Bilkent University
Econ 101 - Fall 2022
Chapter 5: Demand

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Contents

1 What Is Going On?

In this chapter, we will draw a **demand curve** for a particular good and discuss its properties. A demand curve is the first important element of a **market for goods and services**. (The second important element of a market is the **supply curve**, which we will get to in a few lectures). Basically, we are very close to wrapping up our discussion on the **consumer side** of the market.

There are two important insights I'd like you to keep in mind from consumer theory.

1. When a consumer chooses how much to consume among several goods, her decision process resembles the following. **The consumer keeps buying a good until her relative valuation for that good is equal to the relative price of that good.**

   To digest this idea better, let me now focus on the consumption of a single good. Let good 1 be the good that we are focusing on, and let good 2 be the "composite good" that includes every other good and service the consumer can buy in the world. Well, to buy those other goods and services, the consumer will need to use money. Then, let good 2 be the money that the consumer keeps in her pocket to buy all the other goods and services. We will use the framework we developed in the previous chapter to analyze this scenario.[1]

   Now, q_1 is the amount of good that the consumer buys (denominated in kg's, lt's, lb's...) and let q_2 denote the amount of money the consumer "buys" (denominated in TL). Of course, the consumer does not actually buy money with money, but there is nothing wrong with imagining that the consumer can go ahead and buy 1 TL by paying 1 TL. Then,

   - I is the income of the consumer (in TL's),
   - p_1 is the price of good 1 per unit (in TL/kg, TL/lt...)
   - p_2 is the price of good 2 per unit (in TL/TL). By construction, $p_2 = 1$ TL/TL. ("The price of one lira is one lira.")

   The rest is the same. We can just imitate the analysis we made in Chapter 2 and find the optimal amount of good 1 for the consumer. Here, the consumer has preferences between bundles, where a bundle $q = (q_1, q_2)$ consists of q_1 units of the good and q_2 TL's. Given the consumer's preferences, at a bundle q, one can still define $MRS_{2,1}(q)$. This is the answer to the following question:

   > **"Suppose the consumer is endowed with q_1 units of the good. If I take away one unit of the good away from the consumer, how many extra TL's should I give to the consumer, so that she is left indifferent?"**

   If you think a little bit about it, this is a measure of how much the consumer values the marginal good when she already has q_1 units of the good. Let's give this a name:

   Definition 1. *The* **marginal benefit** *(or* **marginal valuation***) of the consumer for the q-th unit is how much money the consumer is willing to pay for the last unit of the good at the margin, when she has q units of the good. This is given by $MB(q)$.*

   To reiterate what I said before: marginal benefit for the q-th unit $\neq$ the benefit for the first q units of the good. The consumer may find her first t-shirt very valuable (i.e., the marginal benefit of the first t-shirt may be very high), but she may not care about the 100-th t-shirt if she already has 99 t-shirts (i.e., the marginal benefit of the 100-th t-shirt may be very low).

   Now, recall that if we have an interior solution ($q_1^* > 0, q_2^* > 0$), the optimal bundle $q^* = (q_1^*, q_2^*)$ satisfies:

$$MRS_{2,1}(q^*) = \frac{p_1}{p_2}$$

[1] Hopefully, this will also convince you on how useful and generalizable this framework is.

In this setup where good 2 is money, we can replace $MRS_{2,1}(q^*)$ with $MB(q_1^*)$. Moreover, $p_2 = 1$. Therefore, the **quantity of good 1 consumed by the consumer when the price is** p_1 satisfies:

$$MB(q_1^*) = p_1$$

Under the optimal quantity, the marginal benefit is equal to the price!

In my experience, imagining the following process is a good way to think about the optimal quantity. The consumer starts by buying small quantities of good 1. At this quantity, if the consumer (marginally) values good 1 more than its price, she buys some more good 1. This will reduce the marginal benefit of good 1 due to the diminishing marginal rate of substitution. If, at the new bundle, she still values good 1 more than its price, she again buys some more good 1. The process goes on like this until the consumer does not want to buy any more good 1. But this is exactly the point where the marginal benefit is equal to the price.

(This process is just a product of our imagination. In this model, the consumer buys the quantity *at once*: the process does not move sequentially. It is not an incremental process. But this is a useful visualization. Moreover, there is nothing wrong with thinking that the consumer "imagines" this process as well.)

Another point: please note the crucial role played by the diminishing marginal rate of substitution in this argument. It ensures that as the consumer buys one more unit of a good, her valuation for the next unit of the same good is lower. This gives the decision process a certain **regularity**. We also believe that this is a reasonable assumption for many goods: most consumers really value the first unit of a good a lot, whereas they do not value 100th unit that much.

2. **If the price of an ordinary good increases, the consumer buys it less.** A reminder: all goods we will consider from now on will be ordinary goods.

   As you recall from Chapter 2, diminishing marginal rate of substitution also plays a very crucial role in this argument. Why? Diminishing marginal rate of substitution ensures that substitution effect works in the "proper" way. That is, it ensures that the substitution effect is such that: if p_1 increases, q_1^* decreases. And as you also recall from Chapter 2, for a good to be an ordinary good, the substitution effect must dominate.[2]

   Now, go back and check the heuristic we developed in the point above. What happens if the price of a good increases? The consumer stops buying the good earlier. Therefore, once we check the quantity consumed by the consumer, we will realize that it is lower. This is consistent with everything we said so far!

2 Demand Schedules

Let us now focus on a single good. Fix the income of the consumer and prices of other goods, and consider an ordinary good 1. We will study the possible prices of good (P) and the quantities of good 1 consumer buys at these prices (q_1^*). That is, we will study the **demand** of the consumer for good 1 at various prices.

Based on the things I reiterated above, let me now construct a **demand schedule**. It is basically an excel sheet of possible prices P and quantities demanded at these prices q_1^*. I construct this by going to the consumer and asking the following question repeatedly:

> "If the price of good 1 per unit is P, what is the quantity you demand q_1^*?"

(This is a hypothetical exercise. I ask this question for different values of P and record the answer on an excel sheet. I am not worried about the consumer lying to me. In real life, we can construct this excel sheet by looking at the data. Suppose, over time, the price of good 1 varies. We can record the amount the consumer buys at different prices. This would construct a demand schedule.)

[2] Good 1 may be a normal good, in which case the substitution and income effects works in the same direction. Or, good 1 may be an inferior good but not a Giffen good, in which case substitution effect is stronger than income effect.

To fix ideas, suppose good 1 is the cups of tea the consumer drinks per day. Let P be the price of a cup of tea for every cup of tea the consumer drinks. Let q_1^* denote the number of cups of tea the consumer drinks per day.

- I may go ahead and ask: "If the price of tea is 7 liras per cup, how many cups of tea would you consume a day?" Suppose the consumer says: "Zero. I am not willing to buy even a single cup of tea if it was 7 liras."

  What does it mean? Based on the process I described in Section 1, what the consumer says means the following. "The value of the first cup of tea is less than 7 liras."

- I then ask: "What if the price is 6 liras per cup?" Suppose the consumer says: "I am willing to buy one cup of tea."

  What does it mean? Based on the process I described in Section 1, what the consumer says means the following. "The valuation of the first cup of tea I consume is more than 6 liras. The valuation of the second cup is less than 6 liras." (This is because the consumer stops before buying the second cup.)

- I then ask: "What if the price is 5 liras per cup?" Suppose the consumer says: "I am still willing to buy one cup of tea."

  What does it mean? Based on the process I described in Section 1, what the consumer says means the following. "The valuation of the second cup is less than 5 liras."

- I then ask: "What if the price is 4 liras per cup?" Suppose the consumer says: "I am willing to buy two cups of tea."

  What does it mean? Based on the process I described in Section 1, what the consumer says means the following. "The valuation of the second cup of tea I consume is more than 4 liras. The valuation of the third cup is less than 4 liras." (This is because the consumer stops before buying the third cup.)

- ...

- I then ask: "What if the price is 0 liras per cup?" (i.e., what if tea was free?) Suppose the consumer says: "I am willing to buy five cups of tea."

  What does it mean? Based on the process I described in Section 1, what the consumer says means the following. "The valuation of the fifth cup of tea I consume is more than 0 liras. The valuation of the sixth cup is less than 0 liras." (This is because the consumer stops before consuming the sixth cup.)

Based on this survey, I can construct a demand schedule (an excel sheet). It looks like this:

P (price per unit)	q_1^* (no. of units bought)
7	0
6	1
5	1
4	2
3	3
2	3
1	4
0	5

Table 1: An Example of a Demand Schedule.

To reiterate:

> The **demand schedule** of a consumer for a good is a relation between the possible prices of the good and the quantities the consumer would like to consume at these prices.

2.1 Another Way of Constructing a Demand Schedule

The demand schedule is simply an excel sheet. I can easily rearrange the columns of this excel sheet. This would correspond to asking the following question repeatedly:

> "If I want you demand q_1^* units of good 1, what should the maximum price P of good 1 per unit be?"

(Once again, this is a hypothetical exercise. I am not worried about the consumer lying to me. In real life, this is coming from data.)

I am not doing much indeed, just reordering the columns. It now looks like this.

q_1^* (no. of units bought)	P (price per unit)
1	6
2	4
3	3
4	1
5	0

Table 2: An Example of the Same Demand Schedule as in Table 1.

Even though I am not doing much, the interpretation of the table now differs. If you are following closely, what I am doing corresponds to the following procedure.

- I go ahead and ask: "If I want you to consume one cup of tea per day, what is the price I should charge per cup of tea?" The consumer says: "Six liras. I am not willing to consume even one cup of coffee if the price exceeds six liras."

  What does it mean? "My valuation of the first cup of tea is six liras."/"The marginal benefit of the first cup of tea is six liras."

- I then ask: "If I want you to consume two cups of tea per day, what is the price I should charge per cup of tea?" The consumer says: "Four liras. I am not willing to consume the second cup of tea if the price per cup of tea exceeds four liras."

  What does it mean? "My valuation of the second cup of tea is four liras."/"The marginal benefit of the second cup of tea is four liras."

- ...

- I then ask: "If I want you to consume five cups of tea per day, what is the price I should charge per cup of tea?" The consumer says: "Zero liras. I am not willing to consume the fifth cup of tea unless tea is free."

  What does it mean? "My valuation of the fifth cup of tea is zero liras."/"The marginal benefit of the fifth cup of tea is zero liras."

To reiterate:

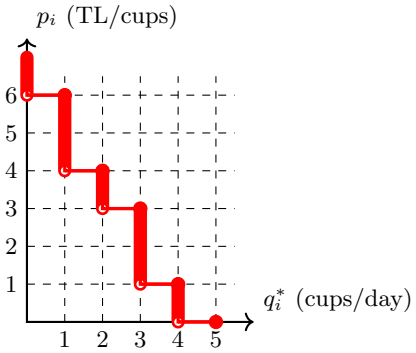
> The *demand schedule* of a consumer for a good is a relation between the quantities of the good and the marginal benefit that the consumer gets from consuming the last unit of the quantity.

3 Individual Demand Curves

As it happens, displaying information in an excel sheet is not the optimal way of illustrating it. We, as economists, prefer graphs! So let me summarize the information I obtained throughout this survey in a graph. This is called an **(individual) demand curve.**

As a convention, I will put the price (P) in the y-axis and the quantity (q_1^*) in the x-axis. NOTE THAT THIS IS A DIFFERENT GRAPH THAN THOSE WE WERE USING EARLIER. The previous one was showing the constrained optimization problem the consumer faces, for a given P. This one shows the outcome of the optimization problem for different values of P.

We end up with a graph looking like this:



Let me just re-emphasize one thing. Because a demand curve contains the same information as in a demand schedule, it also has two interpretations.

1. (From P to q_1^*) It shows, at each price, the quantity demanded by consumer.
2. (From q_1^* to P) It shows, at each quantity q, the marginal benefit of consumer for the q-th unit of the good.

It is important to keep both interpretations in mind, as there are cases when either is useful.

4 Market Demand Curves

Individual analysis is all nice and good, but sometimes we want to analyze economic forces on the aggregate level. To do this, we will go from individual level to **market level**. From Chapter 1:

> A **market** is an infrastructure that facilitates interactions among economic agents.

So we need to define what the market under consideration is. As long as we define it nicely, we can analyze a market. For instance:

- The market may be the Starbucks in the ground floor of Faculty of Business Administration. There is a well defined set of consumers in this market (Bilkent affiliates). We can go ahead and ask every single Bilkent affiliate about their individual demands of cups of tea they demand per day from that Starbucks.
- The market can be the whole Bilkent campus. Once again, there is a well defined set of consumers (Bilkent affiliates). We can go and ask every single consumer: "How many cups of tea you demand per week at the following prices?"
- The market can be the Migros in the Bilkent Center. There is a well defined set of consumers (people who shop from that Migros). We can go and ask: "How many kilograms of tea you demand per year at the following prices?"
- The market can be the whole country. The consumers are the citizens. We go and ask: "How many tons of tea you demand per year at the following prices?"
- The market can be the whole world. The consumers are everyone in the world. "How many tons of tea you demand per year at the following prices?"
- This can be the Shell gas station in the Bilkent road, or the market for gasoline in the whole country...

Regardless of what the market is, as long as we have a clear definition of it, we can obtain the market demand curve. To obtain the market demand curve, we add up the individual demands of every consumer in the market. That is, at every single price, we add up the individual demands of the consumers at the said price. The total we obtain is the **quantity demanded** by the consumers in the market. We will denote this quantity by Q.

Figure 1 is a representative figure where we add up the individual demand curves of two consumers. For more than two consumers, the process is the same.

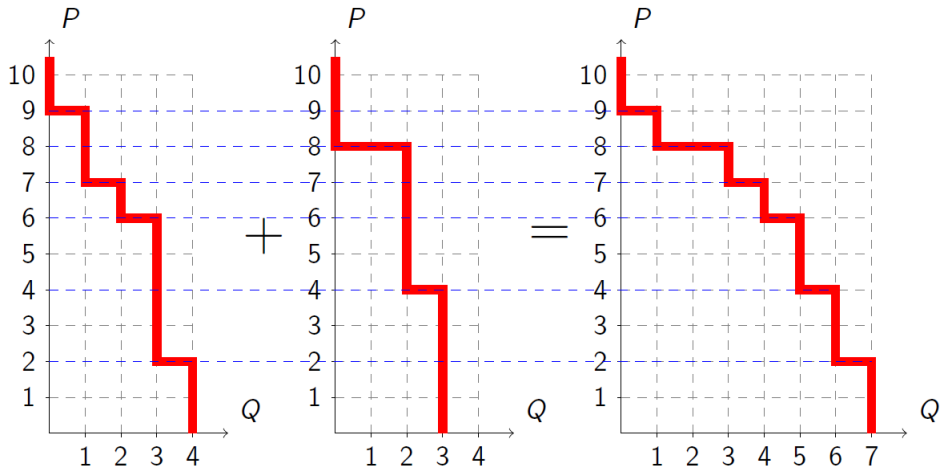


Figure 1: Addition of two individual demand curves.

Let me make one more modification. The demand curves we have so far have a lot of kinks. This is fine as long as you know how they are derived. But in the future, we will give some equations of the demand curves and conduct some mathematical analysis. Giving the equation for a curve with so many kinks is very difficult! To circumvent this problem, I will draw "smoother" individual demand curves. Of course, the market demand curve (which is merely an addition of individual demand curves) will be smooth as well. So it will look like Figure 2.

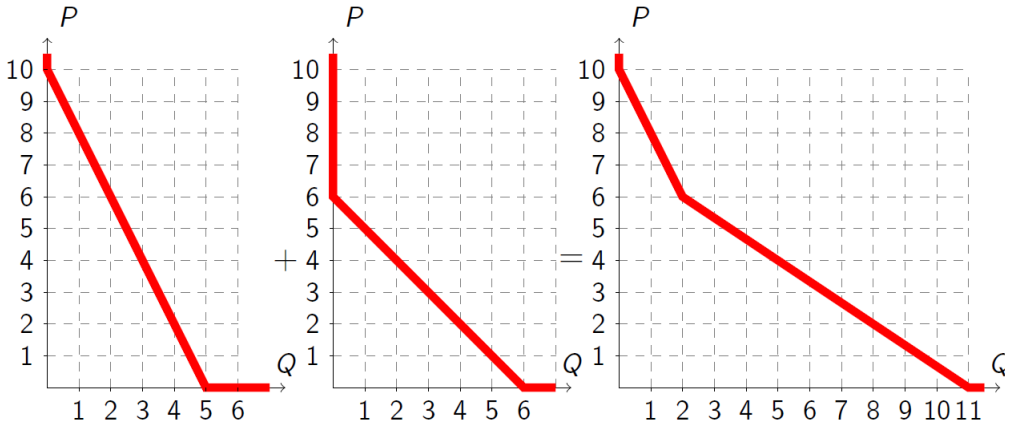


Figure 2: Addition of two (smooth) individual demand curves.

From now on, we will draw "smooth" demand curves. There are at least three ways to defend smooth demand curves.

1. As discussed above, it is easier to write down equations for smooth curves.
2. You can imagine us having finer and finer increments in quantities and prices. Instead of asking for the quantity demanded at each lira, we ask for quantity demanded at each kuruş. We may also have finer increments in quantities: instead of asking in terms of kilograms, we may ask in terms of miligrams etc. Because the increments are finer, the jumps in the demand curve will also be smaller. It will look much more like a smooth curve!
3. You may imagine a smooth curve as an "approximation" to a curve with kinks. As long as we understand what happens in the benchmark case (i.e., the case with smooth curves), the general insights will go through.

From now on, we will draw a demand curve for a market as a smooth one. Figure 3 illustrates a representative demand curve. We will use the letter D to label a demand curve, which stands for "Demand".

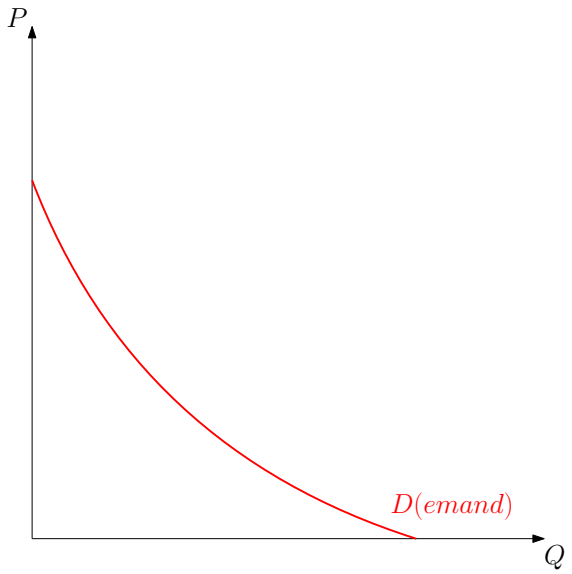


Figure 3: A representative demand curve.

A couple of general points about demand curves follow.

First, realize the convention of labeling the axises. We are using P for the market price and Q for the market quantity of a good. We will keep this notation for the rest of this class (and, to be frank, for the rest of your economics education).

Second, note that the market demand curve is decreasing. This is not surprising: a market demand curve is the summation of individual demand curves. Because we are considering ordinary goods, each individual demand curve is decreasing. Of course, if you add up several decreasing curves, the end result will be a decreasing curve!

The idea that a market demand curve is decreasing (also known as "downward-sloping") will be carried for the rest of this class (and, to be frank, for the rest of your economics education). This is such a widely accepted fact that economists call it a law.

Definition 2. Law of Demand: *holding everything else constant, when the price of a good rises, the quantity demanded falls.*

Third, a simple remark. Most economics textbooks (including ours) draw demand curves as lines, not curves. That is made for the sake of convenience, but that may be misleading sometimes. **A demand curve can be a line, but it does not have to be.** The only requirement we are imposing on the demand curve is being downward-sloping.

Finally, just like an individual demand curve, a market demand curve also has a dual interpretation.

1. (From P to Q) It shows, at each price, the total quantity demanded by the consumers in the market.
2. (From Q to P) It shows, at each quantity Q, the marginal benefit of the marginal consumer.

   This can sometimes be confusing. By the marginal consumer, I mean the following. At the quantity Q and price P, there is a consumer who is at the edge of buying the last unit of the good or dropping her consumption by one unit. If the price increases a tiny bit, this consumer would reduce her consumption by a tiny bit. P, therefore, is exactly this consumer's valuation for that last unit of good.

   Perhaps it is easier to understand through the following example (which is not a general example, but it is illustrating). Consider a good that a consumer buys at most one unit of. For instance, the Econ 101 textbook. Consider the demand for Econ 101 textbooks in Meteksan bookstore in a semester. This is a downward-sloping curve: if the price P is lower, more students will but the textbook, resulting in a higher Q. For the sake of the argument, let $P = 150$ liras and $Q = 147$ textbooks be on this curve. This has two meanings: (i) when the price of textbook is 150 liras, Meteksan will sell 147 textbooks. (ii) If we order the students by their valuation of the textbook, the 147th student has a valuation of 150 liras. Why? At any price higher than 150 liras, Meteksan sells less than 147 textbooks, which means this particular student stops buying the textbook.

5 Variables That Shift the Demand Curve

Recall what we said when we defined law of demand: "Holding everything else constant..."[3] What is "everything else"? It includes income of the consumers, prices of other goods, consumers' tastes etc.

Now, as you can imagine, if we change "other things", demand will change as well. For instance, if the income of the consumers increase, and if the good under consideration is a normal good, the consumer will demand a higher quantity due to income effect. Moreover, demand Q will increase **at every price** P. We will denote it with an outward shift in the demand curve. That is, we draw a new demand curve that has a higher Q at every P. Consider Figure 4. The new demand curve (under higher income) is D'. Note that D' has a higher quantity demanded at each price. For instance, at price P, the quantity demanded is Q', which is higher than Q.

What if the income of the consumers decrease, and the good under consideration is a normal good? Then, the demand curve will shift inwards, representing a decrease in the quantity demanded at each price. Consider Figure 5. The new demand curve (under lower income) is D'. Note that D' has a lower quantity demanded at each price. For instance, at price P, the quantity demanded is Q', which is lower than Q.

One thing I want to emphasize: most textbooks illustrate a shift in the demand curve by *shifting it by the same amount in every price.* That is, *they draw a parallel curve.* That is made for convenience, and it does not have to be that way. For instance, the quantity demanded may increase more in lower quantities (e.g., the income effect may be high in lower quantities), but it may increase less in higher quantities (e.g., the income effect may be low in higher quantities).

Below is a discussion of several variables that may shift the demand curve.

5.1 Income of the Consumers

If the good is a normal good,

- An increase in income will cause an outward shift in the demand curve.
- A decrease in income will cause an inward shift in the demand curve.

If the good is an inferior good,

- An increase in income will cause an inward shift in the demand curve.

[3] If you enjoy plugging in Latin phrases, you can also say *ceteris paribus*. It literally translates as *all else constant*.

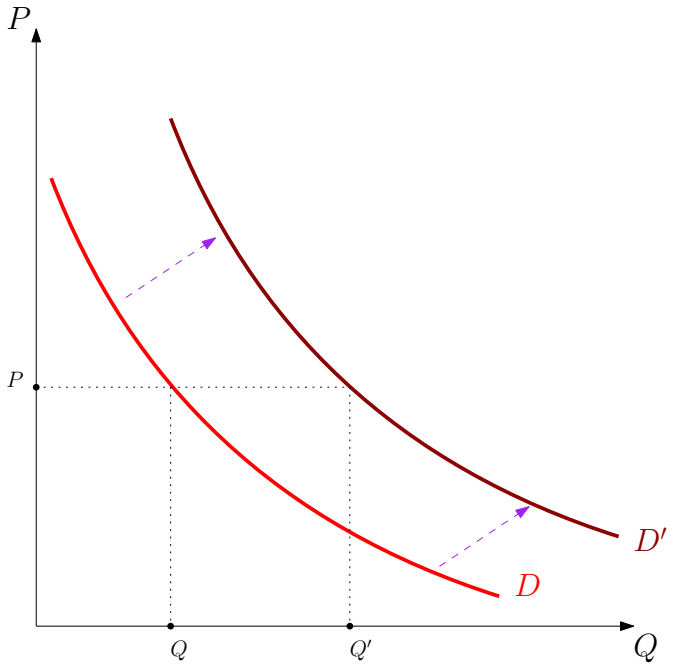


Figure 4: An outward shift in the demand curve. Some resources also call this a "shift upwards" or a "shift in the northeastern direction".

- A decrease in income will cause an outward shift in the demand curve.

5.2 Prices of Other Goods

We say that two goods are **substitutes** if they can be used for the same purpose, i.e. if they can substitute each other easily. Examples of substitutes: tea and coffee, rice and bulgur, hand sanitizer and cologne, Samsung smart phones and iPhones, PlayStation and XBox, blue shirts and white shirts. (Cologne can be used instead of hand sanitizer, I can wear a white shirt instead of a blue shirt.)

Consider the demand curve for good X, and consider another good Y which is a substitute of good X.

- An increase in the price of good Y will cause an outward shift in the demand curve for good X.
- A decrease in the price of good Y will cause an inward shift in the demand curve for good X.

Why? When good Y is more expensive, the consumers will start substituting good Y with good X. This will result in the consumers demanding a higher quantity of good X at every price. For instance, if blue shirts are more expensive, people will start buying white shirts instead, which will result in an outward shift in the demand curve for white shirts.

We say that two goods are **complements** if they are used together, i.e. if they complement each other easily. Examples of complements: coffee and sugar, Chai Tea Latte and cinnamon, cars and gasoline, smart phones and accompanying apps, a game console and accompanying games, tuxedos and bowties. (People consume PlayStation and Fifa 23 together, most people wear a bowtie with a tuxedo.)

Consider the demand curve for good X, and consider another good Y which is a complement of good X.

- An increase in the price of good Y will cause an inward shift in the demand curve for good X.
- A decrease in the price of good Y will cause an outward shift in the demand curve for good X.

Why? When good Y is more expensive, the consumers will start buying it less ("the law of demand"). But since good X and good Y are used together, then the consumer will buy less of good X as well. This will

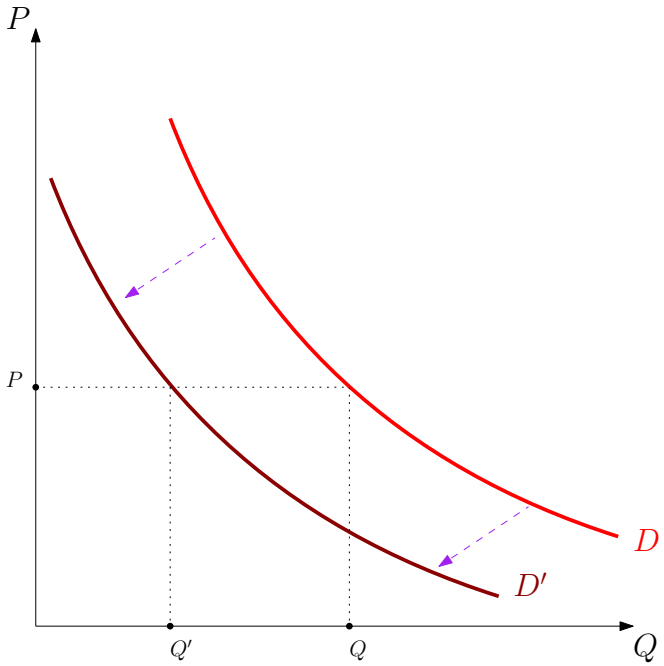


Figure 5: An inward shift in the demand curve. Some resources also call this a "shift downwards" or a "shift in the southwestern direction".

result in the consumers demanding a lower quantity of good X at every price. For instance, if cars are more expensive, people will start buying fewer cars, and as a result they will consume less gasoline as well. This will result in an inward shift in the demand curve for gasoline.

5.3 Changes in Tastes and Preferences

Sometimes people will start preferring a good more due to an exogenous change in tastes and preferences. This will cause an outward shift in the demand of the said good. Advertising is a classical example: it can shift a demand curve outwards. (When Emma Chamberlain drinks 17 cups of iced coffee in her 30-minute videos, a lot of people develop an interest in consuming iced coffee.) On the contrary, a lawsuit against a firm may make people move away from the products produced by that firm, causing an inward shift in the demand. (When Harvey Weinstein scandal blew up, people -rightfully- reduced their consumption of movies produced by the Weinstein Company.)

This does not have to be tastes as well: the external conditions may lead to consumers preferring a good more. Since 2020, there is a pendemic going around. Early on, the pandemic it caused a **tremendous** outward shift in the demand curve for hand sanitizers, face masks, bread machines, and toilet papers. On the other hand, it caused in inward shift in the demand curve for plane tickets, restaurants, haircuts, and souvenirs. See Figure 6.

5.4 Expected Future Prices

If the consumers will expect the price of a good to increase in the future, they will want to buy it now rather than later. This will result in an outward shift in the demand curve. The opposite case happens if the price is expected to decrease.

A classical example is electronic items. If the consumers expect the government to impose a tax on certain electronic items in three months, the demand for those items right now will increase. If, on the other hand, people expect Apple to reduce the price of iPhone 11 in three months, the demand for iPhone 11 right now will decrease.

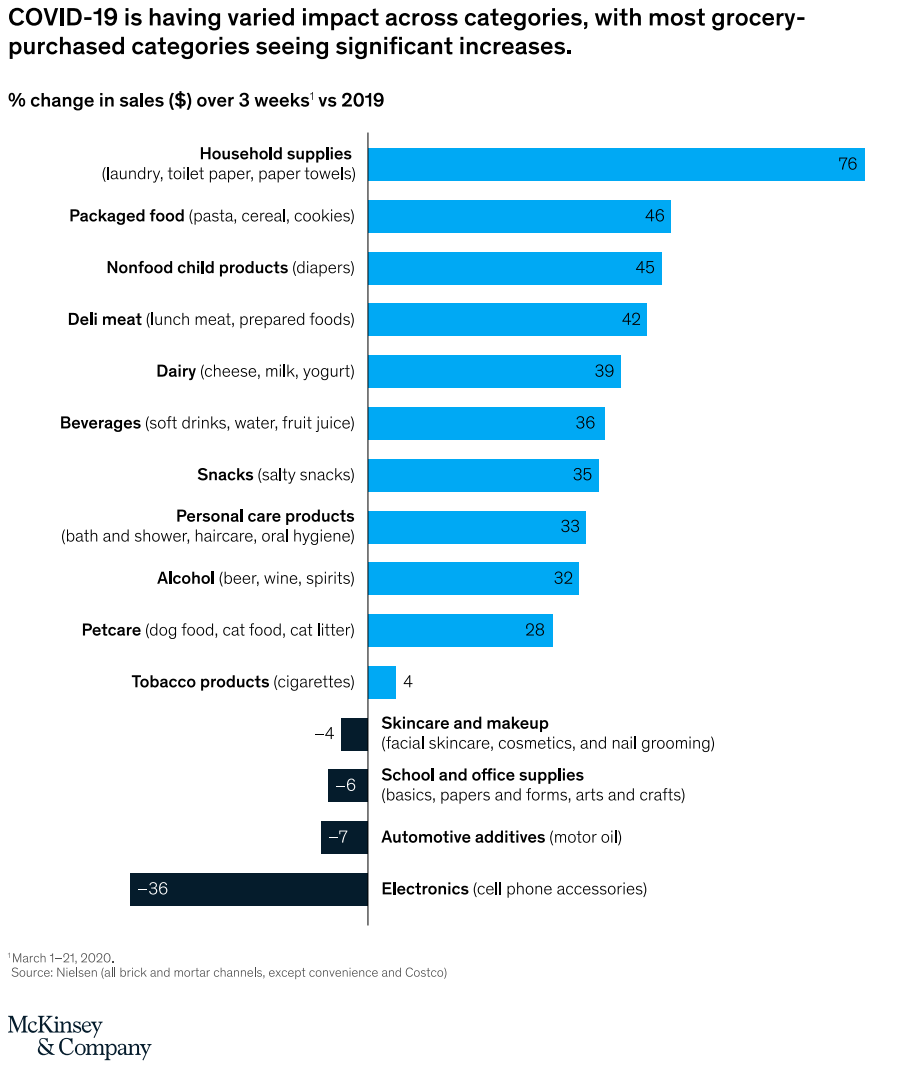


Figure 6: The pandemic caused an outward shift in the demand curves of some goods and services, and an inward shift in others. Source: McKinsey.

5.5 Population and Demographics

The definition of a "market" contains a well-defined group of consumers, but that group may change as well. If Bilkent admits more students, the demand for tea at Starbucks in Business Administration building will increase, and the demand curve will shift outwards. If there is a lot of immigration towards a city, the demand for houses will increase, and the demand curve will shift outwards.

Similarly, the characteristics of the population may change over time. If the population gets older, the demand for adult diapers will increase, causing an outward shift in the demand curve. If the population gets younger, the demand for K-Pop albums will increase, causing an outward shift in the demand curve.

6 Elasticity of Demand

For our next exercise, we will fix a market demand curve (we will not shift it!) and study its properties. Perhaps the most important information a demand curve is the responsiveness of quantity demanded to the price. If the price increases a little bit, we know that the quantity demanded will decrease (this is the *law of demand*.) But how much will it change? By a little, or a lot? How will it compare the change in price? To answer these questions, we will introduce the notion of **elasticity of demand**.

- Informally: elasticity of demand (to be more precise, **own price elasticity of demand**) is a measure of responsiveness of quantity demanded to changes in price.
- A bit more formally: (own price) elasticity of demand is a measure of percentage change in quantity demanded in response to a percentage change in the price.
- Most formally: (own price) elasticity of demand is the rate at which the percentage change in quantity demanded changes in response to a percentage change in the price of the good resulting from a "small" change in the price of the good.

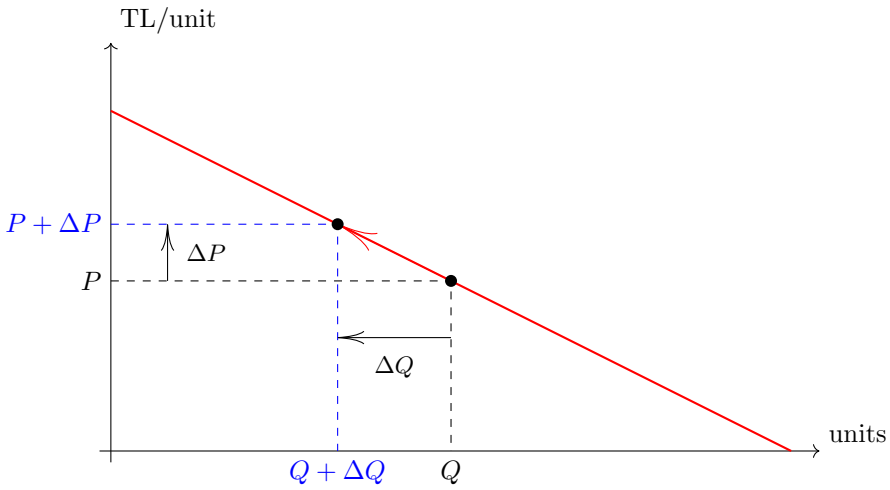
So we are looking for an answer to the following question: "If the price of a good increases by one percent, by what percent the quantity demanded will decrease?"

Here is some notation to get us going:

- P: the price of the good at which we would like to find the elasticity.
- Q: the quantity demanded at the price P.
- ΔP: the change in price.
- ΔQ: change in quantity demanded in response to change in the price.

  Therefore, $Q + \Delta Q$ is the quantity demanded at price $P + \Delta P$. Note that when $\Delta P > 0$, we have $\Delta Q < 0$ (by the law of demand).

The figure below illustrates:



Percentage change in price is

$$\frac{\Delta P}{P} 100$$

Percentage change in quantity demanded is

$$\frac{\Delta Q}{Q} 100$$

The measure of the responsiveness of quantity demanded to a change in price is simply the ratio of these two:

$$\begin{aligned}\text{(own price) elasticity of demand} &= \frac{\text{percentage change in quantity demanded}}{\text{percentage change in price}} \\ &= \frac{\frac{\Delta Q}{Q} 100}{\frac{\Delta P}{P} 100} \\ &= \frac{P}{Q}\frac{\Delta Q}{\Delta P}\end{aligned}$$

Let me make one more modification to this formula. I want to ensure that I consider "small" changes in price. Therefore, I will consider small ΔP's (which implies small ΔQ's as well).

Definition 3. (Own price) elasticity of demand at price P, *denoted* $\epsilon(P)$, *for a good is therefore defined as:*

$$\epsilon(P) = \lim_{\Delta P \to 0} \frac{P}{Q}\frac{\Delta Q}{\Delta P}$$

A couple of notes:

- By the law of demand, $\epsilon(P)$ is always negative. This is because P and Q are positive, and ΔP and ΔQ have the opposite sign.
- However, when we talk about about whether an elasticity is large or small, we typically talk about its absolute value. A large absolute value of an elasticity means that quantity demanded is more responsive to changes in price. A small absolute value means that the quantity demanded is less responsive to changes in price.

  Indeed, we classify elasticities based on their absolute value as follows.

If	then we say
$\lvert\epsilon\rvert$ ...	demand is...
< 1	inelastic
> 1	elastic
$= 1$	unit elastic
$= 0$	perfect inelastic
$= \infty$	perfectly elastic

- What is the interpretation of this number? An elasticity of $\epsilon(P) = -3$ means that if the price increases by one percent, the quantity demanded will decrease by approximately three percent. Similarly, if the price decreases by one percent, the quantity demanded will increase by approximately three percent.

  This is extremely useful information for business owners. If you are operating a hot dog stand, you want to know the own price elasticity of demand for your hot dogs. Why? Because it tells you how many customers you will gain if you cut your prices a little bit. Of course, it also tells you how many customers you will lose if you increase your prices a little bit. If you care about maximizing your revenue (quantity demanded times price), you should follow this rule of thumb:

  > "When the absolute value of own price elasticity is less than one, increase the price. When the absolute value of own price elasticity is larger than one, decrease the price."

  This is because if $|\epsilon(P)| = 3$, a one percent reduction in price leads to a three percent increase in quantity demanded. You can sell much more hot dogs by cutting your price, and the net effect on revenue is positive! You should reduce your price. On the other hand, if $|\epsilon(P)| = 0.2$, a one percent increase in price leads to a 0.2 percent reduction in quantity demanded. You can charge a higher price for your hot dogs, and it is true that the quantity demanded is lower, but it is lower by a small amount! You should charge a higher price for your hot dogs.

  Let me just repeat the rule of thumb using the terminology.

  > "If you want to increase your revenue: When the demand is inelastic, increase the price. When demand is elastic, decrease the price."

- You can go one step further and calculate approximately how much you need to increase (or decrease) your price. Suppose you are selling your hot dogs at 9TL per hot dog, and you are selling 50 hot dogs per day. Suppose you hire an economist to study the market you operate in. The economist runs some calculations and tells you that the own price elasticity of hot dogs at the price of 9TL is -5.

  Given this information, if you want to sell 60 hot dogs per day instead of 50, what should be the price approximately be?

  Let's express this in our notation. We have:

$$\begin{aligned} P &= 9 \\ Q &= 50 \\ \epsilon(P) &= -5 \\ Q + \Delta Q &= 60 \end{aligned}$$

  First, note that $\Delta Q = 60 - 50 = 10$. Then, using the formula for elasticity,

$$\begin{aligned} \epsilon(P) \approx \frac{P}{Q}\frac{\Delta Q}{\Delta P} &\implies -5 \approx \frac{9}{50}\frac{10}{\Delta P} \\ &\implies \Delta P \approx \frac{1}{-5}\frac{9}{50}10 = -0.36 \end{aligned}$$

  Therefore, if you reduce your price by 0.36 TL (i.e., sell you hot dogs at $9 - 0.36 = 8.64$TL per hot dog), you will approximately sell 60 hot dogs per day! $P + \Delta P = 9 - 0.36 = 8.64$.

  Intuitively, you want to increase quantity demanded from 50 to 60, which is a 20 percent increase. Because the quantity demanded is five times as responsive to price changes, only a four percent reduction in price is sufficient. This corresponds to a reduction of 0.36TL in price.

6.1 Geometric Interpretation of Elasticity

Let's rearrange the formula:

$$\epsilon(P) = \lim_{\Delta P \to 0} \frac{P}{Q} \frac{1}{\Delta P / \Delta Q}$$

But $\lim_{\Delta P \to 0} \Delta P / \Delta Q$ is the slope of demand curve at P. Therefore,

$$\epsilon(P) = \frac{P}{Q} \frac{1}{\text{slope of demand curve at } P}$$

What does it mean, geometrically?

- If the demand curve is *steeper*, the absolute value of its slope is higher. Thus, the absolute value of elasticity is lower. The demand is *more inelastic!*
- If the demand curve is *flatter*, the absolute value of its slope is lower. Thus, the absolute value of elasticity is higher. The demand is *more elastic!*

Indeed, you can compare the elasticities of two demand curves that pass through the same point just by looking at them. The steeper one is more inelastic. For instance, consider Figure 7. Here, D_1 (the red demand curve) pass through the same point as D_2 (the dark red demand curve). Let s_1 denote the slope of the red demand curve at P, and let s_2 denote the slope of the dark red demand curve at P. Because the red demand curve is steeper, $|s_1| > |s_2|$.

For the red curve, the own price elasticity of demand at price P is:

$$\epsilon_1(P) = \frac{P}{Q} \frac{1}{s_1}$$

and for the dark red curve, the own price elasticity of demand at price P is:

$$\epsilon_2(P) = \frac{P}{Q} \frac{1}{s_2}$$

Because $|s_1| > |s_2|$, $1/|s_1| < 1/|s_2|$ and

$$|\epsilon_1(P)| < |\epsilon_2(P)|$$

so the red curve is more inelastic (or less elastic, or have lower elasticity, or its elasticity is low, depending on how you want to call it) at price P. It actually makes a lot of sense: the dark red curve is more responsive to changes in price than the red curve. For instance, consider a reduction in price from P to P_1. If the demand curve is D_1, the quantity demanded increases from Q to Q_1: a modest increase. But if the demand curve is D_2, the quantity demanded increases from Q to Q_2: a larger response to the same reduction in price.

NOTE: this comparison is only about the relative magnitudes. For instance, both D_1 and D_2 may be elastic at P (i.e., we may have $|\epsilon_1(P)| > 1$ and $|\epsilon_2(P)| > 1$). Still, D_1 is more inelastic than D_2 at price P.

So, overall, flatter curves are more elastic. Two extreme cases:

- If the demand is perfectly elastic, the slope of demand curve is zero: it is a horizontal line. In this case, if the price increases by a tiny bit, the quantity demanded reduces all the way to zero.
- If the demand is perfectly inelastic, the slope of demand curve is infinity: it is a vertical line. In this case, no matter what the price is, the quantity demanded is the same. (No responsiveness at all.)

6.2 Determinants of the Price Elasticity of Demand

What makes a demand more elastic or less elastic? In other words, under what conditions are quantities demanded more responsive to price changes? Here are some characteristics of the good and the market that play a role in the determInation of elasticity.

1. **Availability of Close Substitutes:** If a good has many substitutes available, the demand for the good will be more elastic.

   Intuitively, when there are many substitutes available, the consumer would responds to a small increase in prices by switching to alternatives, and reducing the quantity demanded a lot.

   Example: consider the demand for regular car tires manufactured by Pirelli. There are many alternative brands of car tires, so the demand is very elastic. In contrast, Pirelli is the sole producer of Formula 1 car tires. Therefore, the demand for Formula 1 car tires manufactured by Pirelli is very inelastic.

2. **Time:** If the time frame is longer, the demand will be more elastic.

   This is because over time, people can adjust their purchasing habits more easily. This results in a higher ability to respond to price changes over the long run.

   Example: the demand for gasoline is quite inelastic in the short run. If the price for gasoline decreases, people will start driving their cars more, but not that much. Yet, over a longer time frame, consumers can buy their own cars etc. so the response to a decrease in gasoline price will be stronger.

3. **Luxuries versus Necessities:** If a good is a necessity, the demand will be more inelastic.

   Intuitively, if the good is a necessity, consumers have to buy it no matter what the price is. If it is a luxury, we can live without buying it. This gives the ability to respond to price changes.

   Example: the demand for certain medicines is quite inelastic (a diabetes patient has to use insulin no matter what the price is). In contrast, the demand for daily vitamin supplements is more elastic.

4. **Definition of the Market:** If the market is more narrowly defined, the demand will be more elastic.

   Narrowly-defined markets include particular brands, particular locations etc. so that consumer have alternatives to switch.

   Example: the demand for Recep Ivedik movies in a particular theater is quite elastic, because consumer can just go to the next theater to watch the same movie. The demand for Recep Ivedik movies overall in the country is more inelastic. The demand for movie tickets overall in the country is even more inelastic.

   Example: The demand for Komili Olive Oil is quite elastic. The demand for Olive Oil is more inelastic.

5. **Share of the Good on a Consumer's Budget:** If the expenditure on a good constitutes a small portion of the consumers' income, the demand will be more inelastic.

   This is because when the consumer is spending only a small amount of money on a good, she will not feel the difference much and will not respond to price changes.

   Example: demand for table salt is inelastic compared to the demand for furniture.

Figure 8 is a table of estimated own price elasticities of some goods, taken from a blog post. You should visit the source at http://econbeh.blogspot.com/2019/05/some-estimates-of-price-elasticity-of.html, and the references there, if you are interested in finding out how these numbers are calculated.

6.3 Changes in Elasticity Along a Demand Curve

As it turns out, the elasticity along a curve is **not** constant even along the same demand curve. This may be because the slope of the curve changes as we move along the curve. But even when the slope is constant (i.e., even when the curve is linear), the elasticity changes because $\frac{P}{Q}$ changes. For higher values of P (and, due to the law of demand, for lower values of Q), the elasticity is higher. As P increases (and, due to the law of demand, Q decreases) along a linear demand curve, the elasticity decreases. The figure below illustrates the elasticities at different points at the same demand curve, whose equation is $Q = 12 - 2P$.

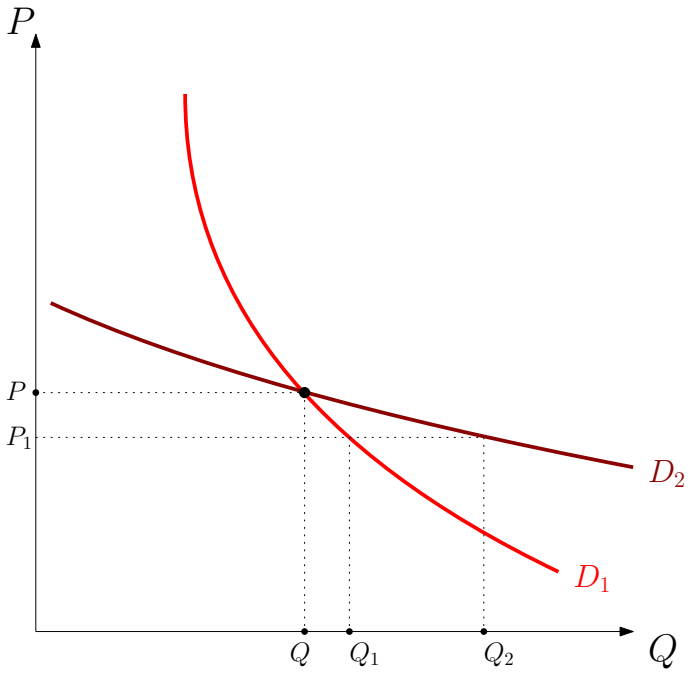
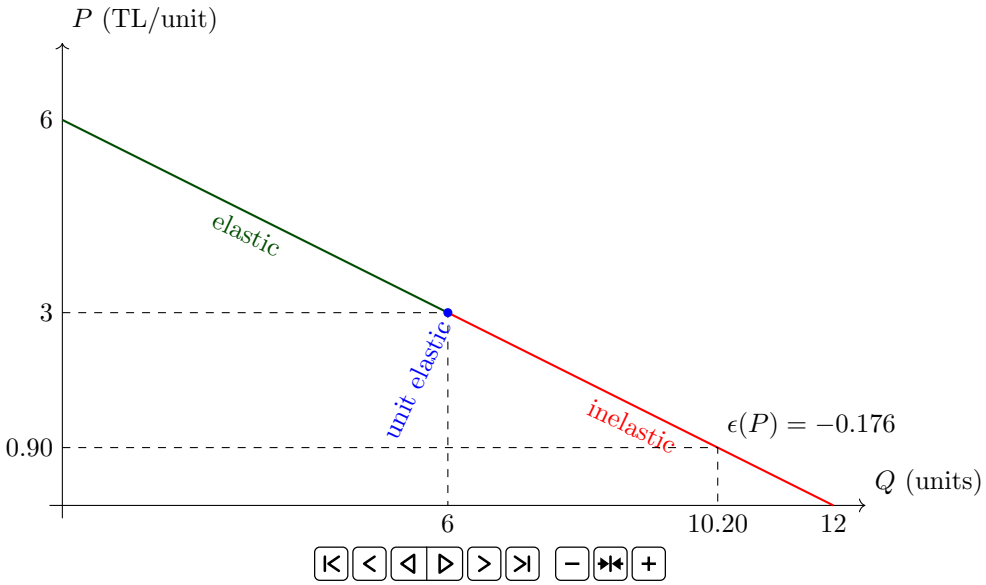


Figure 7: Two demand curves, D_1 and D_2, pass through the same point at price P. Because D_1 is steeper than D_2 at this point, it is more inelastic (less elastic) than D_2 at price P.

Google Play apps	3.7
Cinema	2.5
Apple App Stpre	2
Organic milk	1.8
Air travel for non business	1.5
Vacations	0.8
Beef	0.8
Premier League football	0.7
Fruit	0.7
Air travel business	0.7
Opera	0.7
Milk	0.6
Public transport	0.4
Broadband	0.4
Wine	0.3
Theatre	0.3
Beer	0.2
Car fuel	0.2

Figure 8: (Absolute values of) own price elasticities of some goods. Source: http://econbeh.blogspot.com/2019/05/some-estimates-of-price-elasticity-of.html



Now, combine what you see in this figure with the rule of thumb for the revenue maximization we derived before. If $P > 3$, demand is elastic: decrease the price. If $P < 3$, demand is inelastic: increase the price. It therefore turns out that the price that maximizes revenue is $P = 3$. At this price, $Q = 6$ units are sold and the total revenue is $3 \times 6 = 18$.[4]

Now, this analysis assumes that the seller is interested in maximizing the revenue. Most of the time, the sellers are interested in maximizing **profit** (revenue minus costs), not profit. We will get to profit maximization in the next lecture.

7 Other Demand Elasticities

We can use the "percentage change divided by percentage change" idea to define other measures of responsiveness. Here are some examples.

7.1 Cross Price Elasticity

The **cross-price elasticity** of demand for good i with respect to (the price of) good j is a measure of the responsiveness of the demand for good i to changes in the price of good j.

More formally, the **cross price elasticity** of demand for good i with respect to (the price of) good j, denoted $\epsilon_{i,j}$, is the rate at which the *percentage change in quantity of good i demanded* changes in response to a *percentage change in the price of good j* resulting from a small change in the price of good j.

$$\epsilon_{i,j} = \lim_{\Delta P_j \to 0} \frac{\frac{\Delta Q_i}{Q_i} \cdot 100}{\frac{\Delta P_j}{P_j} \cdot 100} \ .$$

- If good i is a substitute for good j (at the current prices and income level), then the cross price elasticity of good i with respect to good j is positive, i.e.,

$$\epsilon_{i,j} > 0 \ .$$

[4] You could have found this answer by writing down the revenue as $P \cdot Q = P \cdot (12 - 2P)$ and finding the P that maximizes this. But isn't the heuristic more insightful?

- If good i is a complement for good j (at the current prices and income level), then the cross price elasticity of good i with respect to good j is negative, i.e.,

$$\epsilon_{i,j} < 0 \ .$$

7.2 Income Elasticity

Income elasticity of demand is a measure of responsiveness of demand to changes in income.

More formally, the **Income elasticity of demand**, denoted ϵ_I, is the rate at which the *percentage change in quantity of the good demanded* changes in response to a *percentage change in the income* of the consumer resulting from a "small" change her income.

$$\epsilon_I = \lim_{\Delta I \to 0} \frac{\frac{\Delta Q}{Q} \cdot 100}{\frac{\Delta I}{I} \cdot 100} \ .$$

- If the good is an **inferior** good (at the current prices and income level), then the income elasticity of the good is negative, i.e.,

$$\epsilon_I < 0 \ .$$

- If the good is a **normal** good (at the current prices and income level), then the income elasticity of the good is positive, i.e.,

$$\epsilon_I > 0 \ .$$

- If a normal good is a **luxury** good, then the income elasticity of the good is greater than 1, i.e.,

$$\epsilon_I > 1 \ .$$

- If a normal good is **necessity** good, then the income elasticity of the good is less than 1, i.e.,

$$0 < \epsilon_I < 1 \ .$$