,1,1)$ is:

   $$\sigma_2^*(1,1,1)\in\arg\max_{\sigma\in[0,1]}\sigma+(1-\sigma)\cdot\delta_0\cdot\left(1-\bar{\beta}(1)\right)$$

   Given Assumption 2, we conclude that $\sigma_2^*(1,1,1)=1$.

Next, we pin down the strategy of minority-congruent ruler 1 in every history.

1. Consider the case $\hat{s} = s = 0$. If ruler 1 chooses a_1, the probability that ruler 2 chooses $a_2 = 0$ is:

$$(1-q) + q \cdot (1 - \sigma_2^*(0,0,a_1))$$

and the probability that ruler 2 chooses $a_2 = 1$ is:

$$q \cdot \sigma_2^*(0,0,a_1)$$

Therefore, ruler 1's optimal strategy when $\hat{s} = s = 0$ is:

$$\begin{aligned}\sigma_1^*(0,0) \in \arg\max_{\sigma \in [0,1]} \sigma \cdot (((1-q) + q \cdot (1 - \sigma_2^*(0,0,1))) \cdot \delta_0 + q \cdot \sigma_2^*(0,0,1) \cdot (1 - \beta(1,M,\gamma))) \\ + (1-\sigma) \cdot \delta_0\end{aligned}$$

   - If $1 - \beta(1,M,\gamma) > \delta_0$, $\sigma_2^*(0,0,1) = 1$ and thus ruler 1's optimal strategy is:

   $$\begin{aligned}\sigma_1^*(0,0) \in \arg\max_{\sigma \in [0,1]} \sigma \cdot ((1-q) \cdot \delta_0 + q \cdot (1 - \beta(1,M,\gamma))) \\ + (1-\sigma) \cdot \delta_0\end{aligned}$$

   which is maximized when $\sigma_1^*(0,0) = 1$.

   - If $1 - \beta(1,M,\gamma) < \delta_0$, $\sigma_2^*(0,0,1) = 0$ and $\min\{a_1, a_2\} = 0$ for any $a_1 \in \{0,1\}$ in any PBE. We conclude that any $\sigma_1^*(0,0) \in [0,1]$ can be a part of a PBE.

   Note that majority citizens never participate in a revolt, and there are no revolts. Therefore, the payoff of minority-congruent ruler 1 is δ_0 in any PBE.

2. Now, consider the case $\hat{s} = s = 1$. If ruler 1 chooses a_1, the probability that ruler 2 chooses $a_2 = 0$ is:

$$q \cdot (1 - \sigma_2^*(1,1,a_1))$$

and the probability that ruler 2 chooses $a_2 = 1$ is:

$$(1-q) + q \cdot \sigma_2^*(1,1,a_1)$$

Thus, ruler 1's optimal strategy when $\hat{s} = s = 1$ is:

$$\begin{aligned}\sigma_1^*(1,1) \in \arg\max_{\sigma \in [0,1]} \sigma \cdot \big(q \cdot (1 - \sigma_2^*(1,1,1)) \cdot (1 - \bar{\beta}(1)) \cdot \delta_1 + (1-q) + q \cdot \sigma_2^*(1,1,1)\big) \\ + (1-\sigma) \cdot (1 - \bar{\beta}(1)) \cdot \delta_1\end{aligned}$$

But recall that $\sigma_2^*(1,1,1) = 1$. Thus, ruler 1's choice simplifies to:

$$\sigma_1^*(1,1) \in \arg\max_{\sigma \in [0,1]} \sigma \cdot 1 + (1-\sigma) \cdot \delta_1 \cdot (1 - \bar{\beta}(1))$$

Given Assumption 2, we conclude that $\sigma_1^*(1,1) = 1$.

Note that $\sigma_1^*(1,1) \cdot \sigma_2^*(1,1,1) = 1$ in any PBE. That is, when $\hat{s} = s = 1$, the aggregate policy is $A = 1$ with probability one and there are no revolts.

If $1 - \beta(1, M, \gamma) > \delta_0$, $\sigma_1^*(0,0) \cdot \sigma_2^*(0,0,1) = 1$. That is, when $\hat{s} = s = 0$, the aggregate policy taken by two minority-congruent rulers is $A = 1$ with probability one. This is followed with a revolt with probability $\beta(1, M, \gamma)$.

If $1 - \beta(1, M, \gamma) < \delta_0$, $\sigma_1^*(0,0) \cdot \sigma_2^*(0,0,1) = 0$. That is, when $\hat{s} = s = 0$, the aggregate policy is $A = 0$ with probability one and there are no revolts.

When the Issue is Non-Preordained The equilibrium strategy of minority-congruent ruler 2 in any history is characterized by the following equations.

$$\sigma_2^*(\emptyset, 0, 0) \in \arg\max_{\sigma \in [0,1]} \sigma \cdot \delta_0 \cdot (1 - \bar{\beta}(q^*(\emptyset, 0, 1))) + (1 - \sigma) \cdot \delta_0 \cdot (1 - \bar{\beta}(q^*(\emptyset, 0, 0))) \tag{13}$$

$$\sigma_2^*(\emptyset, 0, 1) \in \arg\max_{\sigma \in [0,1]} \sigma \cdot (1 - \beta(q^*(\emptyset, 1, 1), M, \gamma)) + (1 - \sigma) \cdot \delta_0 \cdot (1 - \bar{\beta}(q^*(\emptyset, 1, 0))) \tag{14}$$

$$\sigma_2^*(\emptyset, 1, 0) \in \arg\max_{\sigma \in [0,1]} \sigma \cdot \delta_1 \cdot (1 - \bar{\beta}(q^*(\emptyset, 0, 1))) + (1 - \sigma) \cdot \delta_1 \cdot (1 - \bar{\beta}(q^*(\emptyset, 0, 0))) \tag{15}$$

$$\sigma_2^*(\emptyset, 1, 1) \in \arg\max_{\sigma \in [0,1]} \sigma \cdot (1 - \beta(q^*(\emptyset, 1, 1), M, \gamma)) + (1 - \sigma) \cdot \delta_1 \cdot (1 - \bar{\beta}(q^*(\emptyset, 1, 0))) \tag{16}$$

For the equilibrium strategy of minority-congruent ruler 1, consider two possible histories.

1. Consider the case when $\hat{s} = \emptyset$ and $s = 0$. If ruler 1 chooses a_1, the probability that ruler 2 chooses $a_2 = 0$ is:

   $$(1 - q) + q \cdot (1 - \sigma_2^*(\emptyset, 0, a_1))$$

   and the probability that ruler 2 chooses $a_2 = 1$ is:

   $$q \cdot \sigma_2^*(\emptyset, 0, a_1)$$

   Therefore, minority-congruent ruler 1's policy when $(\hat{s}, s) = (\emptyset, 0)$ is:

   $$\begin{aligned}\sigma_1^*(\emptyset, 0) \in \arg\max_{\sigma \in [0,1]} & \sigma \cdot ((1 - q) + q \cdot (1 - \sigma_2^*(\emptyset, 0, 1))) \cdot \delta_0 \cdot (1 - \bar{\beta}(q^*(\emptyset, 1, 0))) \\ & + \sigma \cdot q \cdot \sigma_2^*(\emptyset, 0, 1) \cdot (1 - \beta(q^*(\emptyset, 1, 1), M, \gamma)) \\ & + (1 - \sigma) \cdot ((1 - q) + q \cdot (1 - \sigma_2^*(\emptyset, 0, 0))) \cdot \delta_0 \cdot (1 - \bar{\beta}(q^*(\emptyset, 0, 0))) \\ & + (1 - \sigma) \cdot q \cdot \sigma_2^*(\emptyset, 0, 0) \cdot \delta_0 \cdot (1 - \bar{\beta}(q^*(\emptyset, 0, 1)))\end{aligned} \tag{17}$$

2. Consider the case when $\hat{s} = \emptyset$ and $s = 1$. If ruler 1 chooses a_1, the probability that ruler 2 chooses $a_2 = 0$ is:

   $$q \cdot (1 - \sigma_2^*(\emptyset, 1, a_1))$$

   and the probability that ruler 2 chooses $a_2 = 1$ is:

   $$(1 - q) + q \cdot \sigma_2^*(\emptyset, 1, a_1)$$

Therefore, minority-congruent ruler 1's optimal strategy when $(\hat{s}, s) = (\emptyset, 1)$ is:

$$\begin{aligned}\sigma_1^*(\emptyset,1) \in \arg\max_{\sigma\in[0,1]} \sigma \cdot q \cdot (1-\sigma_2^*(\emptyset,1,1)) \cdot \delta_1 \cdot (1-\bar{\beta}(q^*(\emptyset,1,0))) \\ + \sigma \cdot ((1-q) + q \cdot \sigma_2^*(\emptyset,1,1)) \cdot (1-\beta(q^*(\emptyset,1,1),M,\gamma)) \\ + (1-\sigma) \cdot q \cdot (1-\sigma_2^*(\emptyset,1,0)) \cdot \delta_1 \cdot (1-\bar{\beta}(q^*(\emptyset,0,0))) \\ + (1-\sigma) \cdot ((1-q) + q \cdot \sigma_2^*(\emptyset,1,0)) \cdot \delta_1 \cdot (1-\bar{\beta}(q^*(\emptyset,0,1)))\end{aligned}$$

The analysis proceeds in a number of claims.

Claim 1. *In any PBE of the game with two rulers, $\sigma_2^*(\emptyset,1,1) = 1$.*

Proof. Suppose, towards a contradiction, that $\sigma_2^*(\emptyset,1,1) < 1$. By Equation (16), this implies:

$$1-\beta(q^*(\emptyset,1,1),M,\gamma) \leq \delta_1 \cdot (1-\bar{\beta}(q^*(\emptyset,1,0)))$$

By Assumption 2, then,

$$1-\beta(q^*(\emptyset,1,1),M,\gamma) < \delta_0 \cdot (1-\bar{\beta}(q^*(\emptyset,1,0)))$$

which, by Equation (14), implies: $\sigma_2^*(\emptyset,0,1) = 0$.

By equations in Section C.3.2, this implies: $q^*(\emptyset,1,1) = 0$. But then, $\beta(q^*(\emptyset,1,1),M,\gamma) = 0$. By Assumption 2, then:

$$1-\beta(q^*(\emptyset,1,1),M,\gamma) > \delta_1 \cdot (1-\bar{\beta}(q^*(\emptyset,1,0)))$$

and therefore $\sigma_2^*(\emptyset,1,1) = 1$, a contradiction. □

Given Claim 1, the beliefs following $(\hat{s}, a_1, a_2) = (\emptyset, 1, 1)$ in any PBE is:

$$q^*(\emptyset,1,1) = \frac{q^2\sigma_1^*(\emptyset,0)\sigma_2^*(\emptyset,0,1)}{q^2\sigma_1^*(\emptyset,0)\sigma_2^*(\emptyset,0,1) + q^2\sigma_1^*(\emptyset,1) + q(1-q)\sigma_1^*(\emptyset,1) + (1-q)q + (1-q)^2} \tag{18}$$

and minority-congruent ruler 1's optimal strategy when $(\hat{s}, s) = (\emptyset, 1)$ is:

$$\begin{aligned}\sigma_1^*(\emptyset,1) \in \arg\max_{\sigma\in[0,1]} \sigma \cdot (1-\beta(q^*(\emptyset,1,1),M,\gamma)) \\ + (1-\sigma) \cdot q \cdot (1-\sigma_2^*(\emptyset,1,0)) \cdot \delta_1 \cdot (1-\bar{\beta}(q^*(\emptyset,0,0))) \\ + (1-\sigma) \cdot ((1-q) + q \cdot \sigma_2^*(\emptyset,1,0)) \cdot \delta_1 \cdot (1-\bar{\beta}(q^*(\emptyset,0,1)))\end{aligned} \tag{19}$$

Claim 2. *In any PBE of the game with two rulers, $\sigma_1^*(\emptyset,1) = 1$.*

Proof. Suppose, towards a contradiction, that $\sigma_1^*(\emptyset,1) < 1$. Then, by Equation (19),

$$\begin{aligned}1-\beta(q^*(\emptyset,1,1),M,\gamma) \leq & \delta_1 \cdot q \cdot (1-\sigma_2^*(\emptyset,1,0)) \cdot (1-\bar{\beta}(q^*(\emptyset,0,0))) \\ & + \delta_1 \cdot ((1-q) + q \cdot \sigma_2^*(\emptyset,1,0)) \cdot (1-\bar{\beta}(q^*(\emptyset,0,1)))\end{aligned} \tag{20}$$

Since the right hand-side of this inequality at most δ_1, and since $\delta_1 < 1$ by Assumption 2, we must have: $\beta(q^*(\emptyset, 1, 1), M, \gamma) > 0$. This means $q^*(\emptyset, 1, 1) > \frac{1}{2}$. By Equation (18), a necessary condition for this is:

$$\sigma_1^*(\emptyset, 0) \cdot \sigma_2^*(\emptyset, 0, 1) > \sigma_1^*(\emptyset, 1) \tag{21}$$

In particular, this requires $\sigma_1^*(\emptyset, 0) > 0$ and $\sigma_2^*(\emptyset, 0, 1) > 0$. We will investigate the implications of these observations separately.

- By Equation (17), $\sigma_1^*(\emptyset, 0) > 0$ implies:

$$\begin{aligned}&\delta_0 \cdot ((1-q) + q \cdot (1 - \sigma_2^*(\emptyset, 0, 1))) \cdot (1 - \bar{\beta}(q^*(\emptyset, 1, 0))) + q \cdot \sigma_2^*(\emptyset, 0, 1) \cdot (1 - \beta(q^*(\emptyset, 1, 1), M, \gamma)) \\ &\quad \geq \delta_0 \cdot ((1-q) + q \cdot (1 - \sigma_2^*(\emptyset, 0, 0))) \cdot (1 - \bar{\beta}(q^*(\emptyset, 0, 0))) + \delta_0 \cdot q \cdot \sigma_2^*(\emptyset, 0, 0) \cdot (1 - \bar{\beta}(q^*(\emptyset, 0, 1)))\end{aligned} \tag{22}$$

- By Equation (14), $\sigma_2^*(\emptyset, 0, 1) > 0$ implies:

$$1 - \beta(q^*(\emptyset, 1, 1), M, \gamma) \geq \delta_0 \cdot (1 - \bar{\beta}(q^*(\emptyset, 1, 0))) \tag{23}$$

By (23), the left-hand side of Equation (22) is bounded above by $1 - \beta(q^*(\emptyset, 1, 1), M, \gamma)$. By (20), this is further bounded above by $\delta_1 \cdot q \cdot (1 - \sigma_2^*(\emptyset, 1, 0)) \cdot (1 - \bar{\beta}(q^*(\emptyset, 0, 0))) + \delta_1 \cdot ((1-q) + q \cdot \sigma_2^*(\emptyset, 1, 0)) \cdot (1 - \bar{\beta}(q^*(\emptyset, 0, 1)))$. Therefore, the following inequality must hold:

$$\begin{aligned}&\delta_1 \cdot q \cdot (1 - \sigma_2^*(\emptyset, 1, 0)) \cdot (1 - \bar{\beta}(q^*(\emptyset, 0, 0))) + \delta_1 \cdot ((1-q) + q \cdot \sigma_2^*(\emptyset, 1, 0)) \cdot (1 - \bar{\beta}(q^*(\emptyset, 0, 1))) \\ &\quad \geq \delta_0 \cdot ((1-q) + q \cdot (1 - \sigma_2^*(\emptyset, 0, 0)) \cdot (1 - \bar{\beta}(q^*(\emptyset, 0, 0))) + \delta_0 \cdot q \cdot \sigma_2^*(\emptyset, 0, 0) \cdot (1 - \bar{\beta}(q^*(\emptyset, 0, 1)))\end{aligned} \tag{24}$$

Recall, by Assumption 2, that $\delta_0 > \delta_1$. Therefore, inequality (24) cannot hold when $1 - \beta(q^*(\emptyset, 0, 0), 1, \gamma) = 1 - \beta(q^*(\emptyset, 0, 1), 1, \gamma)$. We conclude that $1 - \beta(q^*(\emptyset, 0, 0), 1, \gamma) \neq 1 - \beta(q^*(\emptyset, 0, 1), 1, \gamma)$. There are two mutually exhaustive possibilities.

- Suppose $1 - \bar{\beta}(q^*(\emptyset, 0, 0)) > 1 - \bar{\beta}(q^*(\emptyset, 0, 1))$. Then, by Equation (13), $\sigma_2^*(\emptyset, 0, 0) = 0$. Moreover, by Equation (15), $\sigma_2^*(\emptyset, 1, 0) = 0$. Substituting these into (24):

$$\begin{aligned}&\delta_1 \cdot q \cdot (1 - \bar{\beta}(q^*(\emptyset, 0, 0))) + \delta_1 \cdot (1-q) \cdot (1 - \bar{\beta}(q^*(\emptyset, 0, 1))) \\ &\quad \geq \delta_0 \cdot (1 - \bar{\beta}(q^*(\emptyset, 0, 0)))\end{aligned}$$

  But recall that, by Assumption 2, $\delta_0 > \delta_1$. For the above inequality to hold, then, one must have $1 - \bar{\beta}(q^*(\emptyset, 0, 1)) > 1 - \bar{\beta}(q^*(\emptyset, 0, 0))$. This is a contradiction to the case we consider.

- Suppose $1 - \bar{\beta}(q^*(\emptyset, 0, 0)) < 1 - \bar{\beta}(q^*(\emptyset, 0, 1))$. Then, by Equation (15), $\sigma_2^*(\emptyset, 1, 0) = 1$. By equations in Appendix C.3.2, this implies $q^*(\emptyset, 0, 0) = 0$. But then, by Equation (12), $\bar{\beta}(q^*(\emptyset, 0, 0)) = 0$ and $1 - \bar{\beta}(q^*(\emptyset, 0, 0)) \geq 1 - \bar{\beta}(q^*(\emptyset, 0, 1))$, a contradiction to the case we consider.

In any case, we obtain a contradiction, and the result follows. □

Using Claim 2 to substitute $\sigma_1^*(\emptyset, 1) = 1$ into Equation (18) gives that, in any PBE:

$$\begin{aligned} q^*(\emptyset, 1, 1) &= \frac{q^2 \sigma_1^*(\emptyset, 0) \sigma_2^*(\emptyset, 0, 1)}{q^2 \sigma_1^*(\emptyset, 0) \sigma_2^*(\emptyset, 0, 1) + q^2 + q(1-q) + (1-q)q + (1-q)^2} \\ &= \frac{q^2 \sigma_1^*(\emptyset, 0) \sigma_2^*(\emptyset, 0, 1)}{q^2 \sigma_1^*(\emptyset, 0) \sigma_2^*(\emptyset, 0, 1) + 1} \leq \frac{1}{2} \end{aligned}$$

Then, $\beta(q^*(\emptyset, 1, 1), M, \gamma) = 0$. Because $\delta_0 < 1$ by Assumption 2, Equation (14) implies that $\sigma_2^*(\emptyset, 0, 1) = 1$ in any PBE.

Given these observations, Equation (17) simplifies to:

$$\begin{aligned} \sigma_1^*(\emptyset, 0) \in \arg\max_{\sigma \in [0,1]} & \sigma \cdot \delta_0 \cdot (1-q) \cdot (1 - \bar{\beta}(q^*(\emptyset, 1, 0))) \\ & + \sigma \cdot q \\ & + (1-\sigma) \cdot \delta_0 \cdot ((1-q) + q \cdot (1 - \sigma_2^*(\emptyset, 0, 0))) \cdot (1 - \bar{\beta}(q^*(\emptyset, 0, 0))) \\ & + (1-\sigma) \cdot \delta_0 \cdot q \cdot \sigma_2^*(\emptyset, 0, 0) \cdot (1 - \bar{\beta}(q^*(\emptyset, 0, 1))) \end{aligned} \tag{25}$$

Meanwhile, using Claim 2 to substitute $\sigma_1^*(\emptyset, 1) = 1$ into the equation defining $q^*(\emptyset, 0, 0)$ in Section C.3.2 gives that, in any PBE:

$$q^*(\emptyset, 0, 0) = 0$$

Then, by Equation (12), $\bar{\beta}(q^*(\emptyset, 0, 0)) = 0$ in any PBE. Equation (13) simplifies to:

$$\sigma_2^*(\emptyset, 0, 0) \in \arg\max_{\sigma \in [0,1]} \sigma \cdot \delta_0 \cdot (1 - \bar{\beta}(q^*(\emptyset, 0, 1))) + (1-\sigma) \cdot \delta_0$$

This implies:

$$\sigma_2^*(\emptyset, 0, 0) \cdot \delta_0 \cdot (1 - \bar{\beta}(q^*(\emptyset, 0, 1))) + (1 - \sigma_2^*(\emptyset, 0, 0)) \cdot \delta_0 = \delta_0$$

Substituting this into (25), it further simplifies to:

$$\begin{aligned} \sigma_1^*(\emptyset, 0) \in \arg\max_{\sigma \in [0,1]} & \sigma \cdot \delta_0 \cdot (1-q) \cdot (1 - \bar{\beta}(q^*(\emptyset, 1, 0))) \\ & + \sigma \cdot q \\ & + (1-\sigma) \cdot \delta_0 \end{aligned} \tag{26}$$

Where, subtituting our findings so far into the equation defining $q^*(\emptyset, 1, 0)$ gives that, in any PBE:

$$\begin{aligned} q^*(\emptyset, 1, 0) &= \frac{q\sigma_1^*(\emptyset, 1)(1 - \sigma_2^*(\emptyset, 1, 1)) + (1-q)(1 - \sigma_2^*(\emptyset, 1, 1))}{q\sigma_1^*(\emptyset, 1)(1 - \sigma_2^*(\emptyset, 1, 1)) + (1-q)(1 - \sigma_2^*(\emptyset, 1, 1)) + q\sigma_1^*(\emptyset, 0)(1 - \sigma_2^*(\emptyset, 0, 1)) + (1-q)\sigma_1^*(\emptyset, 0)} \\ &= \frac{q \cdot 1 \cdot 0 + (1-q) \cdot 0}{q \cdot 1 \cdot 0 + (1-q) \cdot 0 + q\sigma_1^*(\emptyset, 0)(1 - \sigma_2^*(\emptyset, 0, 1)) + (1-q)\sigma_1^*(\emptyset, 0)} \end{aligned}$$

Note that whenever $\sigma_1^*(\emptyset, 0) > 0$, Bayes' rule applies and $q^*(\emptyset, 1, 0) = 0$. In this case, by Equation (12) and (26), $\sigma_1^*(\emptyset, 0) = 1$. We conclude that there is always a PBE where $\sigma_1^*(\emptyset, 0) = 1$. This completes the description of one PBE.

Remark 3. *There is always a PBE of the game with two rulers where:*

$$\sigma_1^*(\emptyset,0) = \sigma_1^*(\emptyset,1) = 1$$
$$\sigma_2^*(\emptyset,0,1) = \sigma_2^*(\emptyset,1,1) = 1$$

In this PBE, when the issue is non-preordained,

- *If $s = 1$, the aggregate policy is $a = 1$.*
- *If $s = 0$, the aggregate policy is $a = 1$ if and only if both rulers are minority-congruent.*

In any case, there are no revolts.

If $\delta_0 \cdot (1-q) \cdot (1-\bar{\beta}(q')) + q > \delta_0$ for all $q' \in [\frac{1}{2}, 1]$, any PBE of the game with two rulers is a PBE that is described in Remark 3. For the rest of the analysis, suppose $\delta_0 \cdot (1-q) \cdot (1-\bar{\beta}(q')) + q \leq \delta_0$ for some $q' \in [\frac{1}{2}, 1]$. In this case, there is another PBE where $\sigma_1^*(\emptyset, 0) = 0$. Now, Bayes' Rule does not apply to $(\hat{s}, a_1, a_2) = (\emptyset, 1, 0)$, so $q^*(\emptyset, 1, 0)$ can be chosen arbitrarily. Choosing it so that $\delta_0 \cdot (1-q) \cdot (1-\bar{\beta}(q^*(\emptyset, 1, 0))) + q \leq \delta_0$ ensures that $\sigma_1^*(\emptyset, 0) = 0$ is optimal.

Remark 4. *Suppose $\delta_0 \cdot (1-q) \cdot (1-\bar{\beta}(q')) + q \leq \delta_0$ for some $q' \in [\frac{1}{2}, 1]$. There is a PBE of the game with two rulers where:*

$$\sigma_1^*(\emptyset,0) = 0$$
$$\sigma_1^*(\emptyset,1) = 1$$
$$\sigma_2^*(\emptyset,1,1) = 1$$

In this PBE, when the issue is non-preordained,

- *If $s = 1$, the aggregate policy is $a = 1$.*
- *If $s = 0$, the aggregate policy is $a = 0$.*

In any case, there are no revolts on the equilibrium path.

Although the PBE described in Remark 4 is a theoretical possibility, it is a very fragile equilibrium. This is because it relies on the belief $q^*(\emptyset, 1, 0)$ being above $\frac{1}{2}$, even though the scenario where $(\hat{s}, a_1, a_2) = (\emptyset, 1, 0)$ occurs with zero probability. In particular, for this equilibrium to be sustained, the citizens must believe, with high probability, that $s = 1$ following $(\hat{s}, a_1, a_2) = (\emptyset, 1, 0)$. Then, the minority-congruent type of ruler 1 is worried about having $a_2 = 0$ by the majority-congruent type of ruler 2.[3] The reason for this worry is **not** the aggregate action changing. Rather, it is the worry of **revolt**: when citizens encounter $(\hat{s}, a_1, a_2) = (\emptyset, 1, 0)$, they incorrectly infer that state is $s = 1$ with high probability and revolt against aggregate action $A = 0$.

Given the minority-congruent ruler's preference towards $A = 1$ (by Assumption 2), this is a counterintuitive equilibrium. If anything, $(\hat{s}, a_1, a_2) = (\emptyset, 1, 0)$ should make citizens infer that "The state must be $s = 0$, but ruler 1 is minority-congruent and could not resist the temptation of $a_1 = 1$. He is corrected by a majority-congruent ruler 2. But since $A = 0$,

[3]Note that this reasoning falls apart when ruler 1 can observe ruler 2's type, which is the reason why the setup described in the main text does not suffer from equilibrium multiplicity.

I will not revolt against it." The counterintuitivity of PBE described in Remark 4 can be formalized by showing that it fails forward induction. The next result shows this.

Claim 3. *Any PBE described in Remark 4 fails forward induction.*

Proof. Consider a PBE described in Remark 4. In the subgame following $\hat{s} = \emptyset$, the outcome of this PBE is:

$$(s, \hat{s}, a_1, a_2, r) = \begin{cases} (0, \emptyset, 0, 1, 0), & w.p.\ \frac{1}{2}\sigma_2^*(\emptyset, 0, 0), \\ (0, \emptyset, 0, 0, 0), & w.p.\ \frac{1}{2}(1 - \sigma_2^*(\emptyset, 0, 0)), \\ (1, \emptyset, 1, 1, 0), & w.p.\ \frac{1}{2} \end{cases}$$

Our first observation is that the pure strategy of ruler 1 defined as

$$\sigma_1(\emptyset, 0) = \sigma_1(\emptyset, 1) = 1 \tag{27}$$

is a relevant strategy for this outcome. To see this, among the PBE's described in Remark 4, take the one with $q^*(\emptyset, 1, 0)$ such that:

$$\delta_0 \cdot (1 - q) \cdot (1 - \bar{\beta}(q^*(\emptyset, 1, 0))) + q = \delta_0$$

This is the belief that leaves ruler 1 just indifferent between the two actions when $(s, \hat{s}) = (0, \emptyset)$, and such a belief exists due to continuity of $\bar{\beta}(q')$ in q'.

The strategy in (27) is optimal under these beliefs, and therefore it is a relevant strategy. Intuitively, under this PBE, ruler 1 may consider deviating to $a_1 = 1$ when $s = 0$.

Our next observation is that any strategy that includes

$$\begin{aligned} &\sigma_2(\emptyset, 0, 1) = 0, \qquad or \\ &\sigma_2(\emptyset, 1, 1) = 0 \end{aligned}$$

is irrelevant for this outcome. This is because, as discussed above, $q^*(\emptyset, 1, 1) < \frac{1}{2}$ in any PBE. Therefore, $\beta(q^*(\emptyset, 1, 1), M, \gamma) = 0$ in any PBE. By equations (14) and (16), and by Assumption 2, then, $\sigma_2^*(\emptyset, 0, 1) = 1$ and $\sigma_2^*(\emptyset, 1, 1) = 1$ are strict best responses in any PBE. Intuitively, because $a = 1$ is the minority-congruent ruler's favorite outcome, any minority-congruent ruler 2 will not consider deviating to $a_2 = 0$ following $a_1 = 1$.

The discussion above shows that information set $(\hat{s}, a_1, a_2) = (\emptyset, 1, 0)$ is relevant for the outcome under PBE in Remark 4. Moreover, for the outcome to satisfy forward induction, any beliefs in this information set must contain $\sigma_1(\emptyset, 0) = 1$ and rule out $\sigma_2(\emptyset, 0, 1) = 0$ as well as $\sigma_2(\emptyset, 1, 1) = 0$. Under this restriction, the only scenario consistent with $(\hat{s}, a_1, a_2) = (\emptyset, 1, 0)$ occurs when $s = 0$. Therefore, $q^*(\emptyset, 1, 0) = 0$. Under these beliefs, $\bar{\beta}(q^*(\emptyset, 1, 0)) = 0$, and $\sigma_1^*(\emptyset, 0) = 0$ ceases to be optimal. We conclude that any PBE of the type described in Remark 4 fails forward induction. □

In contrast, the outcome of the PBE described in Remark 3 survives forward induction. This is because:

- The strategies that include:

$$\sigma_1^*(\emptyset,0) = \sigma_1^*(\emptyset,1) = 1$$
$$\sigma_2^*(\emptyset,0,1) = \sigma_2^*(\emptyset,1,1) = 1$$

are relevant for this outcome. After all, they are part of the PBE strategies, and thus they are always optimal.

- The information set $(\hat{s}, a_1, a_2) = (\emptyset, 1, 1)$ is always relevant, because they are reached by the strategies above with strictly positive probability.

- In the information set $(\hat{s}, a_1, a_2) = (\emptyset, 1, 1)$, with the relevant strategies specified above, an equilibrium belief such that $q^*(\emptyset, 1, 1) < \frac{1}{2}$ can always be constructed. Then, the relevant strategies mentioned above remain optimal.

Our findings so far imply the following result.

Proposition 11. *In the extended model of institutional constraints, there is a unique outcome that satisfies forward induction of the Perfect Bayesian Nash Equilibrium of the game. In this outcome,*

$$Pr_{(t_1,t_2)}(A = s) = 1, \quad \textit{if } (t_1, t_2) \neq (b, b).$$

Otherwise,

$$Pr_{(b,b)}(A = 1|\hat{s}, s = 1) = Pr_{(b,b)}(A = 1|\hat{s} = \emptyset, s = 0) = 1$$

and

$$Pr_{(b,b)}(A = 1|\hat{s} = s, s = 0) = \begin{cases} 1 & ; \beta(1, M, \gamma) < 1 - \delta_0 \\ 0 & ; \textit{otherwise.} \end{cases}$$

There is a revolt only if $\hat{s} = 0$ and both rulers take action 1. This revolt succeeds with probability $\beta(1, M, \gamma)$. Moreover, the expected policy payoff for a majority citizen is

$$\begin{cases} 1 - q^2(1-p) - \mu & ; \beta(1, M, \gamma) > 1 - \delta_0 \\ 1 - q^2(1 - p\beta(1, M, \gamma)) - \mu & ; \beta(1, M, \gamma) < 1 - \delta_0. \end{cases}$$

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