Bilkent University
Introduction to Microeconomics
Lecture Notes

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Econ 101: Introduction to Microeconomics - Lecture Notes

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Contents

Chapter 1

Introduction

1.1 What is Economics?

Every answer to this question, in one way or another, refers to

scarcity

or

decisions/choices

(Sometimes there is some reference to "allocation/production/distribution/consumption..." These are merely examples of *decisions* made by society/producers/consumers. We should still keep in mind that *all* forms of entities make choices, not just producers and consumers!)

(Sometimes there is some reference to "needs/preferences/desires..." We will get to that in a second.)

Going back... What is the deal with scarcity and decisions?

- Is it possible to have scarcity without decisions?

  Yes, but that wouldn't be an interesting problem to study, that would just be depressing...

- Is it possible to have decisions without scarcity?

  Yes, but that wouldn't be an interesting problem either. Suppose you are studying a decision without scarcity: you can have anything you want. What is the fun of it? There is no real "choice" being made.

> Besides, it's not even realistic! No one can have it all (even Jeff Bezos cannot buy, or do, anything he wants). Everyone is subject to *some* sort of scarcity.

So, I would argue that *scarcity* and *decisions* are closely related to each other. Maybe using both of them in a definition is an overkill.

Let me give my personal definition:

> "Economics is the study of decisions made by economic agents, and how these decisions interact with each other."

I know it is slightly self-referential ("Economics is the science that studies economic agents"). And it does not include scarcity. But hang on! I will embed scarcity in the definition of an "economic agent" and solve both problems.

1.1.1 Economic Agents

An economic agent is a decision maker who faces a **scarcity constraint**. This may be:

- A customer in a supermarket who is deciding whether to buy milk or ayran, and how much to buy. She has a budget constraint, and therefore she cannot buy them all.
- A student who is in the library a night before her economics and sociology exams. She has limited time, and therefore she cannot study them both.
- A driver who is choosing whether to listen to radio or a podcast during traffic. She has limited attention, and therefore she cannot listen to them at the same time.
- A political party who is deciding whether to promise more social services or lower taxes before the election. The two promises are incompatible with each other, and therefore not credible if offered at the same time.

As you see, when the definition of economics includes "people", it actually means something broader than human beings. An economic agent can be a person, an organization, an institution, a state... This is why I prefer to use "agents" rather than "people".

The emphasis on scarcity is a crucial part of the economic analysis. For this reason,[1] economics is sometimes referred as "dismal science". I just want to emphasize that

[1] Okay, actually not for this reason. It turns out there is a depressing story behind the coining of the term. You can read it through here: `https://www.theatlantic.com/business/archive/2013/12/why-economics-is-really-called-the-dismal-science/282454/`. I personally don't think there is anything dismal about economics, but you are entitled to your own opinion.

scarcity captures the broad idea that "the agent cannot have it all." This may be due to a monetary constraint, a physical constraint, or simply because the two options are incompatible with each other.

1.1.2 Preferences

Okay, so we study agents who have to make decisions under a scarcity constraint. How do these agents make their decisions? Answer: according to their **preferences**.

- The customer in supermarket chooses based on whether she likes the taste of milk more, whether she cares about the health effects, what the price of milk and ayran are, what she is eating... These factors, collectively, define her preferences towards milk versus ayran.
- The student in library chooses based on which class she likes more, what her existing grades are, what her future plans are...
- The driver in traffic chooses which broadcast she enjoys more, whether she already has heard about the songs/podcast topic...
- The political party evaluates its election prospects, the feasibility of different policy options, future elections...

The bottom line of these examples is: Preferences include more than "what the economic agent likes". They can include costs and benefits as well as social norms, ethical concerns, consistency with past behavior, future considerations, etc. So, when we say "an economic agent acts according to her preferences", we do *not* necessarily mean that the agent is acting selfishly!

Specification of an economic agent's preferences is an essential premise of the economic analysis. We cannot do economic analysis without specifying the preferences (and we don't want to.) **An economic agent is defined by her preferences.** This aspect of the analysis is what distinguishes economics from other social sciences, such as sociology.[2]

An Aside on "Rational" Decision-Making To recap: every economic agent faces a scarcity constraint, and they make decisions according to their preferences. In other words, an economic agent chooses the option they prefer the most, among the options that are available. We call such behavior the **rational** behavior.

[2] I am not trying to claim that one method is better than the other, I'm merely trying to draw some distinctions.

I would like to pause here and take a deep breath. This naming choice has admittedly created a bit of a PR crisis for economics. This is because the word *rational* has a negative connotation: in colloquial language, it is used to mean *selfish.* So, when we say "an economic agent behaves rationally", it may sound like we believe an economic agent behaves selfishly. In reality, this is not what we mean by rationality. Let me repeat: **WE DO NOT ASSUME THAT PEOPLE ARE SELFISH.** We just assume they act according to some preferences, which may or may not contain selfish interests. Economic agents may act based on purely altruistic motives! As long as these motives correspond to their preferences, such agents are acting rationally.

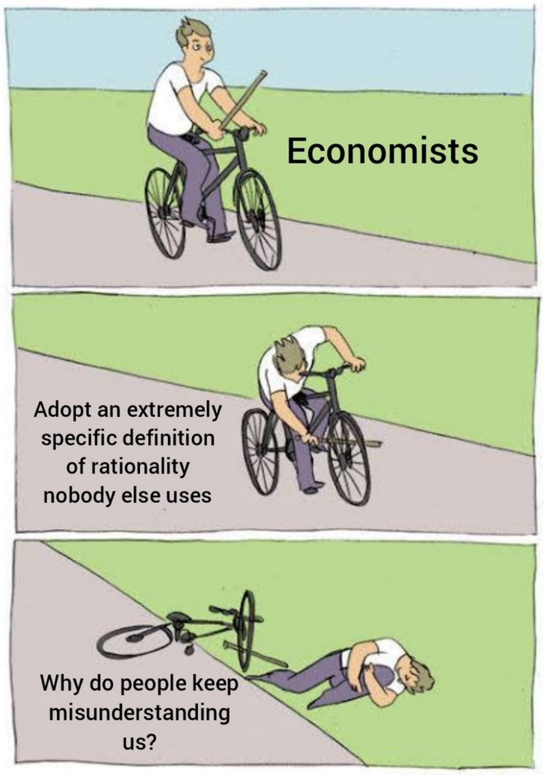


Figure 10.1: :(

Should we have called "acting according to preferences" something other than *rationality*? Perhaps so. Still, this unfortunate choice we made has a silver lining. Sometimes you hear people saying "Well, economics assumes that people are rational, but in reality people are irrational, so it is worthless." If you hear someone genuinely saying this sentence, you can immediately conclude that they have not received a good economics education. (Anyone with a good economics education would have had the discussion we just had.) So, we've got that this going on for us. Which is nice.

One of my objectives for this class is to prevent you from saying "Well, economics assumes that people are rational, but in reality people are irrational, so it is worthless." I cannot emphasize this more: never EVER EVER EVER say this.

Suppose you hear someone making this unsubstantiated claim. There are at least three answers I can come up with on the spot. You can borrow these answers and use at your own discretion.

a. A "well-defined preference" just means that people's preferences follow some broad, regular, somewhat predictable patterns. It provides consistent answers to questions such as: how much milk individuals buy if milk was 40 TL and ayran was 50 TL? What about 60 TL vs 80 TL?... We simply need to have such regularity to be able to conduct economic analysis. If we don't assume people make their purchasing decisions in a broadly consistent way, we cannot even begin reasoning about the milk market! We would just say "well, tomorrow milk producers may be crazy rich and or bankrupt, I can't really know". But then, the whole discipline is worthless.[3] I guess what I am saying is: the scientific method of economics forces us to assume some regularity in a broad manner.

b. On a more philosophical point: What are "well-defined preferences" anyway? These are some premise of the model that are consistent with the observed behavior of economic agents. Suppose I observe you buying Chai Tea Latte for 200 TL every morning. Can I conclude that your preferences just include having Chai Tea Latte for a price of 200 TL every morning? Maybe not, maybe you are just flirting with the barista at the coffee shop and using Chai Tea Latte as an excuse to chat with the barista. Nevertheless, I can still write an economical model that specifies your preferences over Chai Tea Latte, and that model would be consistent with your observed behavior. The baseline assumption of that model would be wrong, of course, but that's okay from a scientific point of view.

   > "All models are wrong, but some are useful."
   > - Statistician George Box.

   Why is such a model still useful? To this model, I can ask a question such as: "How much Chai Tea Latte would be sold if there was a worldwide cinnamon shortage?" If my baseline assumption was wrong, it would produce the wrong answer. But that's fine – if there was an actual cinnamon shortage (hopefully not anytime soon) and if my predictions were wrong, I can easily say "Hmm, I

[3] At this point, your hypothetical friend (who does not know about economics) may resort to claiming that economics is, after all, worthless. This is the same thing as arguing that nothing useful will come out of understanding the milk market. This debate is much easier to win, compared to a debate about rationality. I trust your argumentation skills. :)

guess I had the preferences wrong. Let me think about other preferences that can explain the new outcome I am seeing." This is a healthy thing – this is how science proceeds. Falsifiability FTW! The nice thing about regular preferences is that: it allows me to check what part of my model was wrong and evaluate my model based on the data.

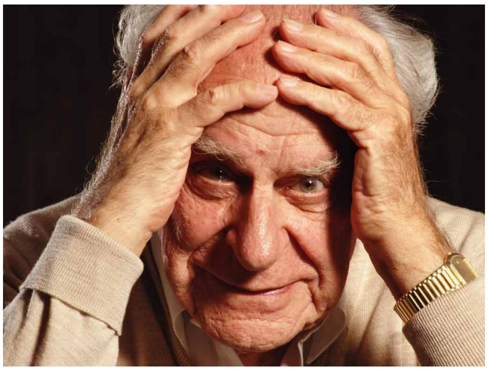


Figure 12.1: Karl Popper after seeing you stop buying Chai Tea Latte once the barista quits their job.

What I am getting at is: when we say "people are rational", it is not due to the belief that every individual has a well-defined set of principles that they follow. It is, rather, due to a requirement that we need to have a model with falsifiable predictions. Every prediction requires some regularity (i.e., some logical steps in producing its results), and "rationality" is an umbrella term we use to contain such regularity.

c. Finally, do people sometimes act in inconsistent, unpredictable ways? Of course. There is a whole field of economics, called **behavioral economics**, devoted to understanding such inconsistencies.[4] Are these inconsistencies actually consistent with something we did not know was relevant? Can we come up with a framework to understand such behavior? How can we represent it, quantify it? These are the questions behavioral economics have been asking for years. Because we are in an introductory-level class, we will not have time to cover behavior economics, and keep the "rational" agents as a benchmark. Think of it as Newtonian physics: we will start by assuming that gravity is the only force of nature. Once we understand how gravity works, we can move on to nuclear physics and electromagnetics. But let's nail this down first and use it as a benchmark. This is Econ 101, and for centuries economist have been working on things we cannot squeeze into a semester. :)

[4]Maybe you have read some books by Daniel Kahneman, Amos Tversky, or Richard Thaler?

1.1.3 Scarcity and Optimization

So far so good: Every economic agent faces a scarcity constraint and tries to act in a way that aligns with her preferences. Here is a paraphrasing of what I just said for the more math-minded: **Every economic agent solves a constrained optimization problem.** Indeed, every single economics model is a constrained optimization problem of some kind. In this problem, the objective function is derived from the preferences, and the constraints are due to scarcity.

Example 13.1 Consider a student who has 8 hours before an econ exam, and she decides on how much to study for the exam. Let us try to express and analyze this decision in an economic model.

We consider the student as an economic agent. This means the student is defined by two things: (1) her constraint, and (2) her preferences. The student chooses the best option according to her preferences, given her constraint. In math terms, the student chooses some hours x, which is a number. Here, x is the number of hours the students spends working on the econ exam.

1. The student's **constraint** is that she cannot work for negative hours, and she cannot work for more than 8 hours. In math terms, her constraint is:

   $$x \in [0, 8].$$

2. What about the student's **preferences**? For the sake of the example, let us assume that the only thing the student cares about is her grade. She wants to have a high grade. This means: the student will choose the x that will give her the highest grade, given her constraint.

   How is the student's grade determined by x? For the sake of the example, let us assume that if the student works for x hours, she receives a grade $g(x)$ from the exam, where:

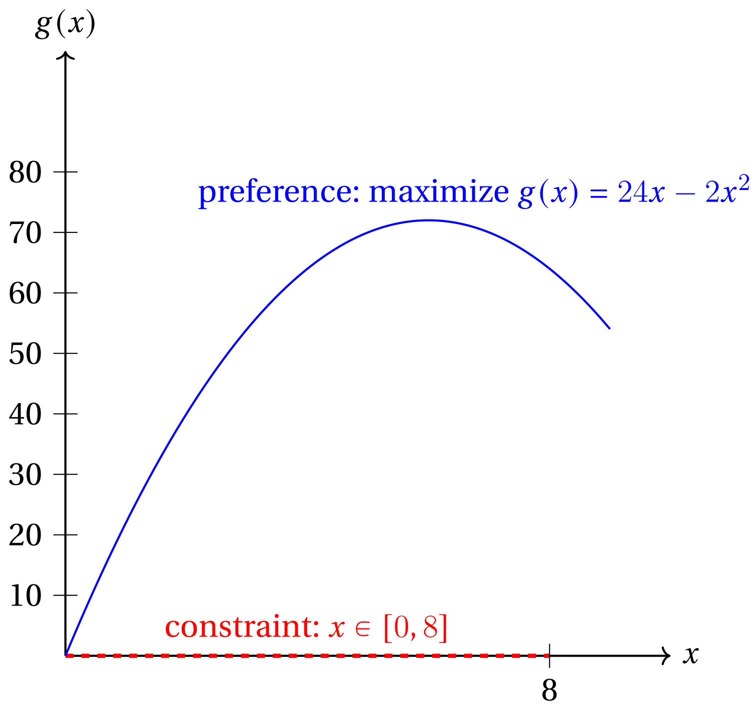
   $$g(x) = 24x - 2x^2$$

   Note that, in this example, $g(x)$ is decreasing in x if x is close to 8. This is meant to capture that if the student works on the exam too much she will have too little sleep, which will affect her performance negatively.

All in all, the student chooses some x to maximize $g(x) = 24x - x^2$, given the constraint that $x \in [0, 8]$. In other words, the student solves the constrained optimization problem:

$$\max_{x \in [0,8]} g(x)$$

Graphically:



How would you find the student's optimal hours (i.e., the most preferred hours by the student, given her constraint)? You can eyeball the graph above, or you can be a little bit more methodological: set the derivative of g equal to zero and find that $x^* = 6$.

You can see that at $x^* = 6$, the derivative of g is equal to zero. In other words, the function g is not decreasing in x around $x^* = 6$, it is also not increasing in x around $x^* = 6$. That is, if you change x *just a little bit* around $x^* = 6$, $g(x)$ would not change in a meaningful way.

It is good to keep the heuristics of taking derivatives in mind. Sometimes we get too methodological and forget the meaning of what we are doing. So let's remember: What is taking a derivative of a function with respect to x and setting it equal to zero? It is: finding the values of x such that *small changes* around x does not change the function's value. So, whenever you take a derivative, you are thinking in terms of *small changes*. And thinking in terms of small changes is a very useful mindset, because it allows us to solve constrained optimization problems. It is such a useful mindset that economists give it a name: *thinking at the margin*.

1.1.4 Thinking at the Margin

In late 19th century, economics as a discipline went through what we call "the marginal revolution". This refers to the idea that economic agents make their decision at the margin. **Marginal thinking** is a particular mindset economic agents use when they make their optimization decisions. A margin is where you are at with the decisions you have already made (it is an "edge" or a "border" than defines your current boundaries). Thinking at the margin is imagining what would happen if you made a small adjustment to your action (i.e., imagining what would happen if you extended your boundaries just a little bit). That is, **marginal thinking** is **thinking in terms of small increments and adjustments**.

Example 15.1 Suppose a student has been working for the economics exam for six hours (her margin). Now she is thinking: "Should I study for an additional hour? An additional minute? What is the extra benefit (the **marginal benefit**) of working for the additional minute? What is the extra cost (the **marginal cost**)? If the marginal benefit is larger than the marginal cost, I will study for the extra minute. Otherwise, I will stop studying." It looks like she is already thinking at the margin – a sign that she will receive a high grade in the exam. :)

Once you think about it, this is basically taking a derivative (i.e. thinking about the marginal change) when solving an optimization problem. This is not really surprising: economic agents solve optimization problems, and taking a derivative is a method (or a mindset) of solving that problem.

One thing I want to emphasize: thinking at the margin is nothing more than a method of solving an optimization problem, there is nothing so deep about it. But on the surface, it is different from "absolute thinking." A student may love economics, but she may be working for the exam for eight hours already, which means she is very sleepy and the marginal cost of studying is very high. At the margin, she prefers sleeping. It doesn't necessarily mean that she prefers sleeping to economics in general! Marginal thinking suggests that we need to pay attention to the existing conditions.

Example 15.2 You may strongly prefer Zeynep Bastık to Müslüm Gürses, but after listening to the same Zeynep Bastık song over and over 80 times, you may say "Hmm, maybe I'll listen to a Müslüm Gürses song this time." This means, at the margin, you prefer Müslüm Gürses over Zeynep Bastık. At another margin, your preferences maybe be otherwise.

1.1.5 Equilibrium

As it turns out, economic agents do not live in a vacuum. They each make their decisions, and more importantly, these decisions interact with each other. But what happens when multiple agents engage in an economic interaction? We need some discipline to analyze and predict how these interactions play out. The concept we use to impose this discipline on our analysis is called **equilibrium**.

Broadly speaking, equilibrium is defined as **the situation in which every agent is optimizing, so nobody would benefit personally by changing her behavior, given the choices of other agents**.

I want to emphasize two things.

First: there is nothing desirable or undesirable about equilibrium *per se*. When we are looking for equilibrium, we are not making any value judgments – equilibrium is just a concept we use to analyze economic interactions. Sometimes you will see equilibrium giving the best outcome for everyone, and sometimes the worst outcome – it really depends on the nature of interactions.

Example 16.1 Suppose you are meeting a friend at noon on a Sunday, and you are deciding where to meet. Let's express this in the language of economics.

You are an economic agent. Your constraint is that you can only be at a single location at noon on Sunday. For the sake of the example, let's say that your preferences are such that: you would prefer to be at a location where your friend is, rather than a location where your friend is not. Your friend is also an economic agent; she has the same constraint as you, and let's say she has the same preferences as you.

What is the **equilibrium** in this economic interaction? Let me postulate that the situation where you meet at your friend's house is an equilibrium. Why? Given that your friend will be at her house, you do not want to change your behavior (i.e., you do not want to be anywhere else), because your preferences are such that you want to be where your friend is. Similarly, your friend does not want to change her behavior.

By the same token, meeting at your house is also an equilibrium. Meeting at Güvenpark is also an equilibrium. Meeting in front of Sincan Municipality is also an equilibrium. Come to think of it, any situation where you and your friend go to the same location is an equilibrium. There are infinitely many equilibria of this economic interaction!

Note that some of the equilibria of this game (e.g., the one where you meet at your friend's house) is more preferred by your friend. Some other equilibria (e.g., the one where you meet at your house) is more preferred by you, and less preferred by your friend. Some others (e.g., meeting at Güvenpark) is a compromise. Some other (e.g., meeting in front of Sincan Municipality) is a compromise that neither of you may like. Nevertheless, it is still an equilibrium.

Second: Equilibrium is a benchmark we use, not necessarily the actual description of the outcome. Sometimes economic systems are not in equilibrium. Sometimes it takes a while to find the equilibrium. Yet, we have an inclination to believe that an economic system *tends* towards an equilibrium.

Example 17.1 Think about the lines in the toll gates at the entrance of a highway. Suppose each driver has to pass through the highway and choose a line (her constraint) and each driver wants to minimize the waiting time (her preferences).

What is the equilibrium of this interaction, i.e., who stays at which line? Think about it a bit and you will realize: If any one of the lines are shorter than the other, you would expect the drivers to switch to that line, until that is not shorter any more. That is, in equilibrium, you would expect the lines to be equally long.

Now, if you have any experience with situations like these, you would notice that this is *approximately* true in reality: the lines are more or less equal, but not always exactly equal. Yet, they have a tendency to equalize, especially over long time intervals. Expecting them to be equal in the benchmark is, therefore, a useful starting point if you want to analyze toll lines.

Let me tie back with a further elaboration of this very simple example, to show how economic analysis is useful. Suppose you are working for the highways department, and someone comes and asks you: "We have 4 toll gates in this highway, and the average waiting time is 5 minutes. If we open an extra toll gate, what would the average waiting time be?" How would you start thinking about this question? First, you need to come up with some assumptions about how people behave (i.e., their preferences and constraints). "Minimizing the waiting time, given the number of toll gates" seems like a reasonable starting point. Second, you need to make some assumptions about how they interact. Equilibrium, as argued above, seems like a good benchmark. Given these assumptions, you can now calculate the expected waiting time with 5 toll gates. (4 minutes?) Voilà! You have set up your first economic model and made your first analysis! Now, if someone comes up and says "But what if we also increase the price of the highway?", you need to make further assumptions about how much people are willing to pay for the highway, how much they are willing to wait, etc. This class will give you some tools to get started with this kind of thinking.

1.1.6 Positive versus Normative Analysis

Now that we have talked about economic analysis, it is a good time to make the following distinction. There are two types of analysis you can conduct using economic models.

- **Positive analysis** is concerned with **what economic agents do**.
- **Normative analysis** is concerned with **what economic agents, including society,** *should* **do**.

Here is an example of a positive statement:

> "If you raise the minimum wage by ten percent, unemployment will raise by three percent."

This is just a statement about the relationship of two variables. It does not make any value judgments. In contrast, here is an example of a normative statement:

> "Therefore, you should not raise the minimum wage."

Why is this a normative statement? Because it is making a value judgment. When the government increases the minimum wage, some people will lose their jobs. Some firms will lose money. On the other hand, some people will be working for higher wages. This statement implicitly says that the losses (unemployed people, less profits) outweigh the benefits (higher wages for those not losing their jobs.) Therefore, it makes a comparison of what the society cares, or should care, more about. This is a question of **ethics**, a subfield of philosophy, and not **economics**.

It is, of course, extremely important to be knowledgeable about ethics. It is a fascinating topic, and every policy maker should have the necessary tools to conduct ethical analysis. All economists should know very well about ethics too. I am just trying to emphasize that it is important to distinguish positive statements from normative statements.

In economics, we try to conduct positive analysis as much as we can. Nevertheless, the line between positive and normative sometimes becomes blurry. Occasionally we refer to concepts such as "efficiency" and "social welfare", which carry a bit of normative reasoning in it. It is good practice to occasionally think about whether out statements carry value judgments.

1.2 Trade-Offs and Opportunity Cost

Read, literally, Chapter 1 any economic textbook to get a grasp of these concepts.

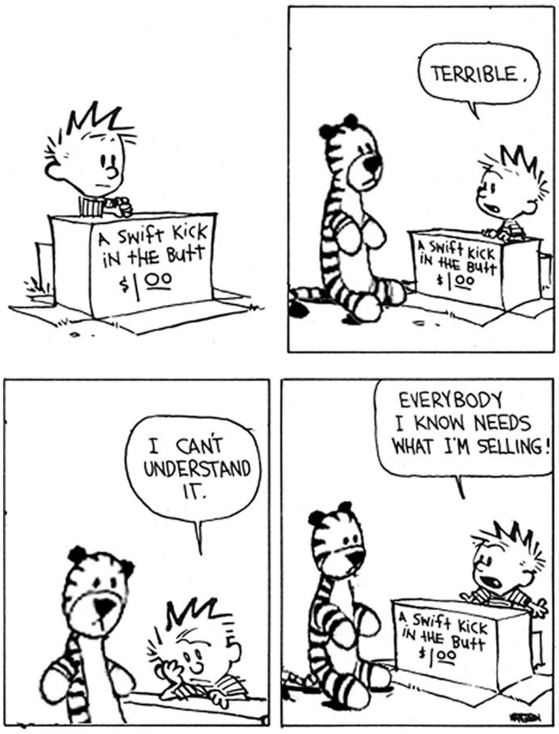


Figure 19.1: Calvin is confused about positive (what people want) and normative (what people *should* want). Don't be like Calvin.

- **Trade-off:** the idea that, because of scarcity, an economic agent needs to give up an something to get something else.
- **Opportunity cost:** The best alternative you give up.

Example 19.1 A student spends the night studying for the econ exam. If she wasn't, she might have been spending time with her friends during that time. Or she might be studying for the sociology exam instead. Therefore, she is calculating what she needs to give up (time spent with friends, better knowledge of sociology, a better grade in the sociology exam) in order to get something else (better knowledge of economics, a better grade in the economics exam.) That is, she is facing a **trade-off**.

So, what is the **opportunity cost** of studying for the economics exam? It depends on what she would be doing in that time instead (remember that she is rational, so she would be spending her time doing the best alternative).

- If her best alternative was spending time with friends, the opportunity cost of studying for the economics exam is the time spent with friends.

- If her best alternative was studying for the sociology exam, the opportunity cost of studying for the economics exam is a better better knowledge of sociology and a better grade in the sociology exam.

At this point, you may be tempted to ask: "Why don't you just say *cost* instead?" The answer is: we want to emphasize that the opportunity cost is different than the explicit financial cost of getting something. The opportunity costs includes the alternatives, and the best use of those alternatives. Here is a standard example: suppose you graduated from college (congratulations!) and received a job offer that pays 500,000 TL per year (well done!). Suppose taking that job was your best alternative, but you opted to pursue a masters degree instead. The masters degree is a one-year program and it costs 800,000 TL. Someone might come up and ask: what is the cost of the masters degree? The standard answer would be: 800,000 TL, and that's not a wrong answer but it is missing the opportunity cost. An economist would say: "The explicit financial cost of the masters degree may be 800,000 TL. But the opportunity cost of the degree includes what you give up by pursuing that degree instead! Remember that you would take a job that would pay you 500,000 TL for the year you would be studying. Therefore, the opportunity cost of pursuing a masters degree is at least 500,000 TL + 800,000 TL = 1,300,000 TL."[5]

Figure 20.1: Kibar Feyzo's mom knows about opportunity costs ("We could buy an ox with that money!").

[5] I am saying *at least*, because this calculation misses some things. **First**, there is the job experience you would gain, the workplace friendships you would have etc. by taking the job. The opportunity cost includes those as well. **Second**, this calculation misses the best use of the 800,000 TL. Suppose if you didn't pay 800,000 TL for a masters degree, you would use it for a week-long vacation in Ibiza (your best alternative use of 800,000 TL.) In that case, the opportunity cost of a masters degree is a week in Ibiza + 500,000 TL.

1.3 On the Role of Math and Models in Economic Analysis

1.3.1 Why Model?

The economic methodology is based on the construction of **models** (in all fairness, every scientific methodology uses models.) A model consists of a group of economic agents, their preferences and constraints, and the environment they interact in. It is made for the purpose of identifying some economic forces ("In such and such environments, what happens to waiting time if I add another toll gate to the highway?"). It is based on some assumptions and, inevitably, an abstraction of reality: it includes the important bits and pieces for the purposes of the question, and rules out some of them. Think of a physics model that assumes the air frictions away because it is interested in other forces. Or an architectural model of a building that doesn't show the tiles on the floor, because it is interested in the structure of the building, not the interior design. Or a subway map. Or: See Figure 21.1.

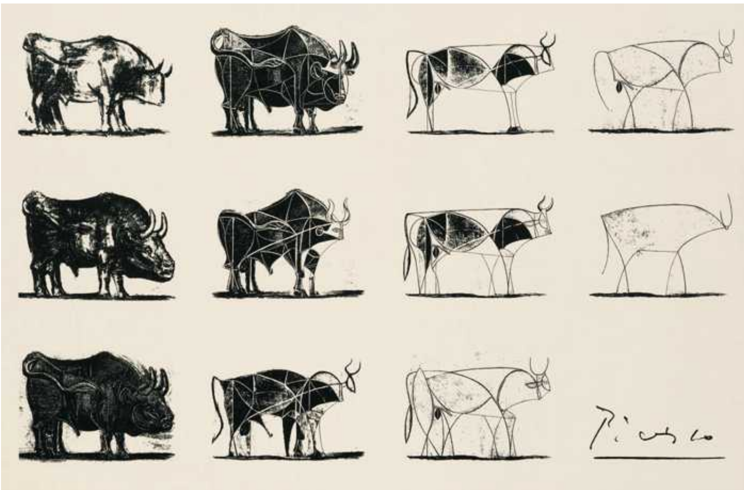


Figure 21.1: Picasso's "model" of a bull, 1945.

Because a model consists of some simplifying assumptions, it is always *wrong*. But, remembering George Box once again, it is *useful* most of the time. Let me give you a reason why.[6] Jose Luis Borges has a short story titled "Funes the Memorious". In that story, a teenage boy falls of a horse and acquires the amazing talent of remembering *everything*. For instance, he remembers every horse he encounters in every single detail. It soon becomes obvious that this talent is actually a curse, because it renders the boy incapable of engaging in *categorical reasoning*. That is, because every horse

[6] Credit to Ivan Werning for bringing this in the collective memory of my classmates.

is different for the child, he cannot group the horses he encounters into a category of "horse". Instead, for him, every single horse means a different category.

You can imagine that such a memory is, after all, useless, because it doesn't allow the person to draw parallels between events. Our purpose while setting up an economic model is precisely this: allowing us to draw parallels between events, drawing some conclusions, having a general understanding about how a market works. Because of this, we will sometimes skip the details of the horse and call it a "horse". This is one reason why we use abstractions of reality, i.e. models.

The idea of models being abstractions of reality will be especially important for this class. Because this is an introductory class, the first set of models we will be setting are going to be extremely simple: they will rule out a lot of stuff. They will be added in your future courses; we are building a base here and just scratching the surface.

1.3.2 Why Math?

Another important purpose of a model, which a lot of textbooks (including yours) emphasize, is to be able to make predictions (also called hypotheses). These predictions can then be taken to the data and we can check if the model holds out well in face of reality (also called hypothesis testing). For instance, we can set up a model of labor market and ask the question of: "What happens if we increase the minimum wage by ten percent?" The model will include workers and their preferences, firms, the market structure etc. It will give an answer like: "The unemployment will rise by three percent." We can then go check the data on the minimum wage and unemployment rates on different locations, compare them, and see whether this prediction is true.

Because all models consist of assumptions, and because the predictions of the model will be extremely important, we must have a discipline to **figure out which assumption causes which prediction**. That discipline is established through **mathematics**. Math is the language that disciplines our thinking and allows us to be rigorous in our analysis. Thanks to the reliance on math, we can say "Let me change this assumption on worker behavior and see how my predictions change." Without the logical consistency provided by mathematics, seeing these connections would be much more difficult.

1.4 Some Other Interesting Things

1.4.1 Subfields of Economics

Traditionally, economics has been thought as having three subfields.

a. **Microeconomics** is the branch of economics that studies smaller units of the economy (consumers, firms, markets). In this class, we will study microeconomics. These ideas will be explored further in Econ 203 and Econ 204.

   Some subfields of microeconomics are:

   - Game theory (Econ 439): study of how agents interact in strategic environments.
   - Contract theory (Econ 448): study of how information asymmetries play a role in economic interactions.
   - Labor economics (Econ 458): study of labor markets.
   - Industrial organization (Econ 433): study of market structures and competition.

b. **Macroeconomics** is the branch of economics that studies the economy as a whole. It studies topics such as inflation, unemployment, and growth. These ideas will be explored in Econ 102, Econ 205 and Econ 206.

   Some subfields of macroeconomics are:

   - Monetary theory (Econ 322)
   - International trade (Econ 331)
   - Growth (Econ 453)

c. **Econometrics** is the branch of economics that focuses in testing the hypotheses brought by economic models. It relies heavily on probability and statistics. Econ 221, Econ 222 and Econ 301 are the courses that will give you the tools.

I have to say, personally, that I find this taxonomy a little unsatisfactory. There is, for instance, a huge field of economics called **public economics** which deals with taxation, redistribution and related matters. Is it micro? Is it macro? Econometrics? I'd say a little bit of all. Same goes with **labor economics**. If it was up to me, I would categorize on the methods they use, and not based on the questions they ask.

Extra Readings for Chapter 1

For a brilliant discussion on how the definition of economics changed over time (moving from subject-matter based definitions, to methodology-based definitions), see:

> Backhouse, Roger E., and Steven G. Medema. "Retrospectives: On the Definition of Economics." *Journal of Economic Perspectives* 23, no. 1 (2009): 221-233.

On why models are useful, see:

> Varian, Hal R. "What Use is Economic Theory?" (1989)

and

> Epstein, Joshua M. "Why Model?" *Journal of Artificial Societies and Social Simulation* 11.4 (2008): 12.

If you feel intrigued by the topic, I suggest you read:

> Rodrik, Dani. *Economics Rules: Why Economics Works, When It Fails, and How to Tell the Difference.* OUP Oxford, 2015.

Exercises for Chapter 1

1) It is commonly observed that the number of applications to Masters and PhD programs increase during recessions. Can you suggest a reason why?

2) Displayed below are the goods offered at a coffee shop and their prices:

Espresso	12 TL/cup
Coffee	9 TL/cup
Tea	6 TL/cup
Pie	6 TL/piece

You are given a coupon worth of 12 TL that you can only use at this coffee shop, and you have to use it now. Suppose you decide to get a cup of coffee with this coupon. For you, what is the opportunity cost of getting a cup of coffee with this coupon?

Chapter 2

Consumer Theory

This chapter introduces the first formal model of the decision made by an economic agent: a **consumer**. We consider the simplest (and possibly the most widespread) form of economic interaction: the decision to buy some goods and services. This is literally the model of a consumer who is deciding to buy some goods in a supermarket. There are n different goods, which have their own prices. The consumer observes the prices and decides how much to buy from each good.

As we discussed in Chapter 1, an economic agent has (i) a constraint and (ii) preferences. This economic agent is no different: the consumer has limited income, denoted by a number I. This is the consumer's constraint: she cannot buy everything she wants. The consumer also has well-defined preferences towards goods. We will discuss what "well-defined" means in a few pages.

Also, as discussed in Chapter 1, each economic model involves some simplifications. Here are a couple of simplifications we assume throughout this chapter:

- This is a one-time interaction.

  This means there is no room for "future" in this model: the customer does not go to the store and say "let me buy this item later". There may be future considerations in her decision to buy some goods (i.e., if she is buying a washing machine she realizes that she will most likely not need a washing machine in the near future). Those are fine – the future considerations, concerns about the quality of the good, uncertainty about its use, social concerns etc. are all captured by her preferences.

  Similarly, there is no "yesterday" either: the consumer does not say "I bought this item yesterday, so I do not need to buy it today". We allow the consumer to have such concerns (i.e., needing something because she did not buy it recently, or enjoying an item because she bought and enjoyed it in the past), but all these concerns are captured by the consumer's preferences.

- The customer observes all the prices perfectly, and knows her income. She can easily calculate the money required to buy a given group of items.
- The customer knows her preferences, and she acts according to her preferences. That is, she is **rational**.

At the risk of being repetitive: are these assumptions sometimes violated? Of course. But we are building a benchmark here.

2.1 The Model

There is a single consumer and n different goods. The consumer decides how much to buy of each good, i.e. she chooses quantities.

Here is the **notation** we will use:

- i: Will be used to denote a generic **good**. We will use natural numbers to denote goods and to index them. Thus, the set of all goods will be denoted by $\{1, 2, \ldots, n\}$. Here n denotes the nth good, and also the total number of goods that is available for consumption.
- q_i: Denotes the **quantity** of good i. The case where the consumer is considering consuming 5 kg of good 2 will be represented with $q_2 = 5$ kg. The quantity can be kilograms, grams, liters, numbers... Whatever the denomination is, we will say it is a **unit**. Note that for any good i we must always have $q_i \geqslant 0$. $q_i = 0$ is allowed, i.e. the consumer may choose not to buy a good.
- $q = (q_1, q_2, \ldots, q_n)$: Denotes a consumption bundle (or simply a **bundle**). This is a list that represents how much of each good a consumer is considering for consumption. As an example, consider a situation where there are 4 possible goods that the consumer can consume (hence $n = 4$). The consumption bundle $(8, 2, 0, 12)$ represents the situation where the consumer is considering 8 units of good 1, 2 units of good 2, none of good 3, and 12 units of good 4 for consumption.
- p_i: Denotes the price of good i per unit. Therefore, if the consumer buys q_i units of good i at price p_i, she pays $p_i q_i$ for that good.
- I: Denotes the income of the consumer (the total wealth of the consumer).

When there are two goods ($n = 2$), each bundle can be illustrated on a two-dimensional graph. Moreover, any point in a two dimensional graph with q_1 on the x-axis and q_2

on the y-axis corresponds to some bundle. See Figure 29.1.

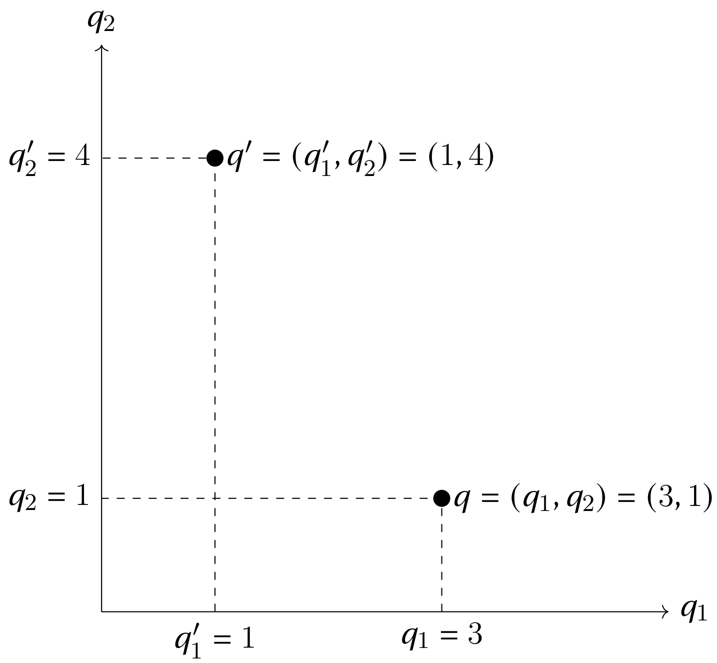


Figure 29.1: A graphical illustration of two bundles, $q = (q_1, q_2) = (3, 1)$, and $q' = (q'_1, q'_2) = (1, 4)$. Any bundle can be illustrated as a point on this graph, and similarly, any point on this graph corresponds to a bundle.

2.1.1 The Constraint

The constraint of the consumer specifies which bundles are affordable (i.e., feasible for the consumer) and which bundles are not.

Definition 29.1 *Given the prices $p_1,\ p_2, \ldots, p_n$ of the goods and the income I of the consumer, a bundle $q = (q_1,\ q_2, \ldots, q_n)$ is* **feasible** *if and only if*

$$\sum_{i=1}^{n} p_i q_i = p_1 q_1 + p_2 q_2 + \cdots p_n q_n \leqslant I \ \ .$$

The set of feasible bundles is also called the **budget set**. *The set of feasible bundles that requires the use of all the income, i.e., the bundles $(q_1, q_2, \ldots, q_n)$ such that $p_1 q_1 + p_2 q_2 + \cdots + p_n q_n = I$ are said to be on the* **budget line** *(they constitute the budget line).*

When there are two goods $(n = 2)$, the set of feasible bundles given the prices p_1, p_2

and income I are the bundles $q = (q_1, q_2)$ that satisfy:

$$p_1 q_1 + p_2 q_2 \leqslant I$$

They can be represented graphically with the orange region shown in Figure 30.1. The budget line is a line with slope $-\frac{p_1}{p_2}$. The (absolute value of) slope of the budget line is an important object we will revisit later. It is important because it is the answer to the following question: "if the consumer gives up one unit of good 1, how many units of good 2 she can buy?" Therefore, $\frac{p_1}{p_2}$ captures the rate at which the consumer can consume good 2 instead of consuming good 1. In other words, $\frac{p_1}{p_2}$ captures the **trade-off** faced by the consumer.

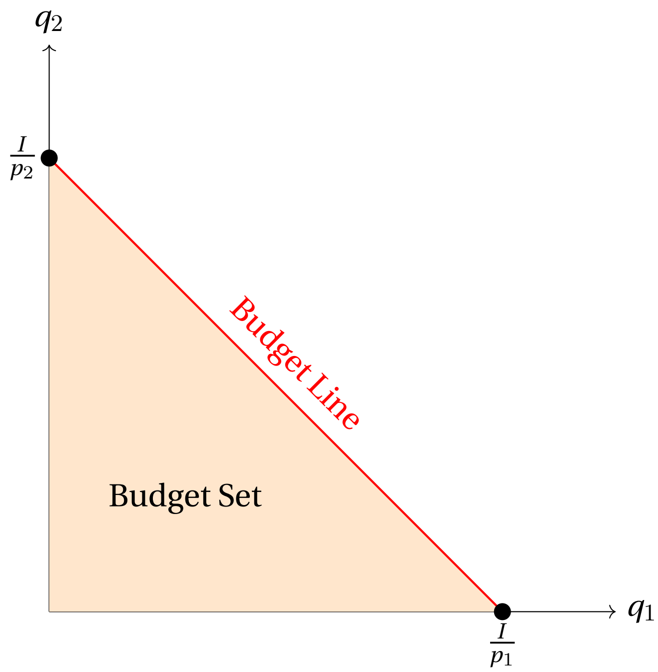


Figure 30.1: A graphical representation of feasible bundles given prices p_1, p_2, and income I.

2.1.2 Preferences

The preferences of the consumer specify the consumer's ranking between any two bundles $q = (q_1, q_2, \ldots, q_n)$ and $q' = (q'_1, q'_2, \ldots, q'_n)$. The consumer may strictly prefer one bundle over the other, or may feel indifferent between them. We capture the consumer's preferences through a preference relation.

A **preference relation** is a relation that compares two pairs of bundles. It is defined on **all** pairs of bundles, including (but not limited to) feasible bundles. Given two bundles q and q', it contains three possible scenarios.

1. The situation where the consumer **(strictly) prefers** bundle $q = (q_1, q_2, \dots, q_n)$ to another bundle $q' = (q_1', q_2', \dots, q_n')$ is represented by:

$$q \succ q'$$

2. The situation where the consumer **(strictly) prefers** bundle $q' = (q_1', q_2', \dots, q_n')$ to another bundle $q = (q_1, q_2, \dots, q_n)$ is represented by:

$$q' \succ q$$

3. If the consumer is **indifferent** between the bundles q and q', then we will write:

$$q \sim q'$$

We say that bundle q is **at least as good as** bundle q' when the consumer prefers q to q' *or* she is indifferent between q and q'. To denote this, we write:

$$q \succsim q'$$

In other words, for any pair of bundles q and q':

$$(q \succsim q') \text{ if and only if } (q \succ q' \text{ or } q \sim q') \tag{31.1}$$

Note that when $q \succ q'$, by definition, it is also true that $q \succsim q'$. This is not really surprising: when a consumer **prefers** q to q', she also thinks q is at least as good as q'.

Similarly: when $q \sim q'$, by definition, it is also true that $q \succsim q'$. This is not surprising either: when a consumer **is indifferent between** q to q', she also thinks q is at least as good as q'.

What I want to point out that two of these scenarios can be satisfied at the same time. This is just like comparisons of real numbers. For the complete analogy: $\succ$ is like the $>$ sign, $\sim$ is like the = sign, and $\succsim$ is like the $\geqslant$ sign. Now, as we know, $5 > 3$ and $5 \geqslant 3$ are both correct. Similarly, $4 = 4$ and $4 \geqslant 4$ are both correct.

Well-Defined Preferences

We want our economic agents to have "well-defined" preferences. A **well-defined** preference relation satisfies the following three conditions:

a. For any pair of bundles q and q',

$$q \succsim q' \quad or \quad q' \succsim q. \tag{31.2}$$

This condition states that the consumer is able compare any pair of bundles. A relation that satisfies this condition is said to be **complete**.

Note that we could have equivalently stated the completeness condition as follows. For any pair of bundles q and q',

$$q \succ q' \quad or \quad q \sim q' \quad or \quad q' \succ q.$$

(You can verify this by writing down equation (31.2) and using the definition of "at least as good as" relationship in equation (31.1).)

b. For any triple of bundles q, q', and q'',

$$\text{if } q \succsim q' \text{ and } q' \succsim q'', \quad \text{then } q \succsim q''.$$

That is, for the consumer, if q is at least as good as q' and q' is at least as good as q'', then q should be at least as good as q''. A relation that satisfies this condition is said to be **transitive**.

There are several implications of transitivity condition, which you derive on your own. The first one is that if the consumer is indifferent between q and q' and if she is indifferent between q' and q'', then she must be indifferent between q and q''. In other words,

$$\text{if } q \sim q' \text{ and } q' \sim q'', \quad \text{then } q \sim q''.$$

Another implication of transitivity is the following. If the consumer strictly prefers q to q' and if she is indifferent between q' and q'', then she must strictly prefer q to q''. In other words,

$$\text{if } q \succ q' \text{ and } q' \sim q'', \quad \text{then } q \succ q''.$$

c. For any pair of bundles $q = (q_1, q_2, \ldots, q_n)$ and $q' = (q'_1, q'_2, \ldots, q'_n)$,

$$\text{if } q_i < q'_i \text{ for all } i \in \{1, 2, \ldots, n\}, \quad \text{then } q' \succ q.$$

This condition states that if the bundle q' has more of each good than bundle q has, then q' is preferred to q. To put it simply, *the consumer prefers more to less.* A relation that satisfies this condition is called **monotonic**.

Throughout this section, we assume that any preference relation is complete, transitive and monotonic, i.e., it is well-defined.

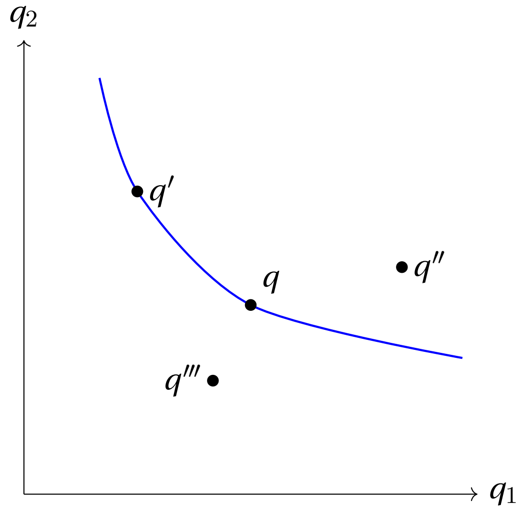


Figure 33.1: A graphical representation of a preference relation.

Indifference Curves

A graphical representation of preference relation can be obtained by drawing curves through bundles that the consumer is indifferent among. Figure 33.1 gives information about a preference relation in which the consumer is indifferent between the bundles q and q' (they are on the same curve).

The curve that passes through q is called **indifference curve** through q. Since q'' and q''' are not on the indifference curve through q, the consumer is not indifferent between q'' and q and also is not indifferent between q''' and q. Since q'' contains more of the (two) goods than q does, by the monotonicity of the preference relation, q'' is preferred to q. Similarly, since the bundle q contains more of the goods than q''' does, by the monotonicity of the preference relation, q is preferred to q'''. Extending this argument, we can conclude that any bundle that lies "above" (in the north east side of) the indifference curve through q is preferred to q and q is preferred to any bundle that lies "below" (in the south west side of) the indifference curve through q.

By now, you may have realized that it is possible to draw multiple indifference curves in the same graph. Consider, for instance, the indifference curve passing through q''. See Figure 34.1.

- By definition, the consumer is indifferent between any bundle on this "higher" indifference curve and q''. For instance, the consumer is indifferent between q'''' and q'':

$$q'''' \sim q''$$

- Recall that, by monotonicity, the consumer prefers q'' to q:

$$q'' \succ q$$

- Once again, by definition, the consumer is indifferent between any bundle on the "lower" indifference curve and q. For instance, the consumer is indifferent between q' and q:

$$q \sim q'$$

- Combining all these stetements and using transitivity, we conclude:

$$q'''' \succ q'$$

Note that this argument can be repeated for any q'''' on the "higher" indifference curve and any q' on the "lower" indifference curve. We conclude: *any bundle on a "higher" indifference curve is preferred over any bundle on a "lower" indifference curve.*

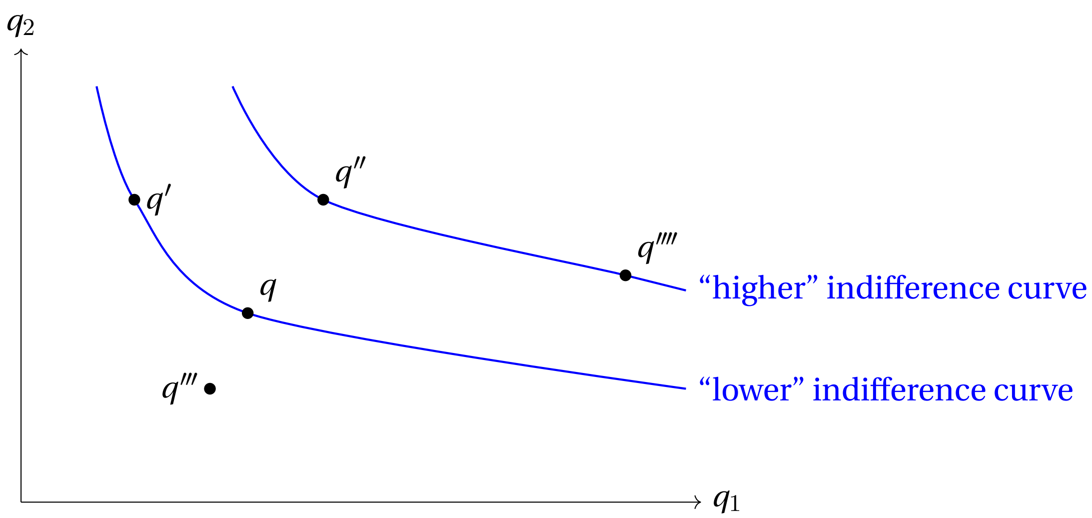


Figure 34.1: Multiple indifference curves. Any bundle on a "higher" indifference curve is preferred over any bundle on a "lower" indifference curve.

There are a couple of other things I want to emphasize about indifference curves:

- Since the preference relation of a consumer is monotonic, the indifference curves must be downward sloping. Suppose, for a contradiction, that a part of an indifference is upward-sloping. Then, there are two bundles q and q' on the same indifference curve such that $q'_1 > q_1$ and $q'_2 > q_2$. By monotonicity, we must have $q' \succ q$. But then, q and q' cannot be on the same indifference curve! A contradiction.

- As long as transitivity and monotonicity are satisfied, indifference curves cannot cross. I am leaving this as an exercise for you to show.

Marginal Rate of Substitution

We will now define a very important object based on the indifference curves. It will provide the very crucial information on how much the consumer "values" good 1 over good 2 at a certain bundle.

The **marginal rate of substitution** of good 2 for good 1 at the bundle $q = (q_1, q_2)$, denoted $\mathrm{MRS}_{2,1}(q)$, is the rate at which good 2 must substitute for a "small" decrease in the consumption of good 1 in order to keep the consumer indifferent to the initial bundle q. More formally (see Figure 35.1):

$$\mathrm{MRS}_{2,1}(q_1, q_2) = \lim_{\substack{\Delta q_1 \to 0^+ \\ (q_1 - \Delta q_1, q_2 + \Delta q_2) \sim (q_1, q_2)}} \frac{\Delta q_2}{\Delta q_1} = |\text{slope of ind. curve at } q| \quad . \tag{35.1}$$

The marginal rate of substitution of good 1 for good 2 at the bundle q, denote $\mathrm{MRS}_{1,2}(q)$, is similarly defined:

$$\mathrm{MRS}_{1,2}(q_1, q_2) = \lim_{\substack{\Delta q_2 \to 0^+ \\ (q_1 + \Delta q_1, q_2 - \Delta q_2) \sim (q_1, q_2)}} \frac{\Delta q_1}{\Delta q_2} = \frac{1}{|\text{slope of ind. curve at } q|} \quad . \tag{35.2}$$

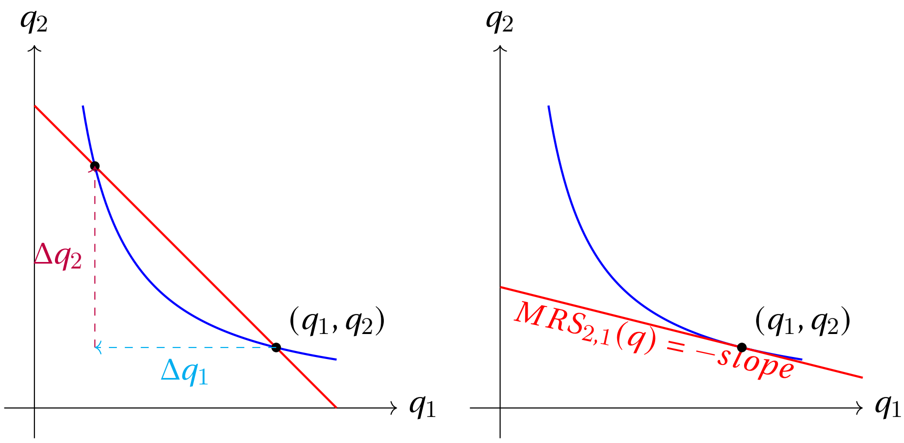


Figure 35.1: Marginal rate of substitution of good 2 for good 1: Taking the limit of $\Delta q_2 / \Delta q_1$ as Δq_1 goes to zero in the figure on left, we obtain the figure on the right.

$\mathrm{MRS}_{2,1}(q)$ is a measure of how much the consumer "values" good 1 in terms of good 2 when he is endowed with the bundle q. Assume that the consumer is endowed with the bundle $q = (q_1, q_2)$. If we ask the consumer to give up a "small" amount of good 1, say Δq_1 units, the consumer would require "approximately" $\mathrm{MRS}_{2,1}(q)\Delta q_1$ units of good 2, to compensate the reduction in the quantity of good 1 in order to be indifferent between the initial bundle and the bundle after the exchange. Similarly, if we asked to consumer to give up a "small" amount of good 2, say Δq_2 units, the consumer would require "approximately" $\mathrm{MRS}_{1,2}(q)\Delta q_2$ units of good 1 to compensate the reduction in the quantity of good 2.

Generally speaking, there are four ways to interpret $\mathrm{MRS}_{2,1}(q)$.

a. **(Mathematical.)** It is the limit of a ratio of two differences: see equation (35.1).

b. **(Verbal.)** It is a measure of how many units of good 2 the consumer must be given, so that she is left indifferent to a decrease in good 1.

c. **(Geometrical.)** It is the (absolute value of) the slope of the indifference curve: the steeper the indifference curve is, the higher $\mathrm{MRS}_{2,1}(q)$ is.

d. **(Economical.)** It is a measure of the value of good 1 in terms of good 2: the more valuable good 1 is, the higher $\mathrm{MRS}_{2,1}(q)$ is.

Diminishing MRS & Smoothness

Let me now define two more properties on the preference, which are common features of many preferences in real life.

- Generally, when a consumer has more of a good (say, good 1), and less of another good (say, good 2), then good 1 becomes less "valuable" for the consumer relative to good 2. We formalize this idea as follows:

  Let q and q' be any two bundles that the consumer is indifferent between (i.e., they are on the same indifference curve). We require preference relations to be such that, if $q_1 > q'_1$, then $\mathrm{MRS}_{2,1}(q) < \mathrm{MRS}_{2,1}(q')$. Also, if $q_2 > q'_2$, then $\mathrm{MRS}_{1,2}(q) < \mathrm{MRS}_{1,2}(q')$.

  Preference relations that satisfy this condition are said to have **(strictly) diminishing marginal rate of substitution**. If a preference relation satisfies the diminishing marginal rate of substitution assumption, the relative value of good 1 in terms of good 2 will be lower as the consumer has more of good 1 and less of good 2. As a result, the indifference curve will get flatter as q_1 increases along the same indifference curve. This means: the indifference curves will be bowed toward the origin (Figure 37.1).

- A preference relation is said to be **smooth** if the indifference curves do not have any kinks. The first two indifference curves displayed in Figure 37.2 are examples of smooth preference relations and the next two are examples of preference

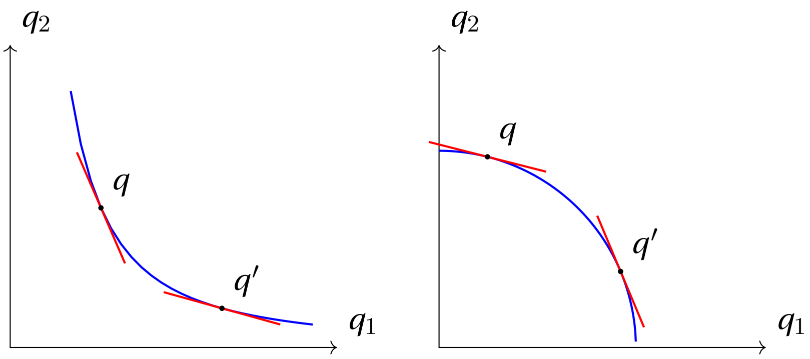


Figure 37.1: The graph one the left displays a indifference curve on which the diminishing marginal rate of substitution assumption holds. The graph on the right displays an indifference curve on which the marginal rate of substitution of good 2 for good 1 increases as we move in the increasing q_1 direction along the indifference curve. Hence the diminishing marginal rate of substitution assumption does not hold for the indifference curve on the right.

relations that are not smooth. If a preference relation is smooth then for any bundle q with positive component (i.e., $q_1 > 0$ and $q_2 > 0$) we have

$$\mathrm{MRS}_{1,2}(q) = \frac{1}{\mathrm{MRS}_{2,1}(q)} \quad . \tag{37.1}$$

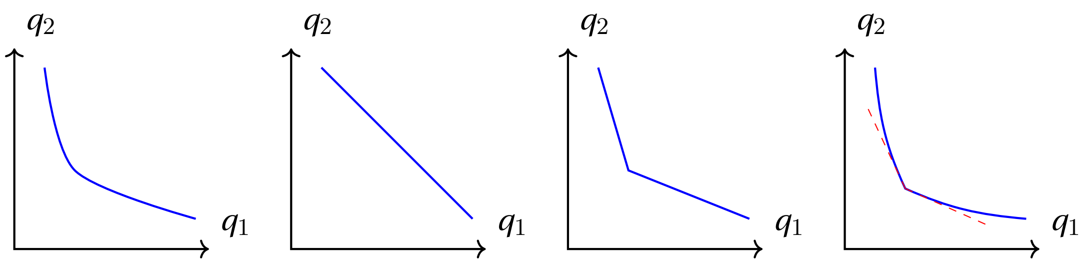


Figure 37.2: The indifference curves at the first two figures are smooth and the indifference curves at the last two have kinks.

2.2 Some Famous Preferences

The preferences we discuss in this section are all well-defined. We will keep assuming that there are two goods ($n = 2$) for the sake of visualization, but the ideas extend to general n.

2.2.1 Perfect Substitutes

Suppose the consumer's preferences are such that: for any two bundles $q = (q_1, q_2)$ and $q' = (q'_1, q'_2)$,

$$\text{if } aq_1 + bq_2 > aq'_1 + bq'_2, \quad \text{then } q \succ q'. \tag{38.1}$$

$$\text{if } aq_1 + bq_2 < aq'_1 + bq'_2, \quad \text{then } q' \succ q. \tag{38.2}$$

$$\text{if } aq_1 + bq_2 = aq'_1 + bq'_2, \quad \text{then } q \sim q'. \tag{38.3}$$

where $a > 0$ and $b > 0$.

(Think of this as the following. For any bundle $q = (q_1, q_2)$, the consumer assigns some number corresponding to that bundle.[1] The number is calculated as $aq_1 + bq_2$. When comparing the two bundles, the consumer acts as if she is comparing the numbers corresponding to these bundles. If the number corresponding to q is strictly larger than the number corresponding to q', the consumer prefers q over q'. If they are equal, the consumer is indifferent.)

How do indifference curves look? Recall that the consumer is indifferent between any two bundles on an indifference curve. Therefore, if q and q' are on the same indifference curve,

$$q \sim q', \qquad \text{or, by (38.3),} \qquad aq_1 + bq_2 = aq'_1 + bq'_2$$

What does it mean? Consider the line defined by the equation:

$$aq_1 + bq_2 = c \tag{38.4}$$

with some $c \geqslant 0$. The consumer is indifferent between any two bundles q and q' on this line, because

$$aq_1 + bq_2 = c = aq'_1 + bq'_2$$

[1] If you have taken an econ course before, you could realize that I am getting very close to muttering the word "utility". I will not mutter this word in this course, because as a ground rule, whenever you can work with preferences you should work with preferences. The notion of utility, in my opinion, is misleading in an introductory micro course. You will learn about this in Econ 203.

Let's make sure that this is actually a line. Rearranging Equation (38.4), we arrive at:

$$q_2 = \frac{c}{b} - \frac{a}{b}q_1$$

which is, geometrically, the equation for a line with intercept $\frac{c}{b}$ and slope $-\frac{a}{b}$.

So the indifference curves in this case are **parallel lines**, each with slope $-\frac{a}{b}$. A higher c means that the consumer is on a "higher" indifference curve, meaning that the consumer prefers the bundles on the indifference curves with higher c to the bundles on the indifference curves with lower c. You can verify this using two alternative methods.

a. Take two indifference curves

$$aq_1 + bq_2 = c$$
$$aq_1 + bq_2 = c'$$

with $c' > c$. Just drawing these indifference curves, you will see that the second indifference curve is higher than the first one (it is further away from the origin.)

Take a bundle $q = (q_1, q_2)$ on the first indifference curve, and another bundle $q' = (q_1', q_2')$ on the second indifference curve. By the equations defining the indifference curves, they following equalities must hold:

$$aq_1 + bq_2 = c$$
$$aq_1' + bq_2' = c'$$

But since $c' > c$, $aq_1' + bq_2' > aq_1 + bq_2$. Then, by (38.2), $q' \succ q$.

b. Just pick two bundles $q = (q_1, q_2)$ and $q' = (q_1', q_2')$ where $q_1' > q_1$ and $q_2' > q_2$. Draw the indifference curves that pass through q and q', and you will see that the one that passes through q' is further away from the origin. By monotonicity,

$$q' \succ q$$

By the definition of an indifference curve, the consumer is indifferent between q and any bundle q'' on the indifference curve passing through q.

$$q'' \sim q$$

Similarly, the consumer is indifferent between q' and any bundle q''' on the indifference curve passing through q'.

$$q''' \sim q'$$

By transitivity,

$$q''' \sim q' \succ q \sim q'' \implies q''' \succ q''$$

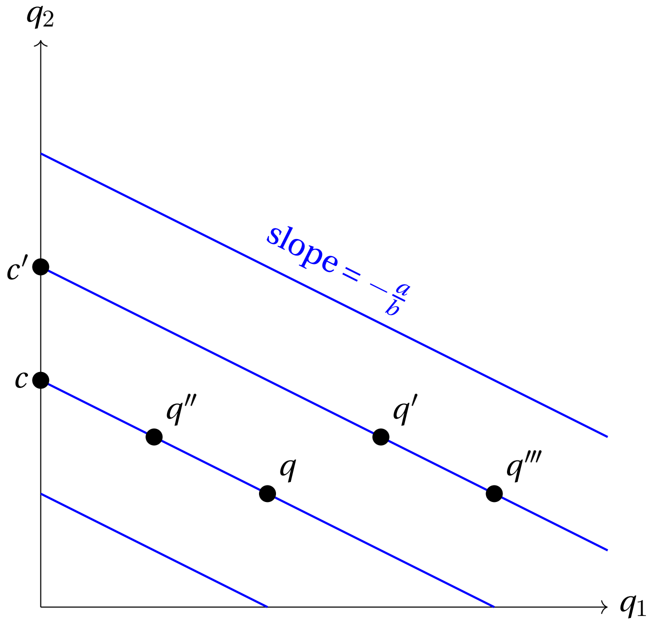


Figure 40.1: Indifference curves for perfect substitutes.

Fine, but what do they mean? Lines have constant slopes. Since the (absolute value of the) slope of the indifference curve is the marginal rate of substitution, it means that the marginal rate of substitution is constant.

$$MRS_{2,1}(q) = \frac{a}{b} \quad \text{for all } q$$

Recall that $MRS_{2,1}(q)$ is a measure of how much the consumer values good 1 in terms of good 2 when she is endowed with bundle q. When $MRS_{2,1}(q)$ is constant, this means that the relative value of good 1 does not depend on the bundle q. No matter how many units of good 1 and good 2 the consumer considers, the relative value is the same. **You can always take away b units of good 1 from the consumer, compensate the consumer by giving a extra units of good 2, and leave the consumer indifferent.** That is, no matter what the consumer is endowed with, a units of good 2 can always **perfectly substitute** b units of good 1. That's why good 1 and good 2 are **perfect substitutes**.

Examples? We typically consider the goods that are very similar in quality to be perfect substitutes. Take Coca Cola and Pepsi, for instance. You may like Coca Cola more than Pepsi, which is fine. In that case, the marginal rate of substitution of Pepsi for Coca Cola will be higher than one. What matters is that if you are willing to substitute one Coca Cola for one Pepsi when you have 10 Pepsis and 0 Coca Colas, then you should be willing to substitute one Coca Cola for one Pepsi when you have 9 Pepsis and 1 Coca Cola. So and so on.

Is the diminishing marginal rate of substitution satisfied? No. The marginal rate of substitution is constant.

Are the preferences smooth? Yes. The indifference curves do not have any kinks. Therefore, $MRS_{1,2}(q) = \frac{1}{MRS_{1,2}(q)} = \frac{1}{a/b} = \frac{b}{a}$ for all q.

2.2.2 Quasi-linear Preferences

Suppose the consumer's preferences are such that: for any two bundles $q = (q_1, q_2)$ and $q' = (q'_1, q'_2)$,

$$\text{if } v(q_1) + q_2 > v(q'_1) + q'_2, \quad \text{then } q \succ q'. \tag{41.1}$$

$$\text{if } v(q_1) + q_2 < v(q'_1) + q'_2, \quad \text{then } q' \succ q. \tag{41.2}$$

$$\text{if } v(q_1) + q_2 = v(q'_1) + q'_2, \quad \text{then } q \sim q'. \tag{41.3}$$

where $v(x)$ is an increasing and concave function. That is, its first derivative is positive and decreasing (i.e., its second derivative is negative). Think of $v(x) = \log x$ or $v(x) = x^\alpha$ for some $\alpha \in (0, 1)$.

How do indifference curves look? If q and q' are on the same indifference curve, $q \sim q'$ or, by (41.3),

$$v(q_1) + q_2 = v(q'_1) + q'_2$$

Consider the curve defined by the equation:

$$v(q_1) + q_2 = c \tag{41.4}$$

This is a typical indifference curve for quasi-linear preferences, where higher values of c correspond to "higher" indifference curves. You can rearrange this to get:

$$q_2 = c - v(q_1) \tag{41.5}$$

Since $v(x)$ is increasing, this curve is downward-sloping. Since $v(x)$ is concave, this curve is convex. For a higher c, we shift this curve upwards.

Fine, but what do they mean? As you can guess by its name, quasi-linear preferences are "kind of" like linear preferences. By "kind of", we mean preferences are linear with respect to one good (in this case, good 2) and not linear with respect to the other good (good 1).

The marginal value that the consumer assigns to good 1 is decreasing in the amount of good 1 the consumer has. It is, however, constant in the amount of good 2 that the consumer has. When the consumer is endowed with more of good 1, she starts liking it less – this is like a usual good we consider. When the consumer is endowed with more of good 2, her attitudes towards good 2 does not change – this is like "linear" preferences.

You can see this feature by checking the marginal rate of substitution. Just take the derivative of Equation (41.5) and take its absolute value to find the slope of the indifference curve:

$$MRS_{2,1}(q) = v'(q_1) \quad \text{for all } q = (q_1, q_2)$$

As you see, this depends on q_1 but **not** on q_2. As long as q_1 remains the same, you can increase q_2 and $MRS_{2,1}(q)$ does not change. This means if you compare two indifference curves and keep q_1 constant, their slopes are the same. Therefore, indifference curves are just shifted versions of each other in the y-axis.

Examples? Good 1 in this example is a standard consumption good, like apples. Good 2 in this example is a good so that the consumer's feelings towards it does not change no matter how much of it she has. Let me give a somewhat radical example. Consider good 2 as **money**. The price of good 2 is $p_2 = 1$. That is, you can spend 1 TL and buy one unit of good 2, which is again 1 TL. Of course, this is just a representation: we are not considering a consumer who spends money to buy money. Instead, we are thinking about a consumer with a certain budget who decides how many apples to buy (q_1) and how much money to keep in her pocket (q_2). The crucial thing is that 1 TL is always 1 TL, no matter how much money you have. So it is reasonable to assume that consumers' feelings towards money does not depend on how much money they have already.[2]

I just want to point out: our earlier discussions made it look like the consumer has to spend all her income when she goes shopping. Now you realize that this framework allows for more general outcomes. It is possible to introduce money saved as another good and conduct the analysis as usual.

[2]This does not have to be universally correct: you can imagine people valuing the extra lira less if they have more money already. That is, the preferences towards money can also satisfy diminishing marginal value. But especially for cash-constrained consumers, having quasi-linear preferences with respect to money seems like a reasonable thing to do.

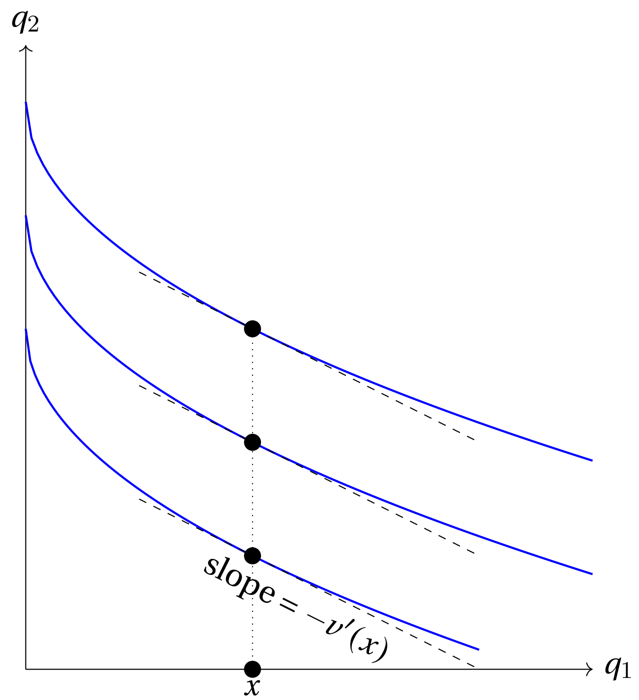


Figure 43.1: Indifference curves for quasi-linear preferences.

Another example: let q_1 denote the time spent on studying for the economics exam, and q_2 denote the time spent on other activities (such as watching more episodes of Aşk-ı Memnu). $v(q_1)$ is the expected grade on the economics exam when the student studies for q_1 hours. The student has a very standard thing to do when she does not study (the satisfaction you derive from Aşk-ı Memnu neither goes up nor goes down as you watch more of it), so the value of the alternative activities does not change at all.

Is the diminishing marginal rate of substitution satisfied? Yes. Recall that $v'(q_1)$ is decreasing.

Are the preferences smooth? Yes, as long as $v(x)$ is a smooth function (it does not have kinks).

2.2.3 Cobb-Douglas Preferences

The following preferences are "invented" by Charles Cobb and Paul Douglas in the first half of 20th century.[3] Suppose the consumer's preferences are such that: for any $q = (q_1, q_2)$ and $q' = (q'_1, q'_2)$,

$$\text{if } (q_1)^\alpha (q_2)^{1-\alpha} > (q'_1)^\alpha (q'_2)^{1-\alpha}, \quad \text{then } q \succ q'. \tag{44.1}$$

$$\text{if } (q_1)^\alpha (q_2)^{1-\alpha} < (q'_1)^\alpha (q'_2)^{1-\alpha}, \quad \text{then } q' \succ q. \tag{44.2}$$

$$\text{if } (q_1)^\alpha (q_2)^{1-\alpha} = (q'_1)^\alpha (q'_2)^{1-\alpha}, \quad \text{then } q \sim q'. \tag{44.3}$$

where $\alpha \in [0, 1]$ is a parameter that measures the "weight" that the consumer attaches to good 1 in her preferences. If α is higher, we have a consumer with a higher inclination towards good 1.

How do the indifference curves look? You can just imitate the arguments in the previous examples to derive the equation for a typical indifference curve:

$$(q_1)^\alpha (q_2)^{1-\alpha} = c \tag{44.4}$$

Once again, higher values of c correspond to "higher" indifference curves. You can rearrange this to get:

$$q_2 = (c)^{\frac{1}{1-\alpha}} (q_1)^{\frac{-\alpha}{1-\alpha}} \tag{44.5}$$

You can check that this is downward-sloping.

Fine, but what do they mean? Not much in particular. Cobb-Douglas preferences are the standard preferences used to capture preferences towards two standard consumption goods. As you will see, these preferences satisfy all the nice properties of a typical preference relation. As you will also see, the optimal bundle satisfies certain nice properties.

With a little bit of messy algebra (which you don't need to know by heart), you can verify that:

$$MRS_{2,1}(q) = \frac{\alpha}{1-\alpha} \frac{q_2}{q_1} \quad \text{for all } q = (q_1, q_2) \tag{44.6}$$

[3] Fun fact: Paul Douglas later went on to serve as a senator in the United States for eighteen years! We, as economists, sometimes wish that he pushed for a legislation that requires every preference to be Cobb-Douglas. :) we're sooo crazzyyy

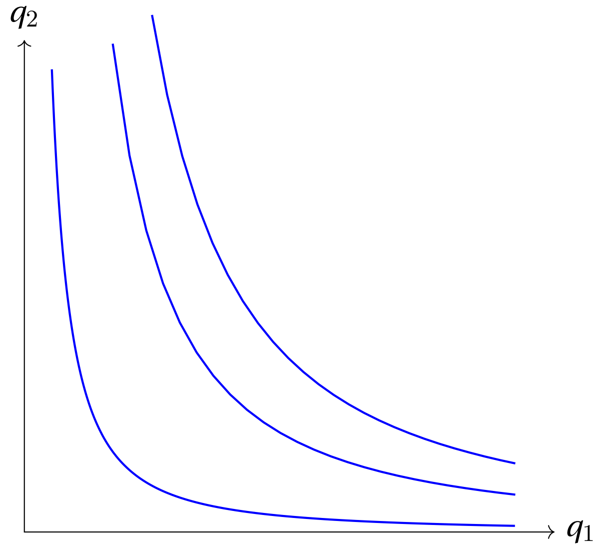


Figure 45.1: Indifference curves for Cobb-Douglas preferences. Note the interesting feature. If you draw a ray passing through the origin, this ray intersects each indifference curve once. Along the ray, $\frac{q_2}{q_1}$ is constant. Therefore, at those intersections, $MRS_{2,1}(q)$ will be the same.

So, the marginal rate of substitution depends **only** on the ratio of q_2 and q_1. Note that if q_1 decreases and q_2 increases, the marginal rate of substitution increases, i.e. good 1 becomes relatively more valuable to the consumer. This is the diminishing marginal rate of substitution!

Also note that the marginal rate of substitution is higher when α is higher, i.e., when the "weight" of good 1 is higher. This makes sense: if the weight of good 1 is higher, the consumer values good 1 more, which translates into a higher $MRS_{2,1}(q)$.

More importantly, as long as $\frac{q_2}{q_1}$ remains the same, the consumer's relative valuation of the good is the same. Suppose the goods are coffee and eggs. When the consumer is endowed with one cup of coffee and three eggs, she has a relative value attached to coffee versus eggs. If the consumer has Cobb-Douglas preferences, then she would have the same relative value when she has two cups of coffee and six eggs.

Examples? Usual consumption goods. Tea versus coffee. Apples versus bananas. White shirts versus blue shirts. Sujuk versus Halloumi cheese.

Is the diminishing marginal rate of substitution satisfied? Yessss.

Are the preferences smooth? Yessss.

2.2.4 Perfect Complements

I will not write this one in much detail. For perfect complements, usually a visual inspection suffices.

Suppose the consumer's preferences are such that: for any $q = (q_1, q_2)$ and $q' = (q_1', q_2')$,

$$\text{if } \min\left\{\frac{q_1}{a}, \frac{q_2}{b}\right\} > \min\left\{\frac{q_1'}{a}, \frac{q_2'}{b}\right\}, \quad \text{then } q \succ q'. \tag{46.1}$$

$$\text{if } \min\left\{\frac{q_1}{a}, \frac{q_2}{b}\right\} < \min\left\{\frac{q_1'}{a}, \frac{q_2'}{b}\right\}, \quad \text{then } q' \succ q. \tag{46.2}$$

$$\text{if } \min\left\{\frac{q_1}{a}, \frac{q_2}{b}\right\} = \min\left\{\frac{q_1'}{a}, \frac{q_2'}{b}\right\}, \quad \text{then } q \sim q'. \tag{46.3}$$

A typical indifference curve is defined by the following equation:

$$\min\left\{\frac{q_1}{a}, \frac{q_2}{b}\right\} = c \tag{46.4}$$

Meaning? a units of good 1 **perfectly complement** b units of good 2. If the consumer has a units of good 1 and more than b units of good 2, the extra units of good 2 are useless. Examples? Left and right shoes, coffee and sugar (if you are drinking coffee only with sugar and if coffee is the only thing you put sugar in).

Is the diminishing marginal rate of substitution satisfied? Depends on how you define "diminishing". Given that we said perfect substitutes do not satisfy diminishing MRS, to be consistent, we will also say that do not satisfy diminishing MRS here.

Are the preferences smooth? Nope.

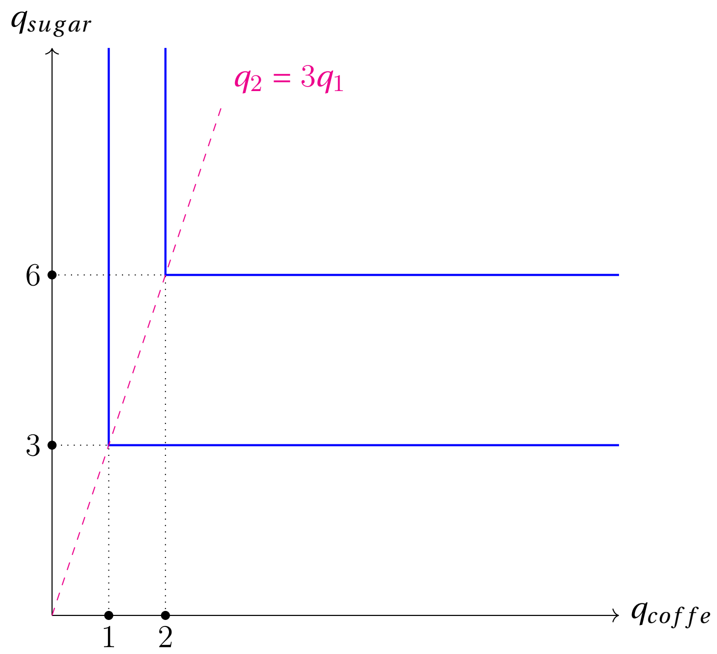


Figure 47.1: An example of indifference curves for perfect complements. Here, good 1 is coffee (quantity is in cups) and good 2 is sugar (quantity is in cubes). The consumer consumes each cup of coffee with three cubes of sugar (I know – she should reduce her sugar consumption.) Therefore, one cup of coffee perfectly complements three cubes of sugar. Thus, $a = 1$ and $b = 3$.

2.3 Optimal Bundle

Now that we have a grasp of the consumer's constraints and preferences, it is time to characterize her decision. The following is a formal definition of the consumer's "favorite bundle among the feasible ones".

Definition 47.1 *Given the prices $p_1, p_2, \ldots, p_n$ and income I, a bundle $q^* = (q_1^*, q^*, \ldots, q_n^*)$ is an optimal bundle if and only if*

- $\sum_{i=1}^{n} p_i q_i^* = p_1 q_1^* + p_2 q_2^* + \cdots + p_n q_n^* \leqslant I$ *(i.e., q^* is feasible), and*
- *for any bundle $q = (q_1, q_2, \ldots, q_n)$, if $\sum_{i=1}^{n} p_i q_i \leqslant I$ (i.e., q is feasible), then $q^* \succsim q$ (i.e., q^* is at least as good as any feasible bundle).*

Thus, a bundle is optimal if and only if it is feasible and is at least as good as (for the consumer) any feasible bundle. Alternatively, a bundle q^* is optimal if and only if any bundle that is preferred to q^* is not feasible.

We will now characterize the optimal bundle q^* when there are two goods (the argument is generalizable to more than two goods). A starting point to develop the intuition is as follows. The optimality conditions imply that the consumer needs to find the **highest indifference curve, given her budget constraint**. Can you try to draw a budget set, the indifference curves, and find the optimal bundle?

Now, let's get more formal. Please note that all the statements below assume that the preferences are "well-defined" (i.e., they satisfy completeness, transitivity and monotonicity). In what follows, I will posit some claims – which are "Claims" in the mathematical sense, so they are correct statements under the assumptions we made. They are not "claims" in the colloquial sense. They have proofs. I am relegating the proofs to the Appendix to make this document more readable. Check them out if you are interested.

Assume that there are two goods and $q^* = (q_1^*, q_2^*) \in \mathbb{R}_+^2$ is an optimal bundle. Here is our first claim.

Claim 48.1 *If q^* is optimal, then $p_1 q_1^* + p_2 q_2^* = I$.*

Informally, Claim 48.1 means that the consumer must exhaust her budget under the optimal bundle. This is intuitively due to monotonocity: more is always better than less, and there is no reason to keep the money in the pocket, so you better just spend the money.

Geometrically, Clam 48.1 means that the optimal bundle must be on the budget line.

Our second claim is a subtle one that relates the marginal rate of substitution to the price ratio.

Claim 48.2 *If q^* is optimal and $q_1^* > 0$, then*

$$\mathrm{MRS}_{2,1}(q^*) \geqslant \frac{p_1}{p_2} \quad . \tag{48.1}$$

Informally, Claim 48.2 says the following: if the consumer is buying good 1 in a strictly positive quantity, then it must be the case that she likes good 1 enough. Otherwise, buying good 1 would not be optimal.

Geometrically, Claim 48.2 means: if $q_1^* > 0$, the indifference curve through q^* has to be steeper than the budget line at q^*. In other words, the indifference curve has to be cutting the budget line *from above* at q^*.

The consumer could also consider consuming a "little" less of good 2 (provided that $q_2^* > 0$). Arguments similar to the above would yield the following claim.

Claim 48.3 *If q^* is optimal and $q_2^* > 0$, then*

$$\mathrm{MRS}_{1,2}(q^*) \geqslant \frac{p_2}{p_1} \quad . \tag{49.1}$$

Informally, Claim 48.3 says: if the consumer is buying good 2 in a strictly positive quantity, then it must be the case that she likes good 2 enough. Otherwise, buying good 2 would not be optimal.

Geometrically, Claim 48.3 means: if $q_2^* > 0$, the indifference curve through q^* has to be flatter than the budget line at q^*. In other words, the indifference curve has to be cutting the budget line *from below* at q^*.

Now, it is time to combine everything we know and have the "big reveal" of consumer theory. That would be the theorem below.

Theorem 49.1 *Given the prices p_1, p_2 and income I, if q^* is an optimal bundle, then $p_1 q_1^* + p_2 q_2^* = I$ and*

- *if $q_1^* > 0$, then*
$$\mathrm{MRS}_{2,1}(q^*) \geqslant \frac{p_1}{p_2} \quad ,$$
- *if $q_2^* > 0$, then*
$$\mathrm{MRS}_{1,2}(q^*) \geqslant \frac{p_2}{p_1} \quad ,$$
- *if $q_1^* > 0$, $q_2^* > 0$, and preference is smooth, then*
$$\mathrm{MRS}_{2,1}(q^*) = \frac{p_1}{p_2} \quad .$$

This is a very neat and intuitive characterization of the optimal bundle. To appreciate its beauty, you should know how to use it.

2.4 The "Cookbook"

How do we put Theorem 49.1 to use? Here are the steps you need to follow, i.e., the recipe for finding the optimal bundle.

First, realize that, by Theorem 49.1, the optimal bundle must be on the budget line. In other words, it must be on the line segment that connects $(\frac{I}{p_1}, 0)$ to $(0, \frac{I}{p_2})$. Then, we have three possibilities.

Case I. Can the optimal bundle be at the bottom right corner? If this is the case, $q^* = (\frac{I}{p_1}, 0)$. Then, $q_1^* > 0$. By Theorem 49.1, we must have: $\mathrm{MRS}_{2,1}(\frac{I}{p_1}, 0) \geqslant \frac{p_1}{p_2}$.

Go ahead and check if the preferences and prices in the problem at hand satisfy this condition. If yes, we cannot rule our $q^* = (\frac{I}{p_1}, 0)$ as an optimal bundle. If no, we rule out $q^* = (\frac{I}{p_1}, 0)$.

Case II. Can the optimal bundle be at the top left corner? If this is the case, $q^* = (0, \frac{I}{p_2})$. Then, $q_2^* > 0$. By Theorem 49.1, we must have: $\mathrm{MRS}_{1,2}(0, \frac{I}{p_2}) \geqslant \frac{p_2}{p_1}$.

Go ahead and check if the preferences and prices in the problem at hand satisfy this condition. If yes, we cannot rule our $q^* = (0, \frac{I}{p_2})$ as an optimal bundle. If no, we rule out $q^* = (0, \frac{I}{p_2})$.

Case III. Can the optimal bundle be in between the two corners? If this is the case, $q_1^* > 0$ and $q_2^* > 0$. If preferences are smooth, by Theorem 49.1, we must have: $\mathrm{MRS}_{2,1}(q^*) = \frac{p_1}{p_2}$.

Go ahead and find the bundle (q_1^*, q_2^*) such that $p_1 q_1^* + p_2 q_2^* = I$ and $\mathrm{MRS}_{2,1}(q_1^*, q_2^*) = \frac{p_1}{p_2}$. This is two equalities and two unknowns, so it should be solvable. Any such bundle is a possible optimal bundle.

Figures 50.1 and 51.1 illustrate preferences and budget sets such that the optimal bundles are at the bottom right and top left corners, respectively.

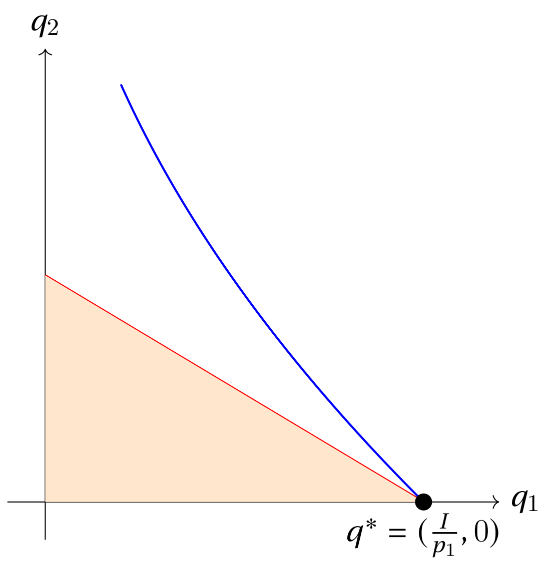


Figure 50.1: An example of a preference relation and budget set such that the optimal bundle is found under Case I.

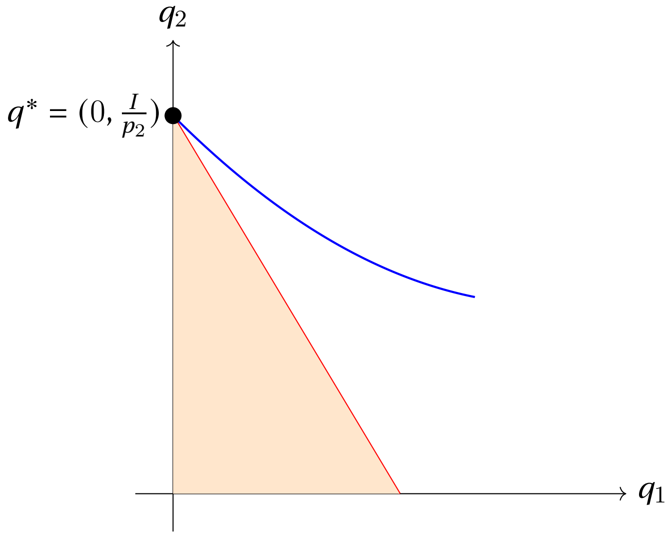


Figure 51.1: An example of a preference relation and budget set such that the optimal bundle is found under Case II.

Figure 51.2 illustrates a preference and budget set such that the optimal bundle satisfies $q_1^* > 0$ and $q_2^* > 0$. Intuitively, it is the point where the indifference curve passing through q^* *barely touches* the budget line, i.e. it is *tangent* to the budget line.

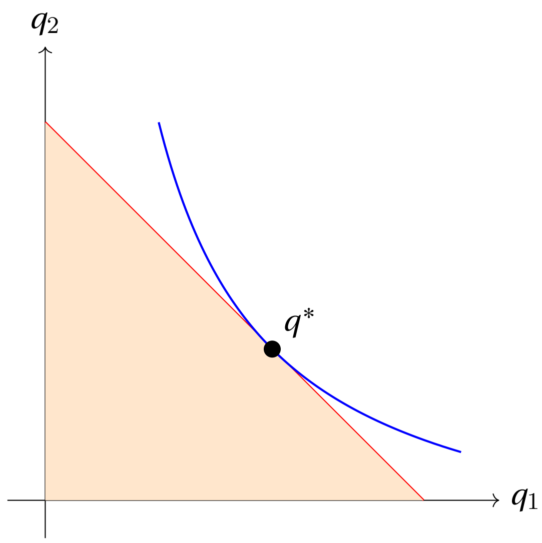


Figure 51.2: An example of a preference relation and budget set such that the optimal bundle is found under Case III.

Take a moment to appreciate the beauty of Figure 51.2! At the optimal bundle, the marginal rate of substitution is exactly equal to the price ratio. That is, suppose you go and ask the consumer in a supermarket:

> "I see your shopping cart, which contains your optimal bundle. Let me ask you a hypothetical question. At the optimal bundle, how much do you value good 1 in terms of good 2?"

And suppose her answer is:

> "Five. You need to give me five units of good 2 for me to give up one unit of good 1."

And then you go ask the cashier in the supermarket:

> "How expensive is good 1 relative to good 2?"

Almost magically, her answer is:

> "Five. You can give up five units of good 2 and buy one unit of good 1 instead."

Isn't this amazing?

What is more amazing is that this applies to *every single consumer* in the supermarket, regardless of their preferences. Another customer may be a fan of good 1, i.e. her $\mathrm{MRS}_{2,1}$ may be higher for every bundle. Fine, she keeps buying more of good 1 and less of good 2, until the marginal rate of substitution decreases and she finds the bundle where the marginal rate of substitution equals the price ratio.

2.5 Optimal Bundle for Some Famous Preferences

2.5.1 Perfect Substitutes

The optimal bundle depends on $\frac{a}{b}$ (marginal rate of substitution, i.e, the consumer's relative valuation of good 1 in terms of good 2) and $\frac{p_1}{p_2}$ (relative price of good 1 in terms of good 2).

- If $\frac{a}{b} > \frac{p_1}{p_2}$, the indifference curves are steeper than the budget line. In this case, the optimal bundle is $(\frac{I}{p_1}, 0)$. (You can go through the three cases in the cookbook and verify this.)

  Intuitively, what is going on? $\frac{a}{b}$ being high means that the consumer values good 1 a lot. Indeed, the consumer's relative valuation of good 1 is higher than the relative price of good 1. The consumer can always buy a units less of good 2. This will save the consumer ap_2. With these savings, the consumer can buy an extra $\frac{ap_2}{p_1}$ units of good 1. Since $\frac{a}{b} > \frac{p_1}{p_2}$, $\frac{ap_2}{p_1} > b$. Thus, the consumer can buy *more than* b units of good 1 with her savings. But remember that b units of good 1 would leave the consumer indifferent! Anything more than b units of good 1 would make the consumer strictly happier! As a result, the consumer keeps reducing her consumption of good 2 until she has no good 2 left in her bundle.

Geometrically, Figure 53.1 is going on. The consumer is finding the highest indifference curve while remaining in the budget set.

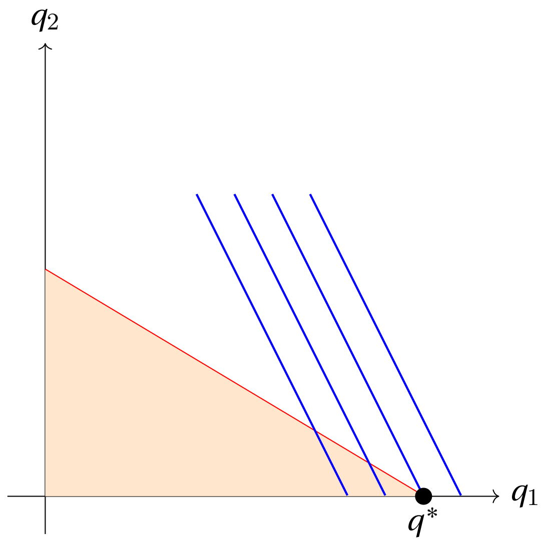


Figure 53.1: Optimal bundle when $\frac{a}{b} > \frac{p_1}{p_2}$. The blue lines are indifference curves and the red line is the budget line.

- If $\frac{a}{b} < \frac{p_1}{p_2}$, the indifference curves are flatter than the budget line. In this case, the optimal bundle is $(0, \frac{I}{p_2})$. (You can go through the three cases in the cookbook and verify this.)

  Intuitively, what is going on? $\frac{a}{b}$ being low means that the consumer does not value good 1 much. Indeed, the relative price of good 1 in terms of good 2 is higher than the relative value of good 1 in terms of good 2. The consumer can always buy b units less of good 1. This will save the consumer bp_1. With these savings, the consumer can buy an extra $\frac{bp_1}{p_2}$ units of good 2. Since $\frac{p_1}{p_2} > \frac{a}{b}$, $\frac{bp_1}{p_2} > a$. Thus, the consumer can buy *more than* a units of good 2 with her savings. But remember that a units of good 2 would leave the consumer indifferent! Anything more than a units of good 2 would make the consumer strictly happier! As a result, the consumer keeps reducing her consumption of good 1 until she has no good 1 left in her bundle.

  Geometrically, Figure 54.1 is going on. The consumer is finding the highest indifference curve while remaining in the budget set.

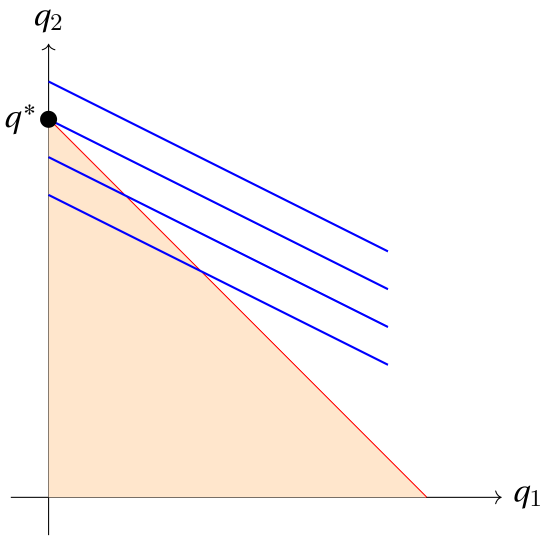


Figure 54.1: Optimal bundle when $\frac{a}{b} < \frac{p_1}{p_2}$. The blue lines are indifference curves and the red line is the budget line.

- Finally, consider the case $\frac{a}{b} = \frac{p_1}{p_2}$. The indifference curves are parallel to the budget line! In this case, there are **many** optimal bundles. Indeed, any bundle on the budget line is optimal. See Figure 55.1.

  This is a somewhat knife-edge case (a consumer whose relative value exactly equals the relative price), but not impossible to find. For instance, suppose $a = b$ and $p_1 = p_2$. This means the consumer is totally indifferent between the goods (take away good 1, give good 2 in equal amounts, doesn't matter), and also the price are equal. This corresponds to cases where the brand of the item does not matter, at all. I am thinking of something like bleach. Does the brand of the bleach matter at all? For many people, no. When I need to buy bleach, I would just go ahead and buy the cheapest one. If the two brands in the supermarket have the same price, I could buy either of them.

2.5.2 Quasi-Linear Preferences

Recall that, under quasi-linear preferences, $\text{MRS}_{2,1}(q_1, q_2) = v'(q_1)$. This means, at the bottom right corner,

$$\text{MRS}_{2,1}\left(\frac{I}{p_1}, 0\right) = v'\left(\frac{I}{p_1}\right)$$

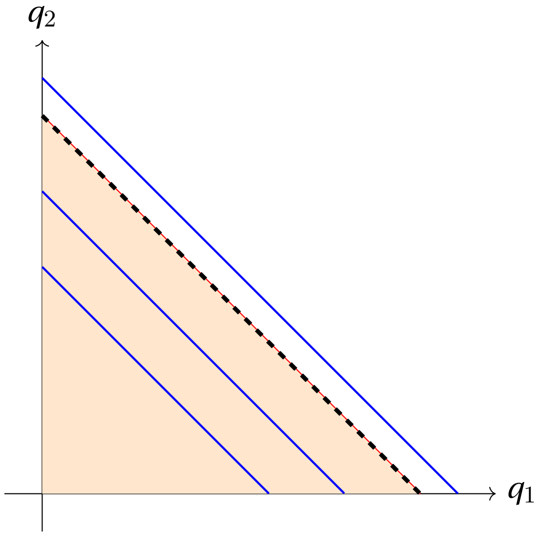


Figure 55.1: Optimal bundles when $\frac{a}{b} = \frac{p_1}{p_2}$. Any bundle on the budget line is optimal.

and, at the top left corner,

$$\text{MRS}_{2,1}\left(0, \frac{I}{p_1}\right) = v'(0)$$

Since v is concave, $v'(0) \geqslant v'\left(\frac{I}{p_1}\right)$.

The optimal bundle depends on $v'\left(\frac{I}{p_1}\right)$, $v'(0)$ and $\frac{p_1}{p_2}$.

- If $v'\left(\frac{I}{p_1}\right) > \frac{p_1}{p_2}$, the optimal bundle is at the bottom right corner. This looks like the case in Figure 50.1.
- If $\frac{p_1}{p_2} > v'(0)$, the optimal bundle is at the top left corner. This looks like the case in Figure 51.1.
- If $v'(0) > \frac{p_1}{p_2} > v'\left(\frac{I}{p_1}\right)$, the optimal bundle is $q^* = (q_1^*, q_2^*)$ such that

$$v'(q_1^*) = \frac{p_1}{p_2}$$

  and $q_2^* = \frac{I - p_1 q_1^*}{p_2}$. This looks like the case in Figure 51.2.

Intuitively, if the consumer's marginal rate of substitution is low to begin with (i.e., if $v'(0)$ is low), she does not buy good 1 at all. Otherwise, the consumer keeps buying good 1 up to the point where her marginal rate of substitution is low enough, so she does not want to buy it any more. The question is: what is the marginal rate of substitution if she, hypothetically, has spent all her income on good 1? At this point, she has bought $\frac{I}{p_1}$ units of good 1. If $MRS_{2,1}(q)$ is lower than $\frac{p_1}{p_2}$, that is more than enough. She should have stopped buying earlier and spend the remaining amount on good 2. If $MRS_{2,1}(q) > \frac{p_1}{p_2}$ at this bundle, she spends all her income on good 1. If she had even more income she would buy even more of good 1, but she is constrained, so she stops here.

The interesting bit about quasi-linear preferences is that: once the consumer chooses some $q_1^* > 0$, even if you gave consumer more income, she would not buy more good 1: she should spend all her extra income on good 2. In economics terminology, this means that quasi-linear preferences do not exhibit *income effect*– this will make more sense in two chapters.

2.5.3 Cobb-Douglas Preferences

Take my word for it when I say that in the optimal bundle $q^* = (q_1^*, q_2^*)$, the consumer has $q_1^* > 0$ and $q_2^* > 0$. (Better yet, don't take my word for it, go through the cases by yourself and see.)

Given that $q_1^* > 0$ and $q_2^* > 0$, we have:

$$\frac{\alpha}{1-\alpha}\frac{q_2^*}{q_1^*} = MRS_{2,1}(q^*) = \frac{p_1}{p_2}$$

Rearrange to get:

$$\frac{q_1^* p_1}{q_2^* p_2} = \frac{\alpha}{1-\alpha}$$

What does this mean? $q_1^* p_1$ is the consumer's total expenditure on good 1. $q_2^* p_2$ is the total expenditure on good 2. Combine this with the equation $q_1^* p_1 + q_2^* p_2 = I$ to derive the following result:

> "At the optimal bundle, the consumer spends α fraction of her income on good 1, and $1-\alpha$ fraction on good 2. Therefore, $q^* = \left(\frac{\alpha I}{p_1}, \frac{(1-\alpha)I}{p_2}\right)$."

Circling back to what we had before: recall that α is the weight of good 1. If α is higher, the consumer allocates a larger share of her budget to good 1!

The interesting bit about Cobb-Douglas preferences is that: if you doubled the consumer's income, she would double her purchase of *both* good 1 and good 2. In economics terminology, this means that Cobb-Douglas preferences are *homothetic* – this will make more sense in two semesters.

2.5.4 Perfect Complements

See Figure 57.1. Theorem 49.1 does not apply in this case because the preferences are not smooth. But a visual inspection suffices.

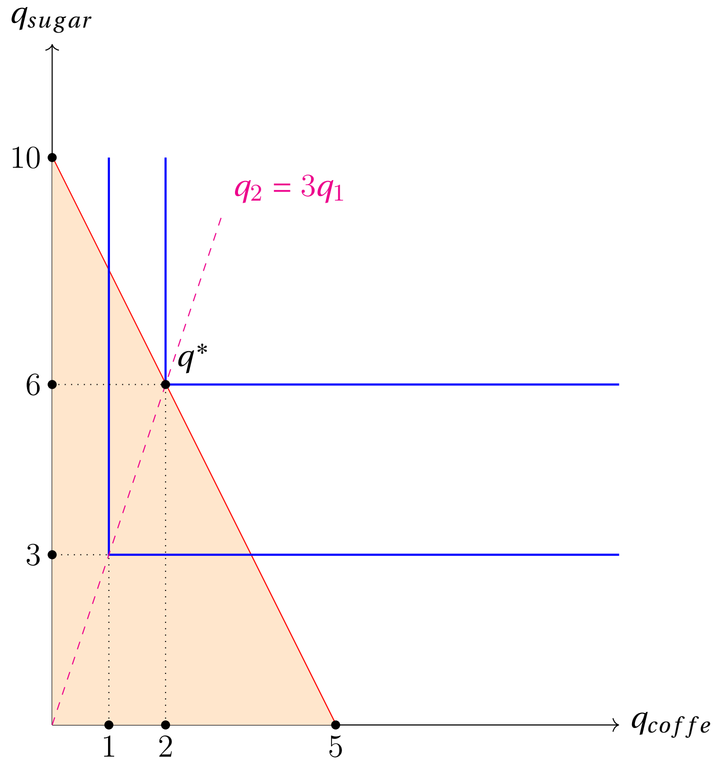


Figure 57.1: Optimal bundle under perfect complements. Suppose $p_1 = 5$ TL, $p_2 = 2.5$ TL and $I = 25$ TL. The consumer could buy 5 cups of coffee, but it will be worthless without the sugar. She could buy 10 cubes of sugar (how expensive is sugar, by the way???), but that would be worthless without the coffee. In the optimal bundle, she buys 2 cups of coffee with 6 sugars, which perfectly complement each other.

Appendix to Chapter 2

Proofs

Proof of Claim 48.1. Suppose, towards a contradiction, that $p_1 q_1^* + p_2 q_2^* \neq I$. There are two possibilities.

- If $p_1 q_1^* + p_2 q_2^* > I$, q^* would not be feasible. This would contradict feasibility of q^*.
- If $p_1 q_1^* + p_2 q_2^* < I$, the consumer can afford another bundle $q' = (q_1^* + \Delta_1, q_2^* + \Delta_2)$, i.e. q' is feasible. Because the preference relation is monotonic, this is preferred to q^*, i.e. $q' \succ q^*$. This contradicts the optimality of q^*.

Since both cases lead to a contradiction, the proof follows. □

Proof of Claim 48.2. Assume that $q_1^* > 0$. Let Δq_1 be a "small" positive quantity such that $q_1^* - \Delta q_1 \geqslant 0$. (Since q_1^* is positive, there is such a positive quantity).

The consumer is considering the bundle q^*. If she consumes Δq_1 units less of good 1 (i.e., consumes $q_1^* - \Delta q_1$ units of good 1 rather than q_1^* units of it), then the consumer would require (approximately) $\mathrm{MRS}_{2,1}(q^*)\Delta q_1$ units of good 2 to substitute for good 1 (so that the she is indifferent between the final bundle and the initial bundle q^*). But if the consumer buys Δq_1 units less of good 1, she would have $p_1 \Delta q_1$ TL to spend on good 2. With this money she can buy $(p_1 \Delta q_1)/p_2$ units of good 2. If

$$\mathrm{MRS}_{2,1}(q^*)\Delta q_1 < \frac{p_1 \Delta q_1}{p_2} \quad , \tag{58.1}$$

then the consumer would become better off by consuming Δq_1 units less of good 1 and $(p_1 \Delta q_1)/p_2$ units more of good 2. That is, if (58.1) holds, then the bundle

$$\left(q_1^* - \Delta q_1,\ q^* + (p_1/p_2)\Delta q_1\right)$$

is feasible and is preferred to the bundle q^*. But this contradicts with q^* being optimal. Thus, if q^* is optimal, then (58.1) can not be true, which means that

$$\mathrm{MRS}_{2,1}(q^*)\Delta q_1 \geqslant \frac{p_1 \Delta q_1}{p_2}$$

must be true. Since Δq_1^* is positive, dividing both sides of the above inequality with q_1^* we obtain:

$$\mathrm{MRS}_{2,1}(q^*) \geqslant \frac{p_1}{p_2} \quad .$$

□

Proof of Claim 48.3 is very similar to that of Claim 48.2, so I am leaving it as an exercise.

Proof of Theorem 49.1. The proof of first two bullet points follow from Claims 48.2 and 48.3. For the last bullet point: If preference is smooth and q^* is an optimal bundle with $q_1^* > 0$ and $q_2^* > 0$, then Claim 48.2, (37.1), and Claim 48.3 imply:

$$\frac{p_1}{p_2} \geqslant \frac{1}{\mathrm{MRS}_{1,2}(q^*)} = \mathrm{MRS}_{2,1}(q^*) \geqslant \frac{p_1}{p_2}$$

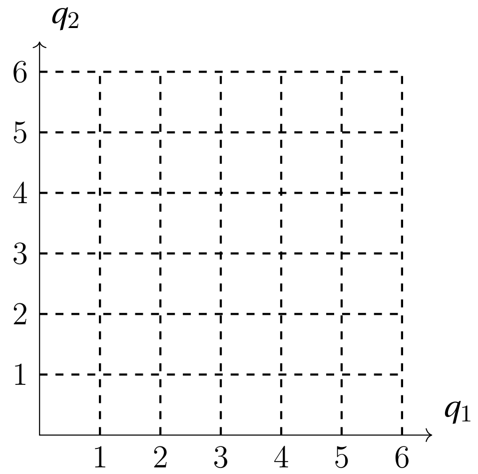
Which in turn implies

$$\mathrm{MRS}_{2,1}(q^*) = \frac{p_1}{p_2} \quad .$$

□

Exercises for Chapter 2

1) Assume that there are two goods ($n = 2$). Good 1 is measured in grams and good 2 is measured in liters. The consumer has an income of $I = 12$ TL, price of good 1 is $p_1 = 2$ TL/gr, and the price of good 2 is $p_2 = 3$ TL/lt. Draw the consumer's budget line:



What is (the absolute value of) the slope of the budget line?

2) Assume that there are two goods ($n = 2$), and let a consumer's preferences over consumption bundles be such that: for any two bundles $q = (q_1, q_2)$ and $q' = (q'_1, q'_2)$,

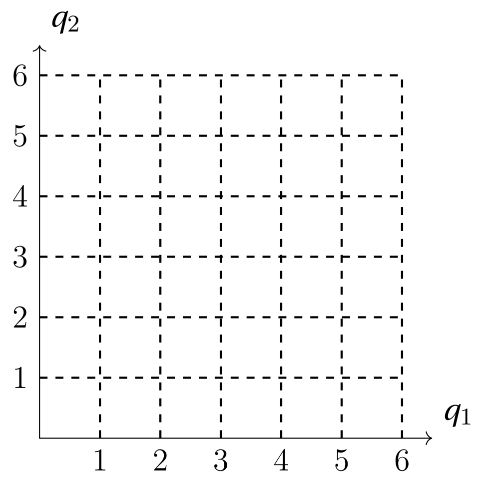
$$\text{if } 2q_1 + q_2 > 2q'_1 + q'_2, \quad \text{then } q \succ q'.$$

$$\text{if } 2q_1 + q_2 < 2q'_1 + q'_2, \quad \text{then } q' \succ q.$$

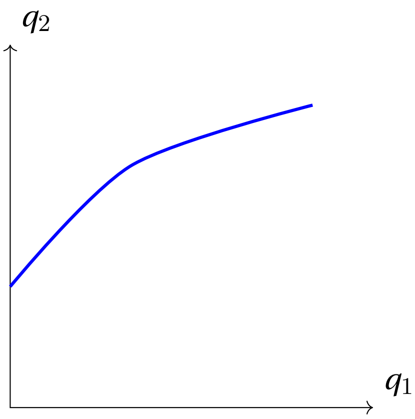
$$\text{if } 2q_1 + q_2 = 2q'_1 + q'_2, \quad \text{then } q \sim q'.$$

(Think of this as the following. For any bundle $q = (q_1, q_2)$, the consumer assigns some number corresponding to that bundle. The number is calculated as $2q_1 + q_2$. When comparing the two bundles, the consumer acts as if she is comparing the numbers corresponding to these bundles. If the number corresponding to q is strictly larger than the number corresponding to q', the consumer prefers q over q'. If they are equal, the consumer is indifferent.)

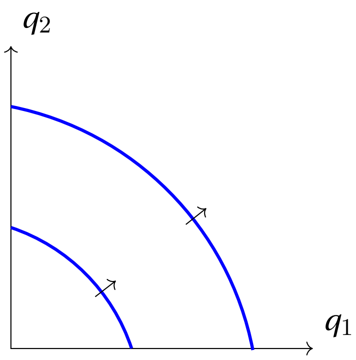
Draw the indifference curve passing through $(2, 1)$:



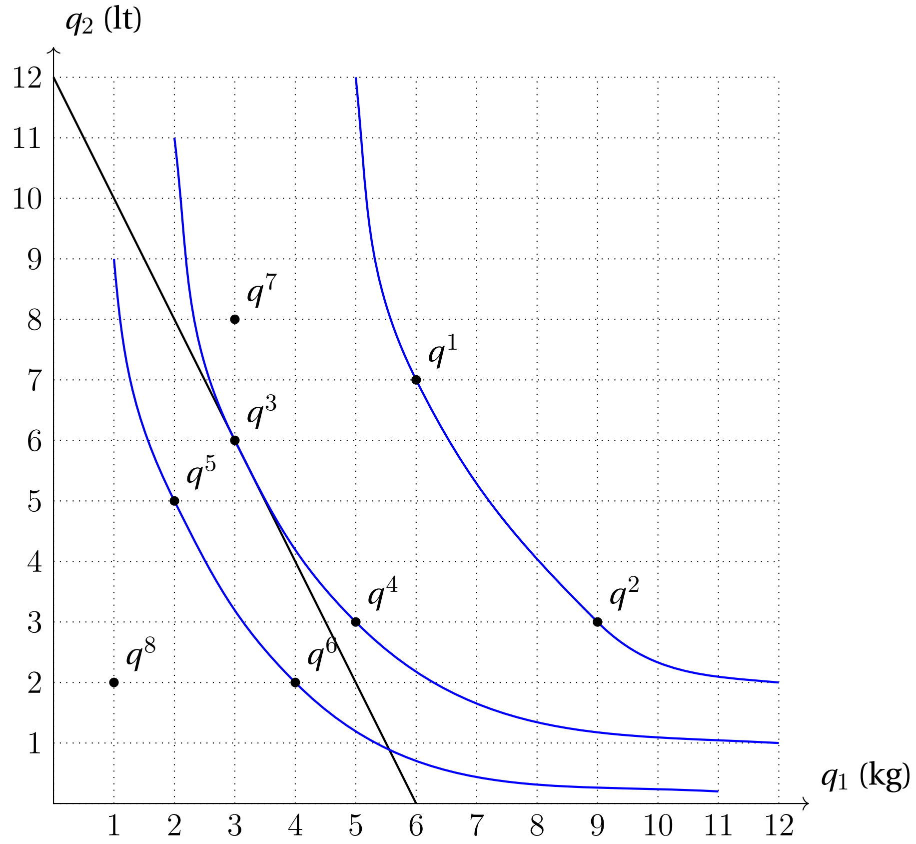
3) Displayed below is an indifference curve for a consumer. Can you figure out which assumption on the preference relation is violated?



4) Displayed below are two indifference curves for a consumer (the arrows on the indifference curves shows the direction of preferred bundles). Can you figure out which assumption on the preference relation is violated?



5) The following figure displays some indifference curves (drawn with blue) and a line.



a. State the preference relation between the bundles q^1, $q^2, \ldots, q^8$ (order them from the most preferred to the least preferred indicating the relation between them accompanied with a reason).

b. Find the marginal rate of substitution of good 2 for good 1 at the bundle q^3. (Note: the line is tangent to the indifference curve that passes through q^3).

c. Find the marginal rate of substitution of good 1 for good 2 at the bundle q^3.

6) A consumer who can only consume two goods has a preference relation over the bundles that are "smooth" and is such that, the marginal rate of substitution of good 2 for good 1 at a bundle (q_1, q_2) is:

$$\text{MRS}_{2,1}(q_1, q_2) = \frac{1}{2\sqrt{q_1}} \quad .$$

a. Is the assumption of diminishing marginal rate of substitution satisfied?

b. What is the marginal rate of substitution of good 1 for good 2 (as a function of (q_1, q_2))?

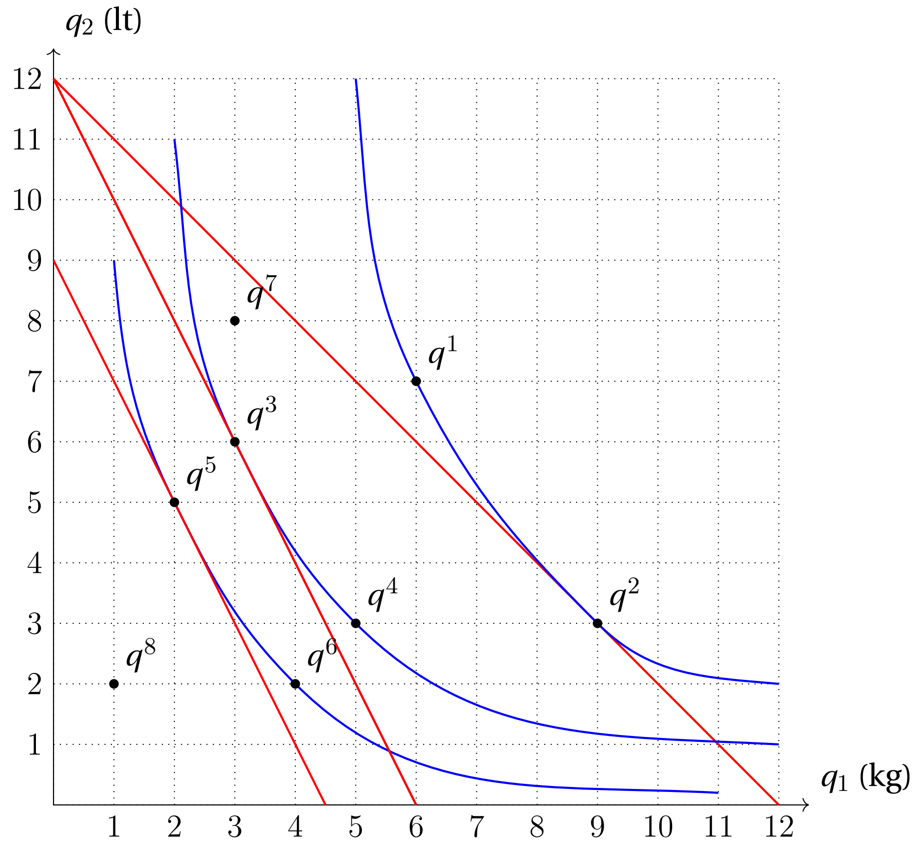
c. What is the marginal rate of substitution of good 2 for good 1 at the bundle $(4, 9)$?

d. By spending all her money the consumer can consume the bundle $(4, 9)$. Given that the consumer is endowed with the bundle $(4, 9)$, if she gives up 0.1 units of good 1, approximately, how much good 2 should she receive so that she is indifferent between her initial bundle $(4, 9)$ and the final bundle which has $4 - 0.1 = 3.9$ units of good 1? Now assume that, rather than giving up 0.1 units of good 1, she gives up 0.1 units of good 2. How much good 1 should she receive in order to be indifferent between his initial bundle and final bundle?

e. The price of good 1 is 4 TL/unit and the price of good 2 is 2TL/unit. Given these prices, if the consumer decides to consume 0.1 units less of good 1 how much more good 2 can the consumer buy (assuming that before and after the exchange she spends all his income)? What if she decides to consume 0.1 units less of good 2, how much more good 1 can she buy?

f. Assume that by buying the bundle $(4, 9)$ the consumer spends all her money. If the price of good 1 is 4 TL/unit and the price of good 2 is 2 TL/units, can the bundle $(4, 9)$ be an optimal bundle?

7) The following figure displays some budget lines and indifference curves (the points labeled with a letter are the point at which the indifference curve is tangent to a budget line).



a. The consumer has an income of 24 TL that she will spend on the two goods. The price of good 1 is 2 TL/kg and the price of good 2 is 2 TL/lt. Given the income and prices, what is the optimal bundle that the consumer will consume?

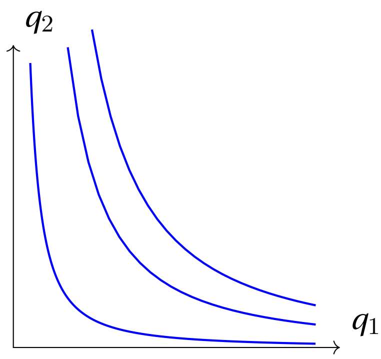
b. Suppose the consumer has an income of 24 TL and the price of good 2 is 2 TL/lt. However, the price of good 1 increases to 4 TL/kg. Given the income and new prices, what is the optimal bundle?

c. Suppose the consumer's income decreases to 18 TL. The price of good 1 is still 4 TL/kg, and the price of good 2 is still 2 TL/lt. Given the new income and prices, what is the optimal bundle?

8) A consumer faces the decision of choosing how much to consume of two goods, say good 1 and good 2. The consumer's preferences are well-defined, smooth, and are such that the marginal rate of substitution of good 2 for good 1 at a bundle $q = (q_1, q_2)$ is:

$$\text{MRS}_{2,1}(q) = \frac{0.4q_2}{0.6q_1} \quad .$$

To give you an idea, several indifference curves are plotted below:

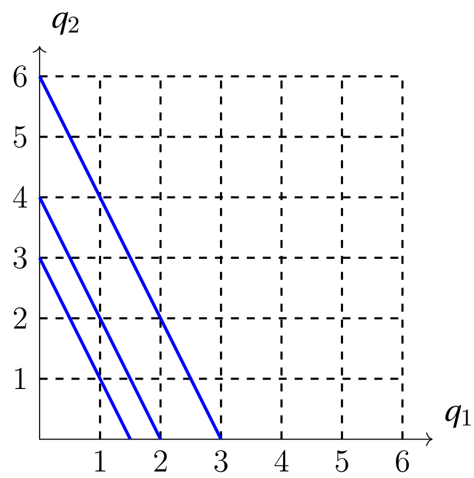


Find how much good 1 and good 2 the consumer consumes, as a function of her income (I), the price of good 1 (p_1), and the price of good 2 (p_2).

9) A consumer faces the decision of choosing how much to consume of two goods, say good 1 and good 2. The consumer's preferences are well-defined, smooth, and are such that the marginal rate of substitution of good 2 for good 1 at any bundle $q = (q_1, q_2)$ is:

$$\text{MRS}_{2,1}(q) = 2$$

To give you an idea, several indifference curves are plotted below:



For the rest of the question, suppose the consumer has an income of $I = 30$ TL.

a. Suppose $p_1 = 15$ TL$/kg$ and $p_2 = 5$ TL$/kg$. Find how much good 1 and good 2 the consumer consumes.

b. Suppose $p_1 = 10$ TL$/kg$ and $p_2 = 10$ TL$/kg$. Find how much good 1 and good 2 the consumer consumes.

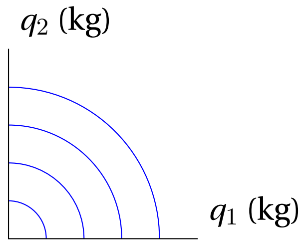
10) **[Fall 2019 Midterm 1, Question 7.]** Consider a consumer, with smooth preferences,

who has to decide on how much of good 1 and good 2 to consume. The price of good 1 is p_1, the price of good 2 is p_2 and the income of the consumer is I. If the consumers preference relation is such that, for any bundle $q = (q_1, q_2)$,

$$\mathrm{MRS}_{2,1}(q) = \sqrt{\frac{q_2}{q_1}}$$

what can you say about the bundle that the consumer will choose (i.e., find the optimal bundle in terms of I, p_1, and p_2)?

11) **[Fall 2019 Midterm 1, Question 8.]** Consider a consumer whose preference is such that the indifference curves are circles with center at the origin (hence the assumption of diminishing marginal rate of substitution is violated but monotonicity is still satisfied). Some of the indifference curves are shown below:



If the income of the consumer is 4 TL, the price of good 1 is 1 TL/kg, and the price of good 2 is 2 TL/kg what is the optimal bundle for this consumer? Explain.

Chapter 3

Equilibrium in an Exchange Economy

In Chapter 2, we analyzed the choice made by a consumer in isolation (i.e., there were no other consumers). Now, we will put two consumers together and see what happens.

We should note that this is a chapter that most introductory economics textbooks do not cover. (Make no mistake, however; what you are about to see contains some of the most canonical ideas of economics, and you *will* encounter them in the future.)

"Why are we covering this?" you might ask. Here are some answers:

a. We want you to get a sense of *equilibrium* as early as possible.

b. In Chapter 2, we took the prices as given. This chapter will give you a sense of how the prices are determined, and the role they play in equilibrium.

c. This is a good time to draw a distinction between *positive* and *normative* analysis – which we will do.

d. You will need this in introductory macro. (Always a great reason.)

Anyway, without further ado, here is a description of an exchange economy.

3.1 An Exchange Economy

In this section, I describe an **exchange economy**. The reason why there is an emphasis on *exchange* is: there is no *production* taking place in this economy. Each consumer in this economy owns some goods, and they just exchange them. Where do these goods come from? At this stage, just imagine they fall from sky. People have some goods to begin with, and they may be willing to exchange them. For the sake of a motivation, you may imagine a bazaar where every agent has her own stand. Some agents bring their old CDs, some bring their old DVDs, and they set up their own stands. Each agent can go to another stand and may exchange CDs for another agent's DVDs.

To keep the matters simple, we will analyze a simplified model where there are only two agents and two goods. Keep in mind that the conclusions we derive here will generalize to cases with more than two agents and more than two goods. But that general case becomes too intractable and is definitely beyond the scope of undergraduate economics.

So, there are two agents and two goods. Each agent comes to market with a bundle. Another way to say this is: *each agent is endowed with a bundle.*

For convenience, let me refer to agent 1 as **Robinson** and agent 2 as **Friday**. Similarly, let me refer to good 1 as **Apple** and good 2 as **Banana**. This will help us remember the storyline of our exchange economy:

> Suppose there are only two agents on an island: Robinson and Friday. The island is divided into two: Robinson's side and Friday's side. There are only two types of trees on the island: Apple trees and Banana trees. Every agent owns the trees (and hence apples and bananas on these trees) on their side of the island.

Suppose Robinson is endowed with e_A^R apples and e_B^R bananas at the beginning. That is, Robinson has an initial endowment, which is a bundle $e^R = (e_A^R, e_B^R)$. Similarly, Friday's endowment is $e^F = (e_A^F, e_B^F)$.

The total number of apples on the island is e_A, i.e., $e_A = e_A^R + e_A^F$. Similarly, the total number of bananas on the island is e_B, i.e., $e_B = e_B^R + e_B^F$.

Here is the catch, though: Robinson and Friday **do not** have to consume their endowments. Instead, they can go on to exchange some apples and bananas. That is, their consumption bundles do not have to be the same as their endowments. After all, we are in an exchange economy!

You may ask: how do we represent consumption bundles? The answer is: the same way we represented consumption bundles in consumer theory. A consumption bundle for Robinson is $q^R = (q_A^R, q_B^R)$, where q_A^R is the quantity of apples Robinson consumes, and q_B^R is the quantity of bananas Robinson consumes. Just as in the consumer theory, Robinson has preferences over q^R's. Let these preferences be denoted by $\succsim^R$. Throughout this section, we will make the assumptions that preferences are well-defined (complete, transitive, monotonic), preferences are smooth and satisfy diminishing marginal rate of substitution.

Similarly, Friday's consumption bundle specifies the quantity of apples Friday consumes and the quantity of bananas Friday consumes, i.e., $q^F = (q^F_A, q^F_B)$. Friday has preferences over q^F's, denoted by $\succsim^F$, and the preferences satisfy the usual assumptions.

An **allocation** q is a pair of consumption bundles, which specifies the bundle Robinson gets and the bundle Friday gets, i.e., $q = (q^R, q^F) = ((q^R_A, q^R_B), (q^F_A, q^F_B))$. Keep in mind that for this model, an allocation is represented by four numbers. An allocation q is **feasible** if it distributes all the apples and bananas in the island between Robinson and Friday. In other words, an allocation q is feasible if it satisfies the following equalities:

$$
\begin{aligned}
q^R_A + q^F_A = e^R_A + e^F_A &= e_A \\
q^R_B + q^F_B = e^R_B + e^F_B &= e_B
\end{aligned}
$$

Here are some broad level questions we will try to answer:

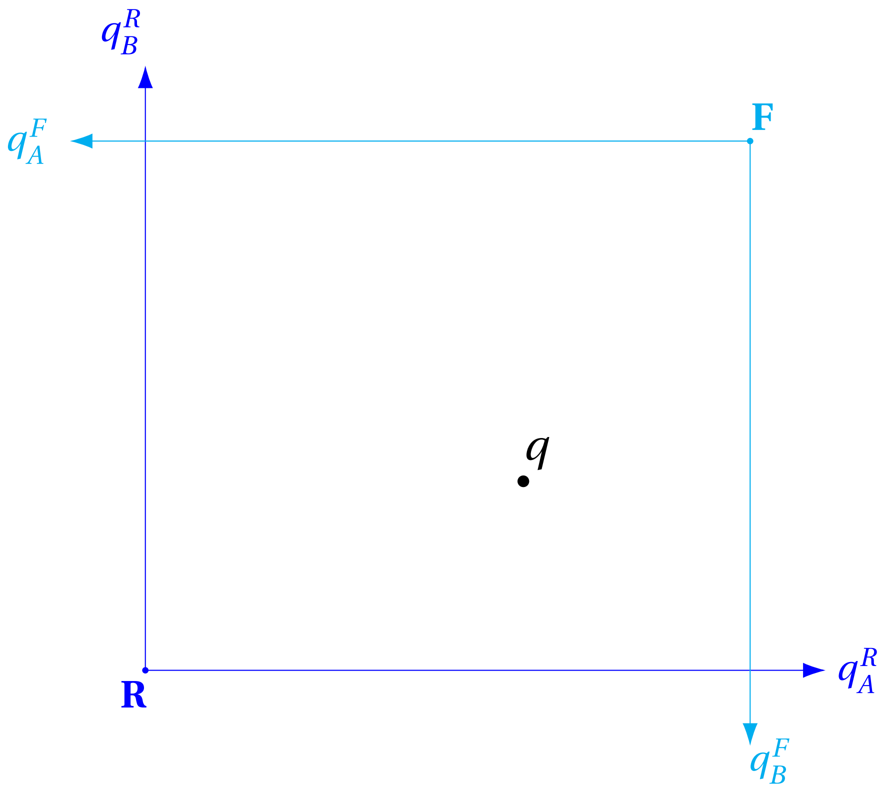
(i) What are some *desirable* allocations in this economy? That is, is there are desirable consumption bundles for Robinson and Friday, possibly different from their endowments?

(ii) How can we achieve that desirable allocation? Is there a *mechanism* that achieves this allocation without requiring too much information?

Before we move on to these questions, let us discuss a very neat way to illustrate this economy.

3.1.1 The Edgeworth Box

Thanks to Irish philosopher and economist Francis Y. Edgeworth (1845-1926), we have a very intuitive to visualize/graphically represent this exchange economy. The **Edgeworth Box** (sometimes referred to as Edgeworth-Bowley Box), depicted below, is a visualization of an exchange economy where there are only two agents and two goods.

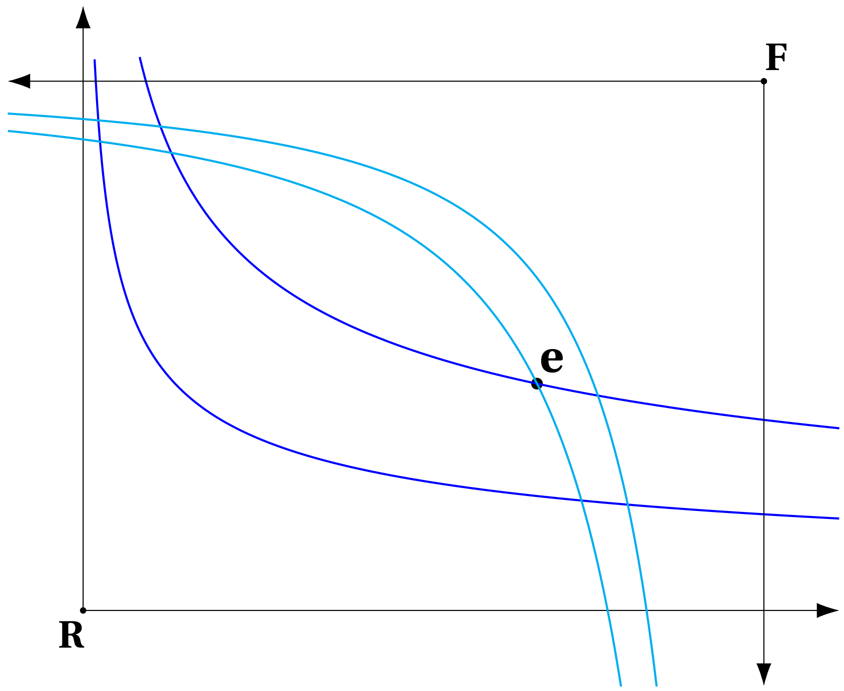


The Edgeworth Box is a rectangle with a width e_A and height e_B. Take any feasible allocation in this exchange economy. It can be represented as a point on the Edgeworth Box. Moreover, any point in this Edgeworth Box corresponds to a feasible allocation.

The figure looks like the figures we had in the previous chapter, where we covered consumer theory. Take any point in this box q. If you take point R as the origin, the x-axis represents Robinson's apple consumption (q_A^R) and the y-axis represents Robinson's banana consumption (q_B^R). The reason why there is a **box** is that Robinson's allocation has to be feasible: $q_A^R \leqslant e_A$ and $q_B^R \leqslant e_B$. But as long as these two inequalities are satisfied, such an allocation is feasible. Therefore, the Edgeworth Box is a box with width e_A and height e_B. Any point in this box is a feasible allocation for Robinson.

But... Here is the beauty of Edgeworth Box. If you rotate this figure by 180-degrees, you will see that the exact same reasoning applies to Friday! Do me a favor and rotate the figure upside down. Take point F as the origin. Now, the x-axis is $e_A - q_A^R$, which is equal to q_A^F, because q is a feasible allocation! Moreover, the y-axis is $e_B - q_B^R$, which is equal to q_B^F. Therefore, just a single point in this box is sufficient to represent four different numbers in an allocation $q = ((q_A^R, q_B^R), (q_A^F, q_B^F))$.

Two more things. **First**, note that the endowment $e = (e^R, e^F) = ((e_A^R, e_B^R), (e_A^F, e_B^F))$ is also a feasible allocation. Therefore, we can represent the endowment e as a point in the Edgeworth Box. **Second**, just as we drew the indifference curves of a consumer in consumer theory, we can draw the indifference curves of Robinson and Friday in this graph. We just need to be a little careful: when drawing Robinson's indifference curves, we take R as the origin (so that Robinson's higher indifference curves are in the northeastern direction). When drawing Friday's indifference curves, we rotate the figure and take F as the origin (so that Friday's higher indifference curves are in the southwestern direction). In the figure below, we illustrate two indifference curves of each agent, including the ones passing through the endowment point e. Robinson's indifference curves are depicted with dark blue, and Friday's indifference curves are depicted with light blue.



3.2 "Desirable" Allocations

Now we are ready to discuss the question: what are the desirable allocations in this Edgeworth Box? A HUGE WARNING that I have just asked a **normative** question. Yes, for the first time in this class, we have stepped into normative analysis. We are about to say something about what *should happen*, instead of what *happens*. To be honest, this is making me a little uneasy. After all, I do not feel comfortable making statements about what should happen between Robinson and Friday. Luckily for us, Vilfredo Pareto (1848-1923) felt less uncomfortable years ago, and invented a criterion to assess *desirability* of an allocation. We will use his criterion of assessment, which is called **Pareto Efficiency**.

Weirdly, I find it a better order when we first describe Pareto **in**efficiency before describing Pareto efficiency. So let's start with that. In words, an allocation is **Pareto inefficient** if it is possible to make an agent strictly better off without making another agent worse off. Formally,

Definition 72.1 *An allocation $q = (q^R, q^F)$ is* ***Pareto inefficient*** *if there exists another allocation $\tilde{q} = (\tilde{q}^R, \tilde{q}^F)$ such that:*

(i) $\tilde{q}^R \succsim^R q^R$ and $\tilde{q}^F \succsim^F q^F$,

(ii) either $\tilde{q}^R \succ^R q^R$, or $\tilde{q}^F \succ^F q^F$, or both.

Intuitively, a Pareto inefficient allocation is so undesirable that there is an obvious improvement: we can reallocate the goods so that one agent will be happier, and the other agent would not object to the reallocation. From a normative point of view, it is easy to see why such an allocation is undesirable. Simply put, the economy has not achieved *efficiency*: an agent can be made happier at no cost (i.e., without hurting the other agent).

To see a visualization, consider the point e in the picture above. We can move to an allocation in the "lens" between the indifference curves that pass through e. Such an allocation would make both agents better off, and at least one of them strictly better off (because they are both moving to "higher" indifference curves). Therefore, if the economy is stuck at point e, that would be really undesirable. An implication is, at point e, there is room for *mutually beneficial exchange* between Robinson and Friday.

Moving on to Pareto efficiency, you can guess where I am going. In words, an allocation is **Pareto efficient** if it is not Pareto inefficient. That is, it must be impossible to make an agent better off without making the other one worse off. Formally,

Definition 72.2 *An allocation $q = (q^R, q^F)$ is* ***Pareto efficient*** *if there does not exist another allocation $\tilde{q} = (\tilde{q}^R, \tilde{q}^F)$ such that:*

(i) $\tilde{q}^R \succsim^R q^R$ and $\tilde{q}^F \succsim^F q^F$,

(ii) either $\tilde{q}^R \succ^R q^R$, or $\tilde{q}^F \succ^F q^F$, or both.

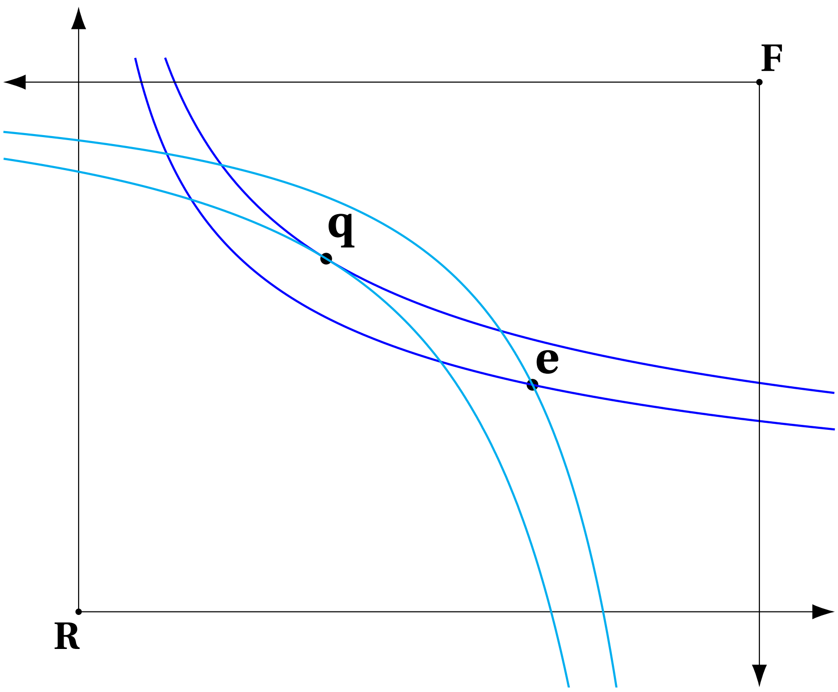
Once we are in a Pareto efficient allocation, if we want to make one agent happier by reallocating the goods, the other agent would object to such a reallocation. This means we have reached the point where the economy achieved some efficiency: there is no more "free happiness" one agent can attain without the other agent being hurt.

Let's move on to the graphical interpretation. If an allocation q is Pareto efficient, then there should be no "lens" between the indifference curves that pass through q. Therefore: an allocation q is Pareto efficient if the indifference curves of Robinson and Friday passing through q are tangent to each other. In other words, if a feasible allocation $q = (q^R, q^F)$ is Pareto efficient, then the following equality must hold:

$$MRS^R_{B,A}(q^R) = MRS^F_{B,A}(q^F) \tag{73.1}$$

In economic terms, at a Pareto efficient allocation, Robinson's valuation of apples in terms of bananas is equal to Friday's valuation of apples in terms of bananas. Why? Suppose, for the sake of the mental exercise, that they are different; for instance, suppose Robinson's valuation of apples is higher than Friday's. This means, at the margin, Robinson likes apples more than Friday does. Conversely, at the margin, Friday likes bananas more than Robinson does. But then, Robinson and Friday would find a mutually beneficial exchange: Robinson would give up some bananas and receive some apples from Friday in return. This would make both agents happier, which means the allocation is Pareto inefficient.

If you are in need of a visualization, see the next figure, where q is a Pareto efficient allocation and e is a Pareto inefficient allocation (as before).



It is time for the next exercise. Using the observations we made, one can find all the Pareto efficient allocations (by checking all the points where the indifference curves passing through are tangent). We can give a name to it:

Definition 74.1 *The set of all Pareto efficient in an Edgeworth Box is called the* **Pareto set** *or the* **contract curve**.

The notion of a "contract curve" captures the idea that once these agents find a Pareto efficient allocation, they will be able to sign self-enforcing contracts that allow them to stay there. After all, in a Pareto efficient allocation, if one agent wants to "break" the contract and offer a new one, the other agent will object to it.

I would like to conclude this section by making a few remarks about Pareto efficiency.

- Do not forget that it contains a normative judgment (i.e., a judgment about what *should* happen)! Even as a write this document, my mind sometimes slips and I use phrases such as "an agent is happier..." etc. These are dangerous sentences, and we should not be talking about "happiness" unless we know what we mean by it.[1] We evaluate Pareto efficiency based on the agents' preferences. But remember what I told when we started covering consumer theory: these preferences contain anything (social concerns, disregard for others, future considerations...) So they may not equal "happiness", or one person's happiness may contain the other person's unhappiness. In general, it is a good idea to be careful not to equate preferences with happiness. This is the primary reason why we should stick with positive analysis whenever possible. But life is life: sometimes, we find ourselves in a position to conduct normative analysis. In such cases, it is a good idea to at least remind ourselves about the perils of conducting normative analysis.

- Pareto efficiency, as it is clear by its name, is an *efficiency* concept. It says very little about *fairness*. What I mean by this is: an allocation can be Pareto efficient, but very unfair. Consider, for instance, the point F (the upper right corner of the Edgeworth Box). That allocation is Pareto efficient: if you wanted to make Friday happier by moving to another point in the Edgeworth Box, Robinson would object to it. Note, however, that this is a *terribly unfair* allocation. Robinson has everything, and Friday has nothing! That is a reason why you may find that allocation "undesirable", even though it is Pareto efficient.

  What I want to arrive at is: just because an allocation is Pareto efficient does not mean that we, as the society, would find it "desirable". There may be other grounds to find it "undesirable" as a society. This is the reason why many people

[1] We do not, in fact, know what we mean by it.

find Pareto efficiency a *too permissive* concept: it allows for *too many* allocations to be efficient, including some which are very unfair.

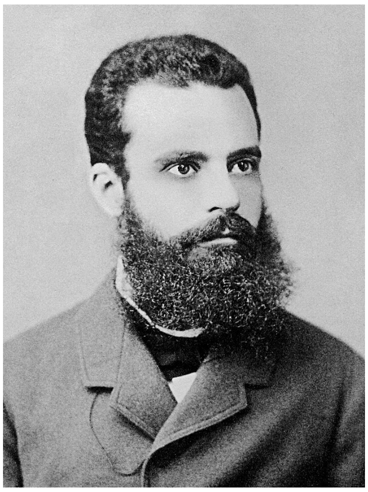


Figure 75.1: Vilfredo Pareto turning his head away from fairness and looking towards efficiency.

The flip side of the coin is that: when an allocation is Pareto inefficient, there are usually grounds to find it "undesirable". I mean, obviously, there are ways to improve upon such allocations. So most people think of Pareto efficiency as a *necessary condition* for an allocation being "desirable". That is, for an allocation to be "desirable", it must be at least Pareto efficient. But Pareto efficiency is not a *sufficient condition*: even when an allocation is Pareto efficient, it may still be "undesirable".

Subject to the caveats discussed above, we have finally defined what the "desirable" allocations are. The next question is: is there a mechanism to guarantee that the economy will end up in an allocation in the contract curve? This is the question we will explore in the next section.

3.3 Allocation Mechanisms

Up to this point, we discussed *what should* happen, and have not discussed *how it should* happen. That is, we have not introduced a mechanism to find a Pareto efficient allocation. Such a mechanism is called an **allocation mechanism** (for obvious reasons), and you can think of many allocation mechanisms.

An obvious possibility is having a *central planning* mechanism. Suppose there is a social planner who can seize and reallocate the goods. Both Robinson and Friday report their endowments and their preferences to the social planner. The social planner draws the Edgeworth Box based on the endowments, draws the contract curve based on their preferences, chooses an allocation on the contract curve and reallocates. This is certainly a way of achieving Pareto efficiency. Note that this mechanism does not use prices or such: it is purely a central planning mechanism. It should be noted, however, that it uses a lot of *information*: it requires the central planner to know Robinson and Friday's endowments and preferences. Endowments may be verifiable (even though that would be a difficult and time-consuming process), but preferences are just impossible to know. Therefore, the central planner must ask Robinson and Friday their preferences, and it should rely on the agents being truthful. But if the agents, for some reason, realize that they can lie to the central planner and receive a better allocation, there is nothing stopping them from doing so.[2] Therefore, the informational requirements of a central planning mechanism makes it a fragile allocation mechanism.

You can consider other allocation mechanisms as well. For instance, you can just rely on Robinson and Friday's ability to sort it out between each other. That is, Robinson and Friday can bargain over possible allocations and maybe reach a desirable allocation through bilateral negotiations and exchange. This would be called a *bargaining mechanism*, but guess what: it also suffers from informational issues (what if Robinson knew about Friday's preferences but Friday didn't know Robinson's preferences?), also we have to ensure that the bargaining is fair (i.e., Robinson cannot bully Friday into accepting an offer etc.)... Informational issues again.[3]

This brings us to our next question: is there an allocation mechanism that does not rely heavily on the information conveyed by agents? The answer to this question is the market mechanism.

3.3.1 The Market Mechanism

Here is another obvious possibility: why don't open a **market** for apples and bananas, and allow Robinson and Friday to buy/sell as many apples and bananas as they want? Opening a market means that there is a *market price* for apples and bananas, and the agents will be able to buy or sell at these prices. Let's denote the market price of apples with p_A, and the market price of bananas with p_B.

[2]The search for mechanisms that ensure the agents do not lie to the central planner is a long-lasting pursuit. There is a field of microeconomic theory, called *mechanism design*, which specializes on this question. In 2007, a trio of economists consisting of Leonid Hurwicz, Eric Maskin and Roger Myerson won the Economics Nobel Prize for their contributions to this field. If you find mechanism design interesting, you should check out Econ 448 (Economics of Information) offered in our department.

[3]If you find bargaining interesting, you should check out Econ 444 (Bargaining Theory and Experiments in Economics) offered in our department.

An Aside on Markets

When economic agents interact with each other, this typically happens in a **market**. In Turkish, market means something different: it means a department store. Due to this, I want to dedicate some time on defining what a market is when we use it in an economic sense.

A market is a broad term: **it is an infrastructure that facilitates interactions among economic agents**. Even a small group of agents who engage in economic interactions can be called a market (and some textbooks call it that way: "A market is a group of economic agents who are interacting.") Of course, a department store is a particular example of a market: it facilitates the trade of goods between consumers and producers. I just want to emphasize that the idea of a market is broader.

Competitive Markets

Let me make an observation we will revisit multiple times this semester. In the market I consider, Robinson and Friday **take the prices as given** and act accordingly. That is, the agents in this exchange economy are **price-takers**.

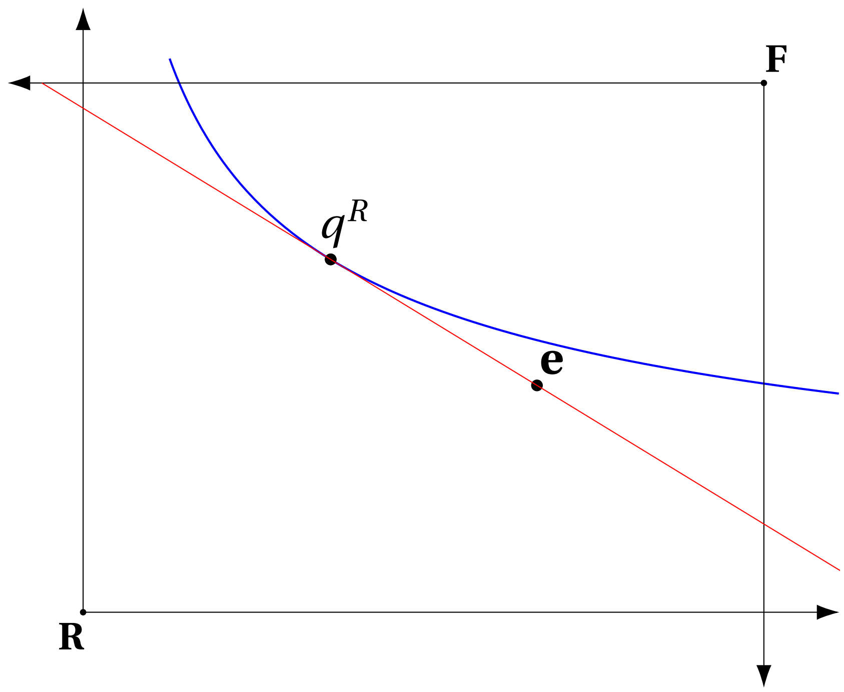
A market where every agent acts as a price-taker is frequently referred to as a **competitive market**. The underlying story here is that there are many agents, so that no single agent is powerful enough to affect the price. As you probably realize, there is a bit of cognitive dissonance here: in our exchange economy, there are only two agents – this can hardly be called an economy with "many agents". Nevertheless, I should once again remind you that all the results we discuss here will extend to multiple agents. Here we have only two agents, but that's for analytical convenience.

So, how does an agent act in a competitive market? Just like the agent we covered in the previous chapter: the agent consumes the *optimal bundle*. That is, the agent consumes the most preferred bundle among the feasible bundles. As a reminder, the feasible bundles are those affordable under the prices p_A, p_B and income I.

But... What is the income of an agent? Quite simply, the income of an agent is *the monetary worth of her endowment*. This corresponds to the following scenario: an agent can sell all her endowment, receive some money in return, and then spend that money on the optimal bundle. Therefore, when prices are p_A and p_B, Robinson's income is:

$$I^R = p_A \cdot e_A^R + p_B \cdot e_B^R$$

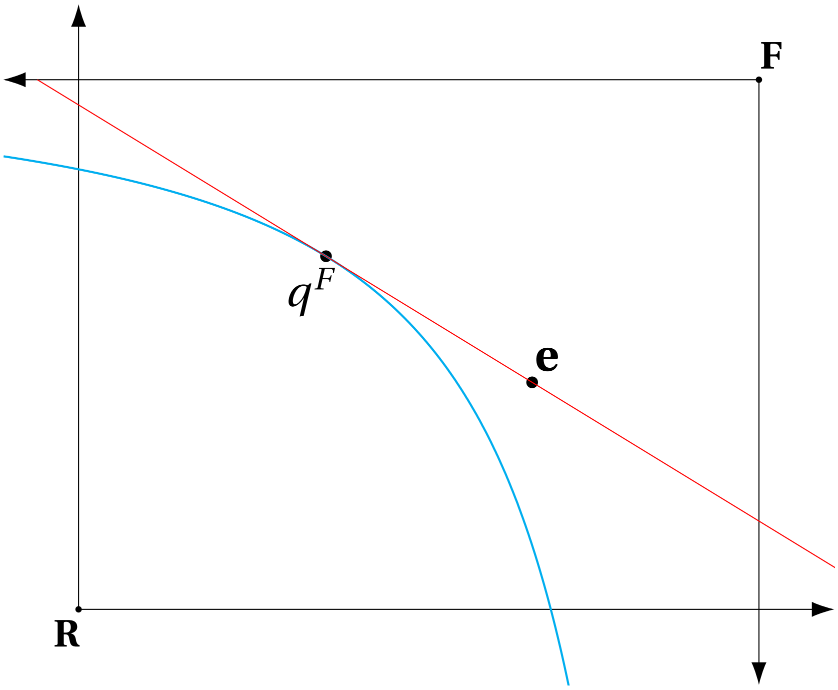
Under this income and prices, Robinson chooses the optimal bundle. Therefore, Robinson has a budget line with the (absolute value of) slope $\frac{p_A}{p_B}$. Moreover, the budget line passes through Robinson's endowment (because Robinson's endowment, as a bundle, has a price equal to Robinson's income). We simply repeat the exercise in the consumer theory chapter: we draw Robinson's budget line and find the optimal bundle under this budget line. Below is a representative figure showing Robinson's optimal bundle $q^R = (q_A^R, q_B^R)$. In this figure, the red line is Robinson's budget line (it passes through e and its slope has an absolute value of $\frac{p_A}{p_B}$).



Things are not really different for Friday. We just need to rotate the figure upside down and repeat the same analysis. When prices are p_A and p_B, Friday's income is:

$$I^F = p_A \cdot e_A^F + p_B \cdot e_B^F$$

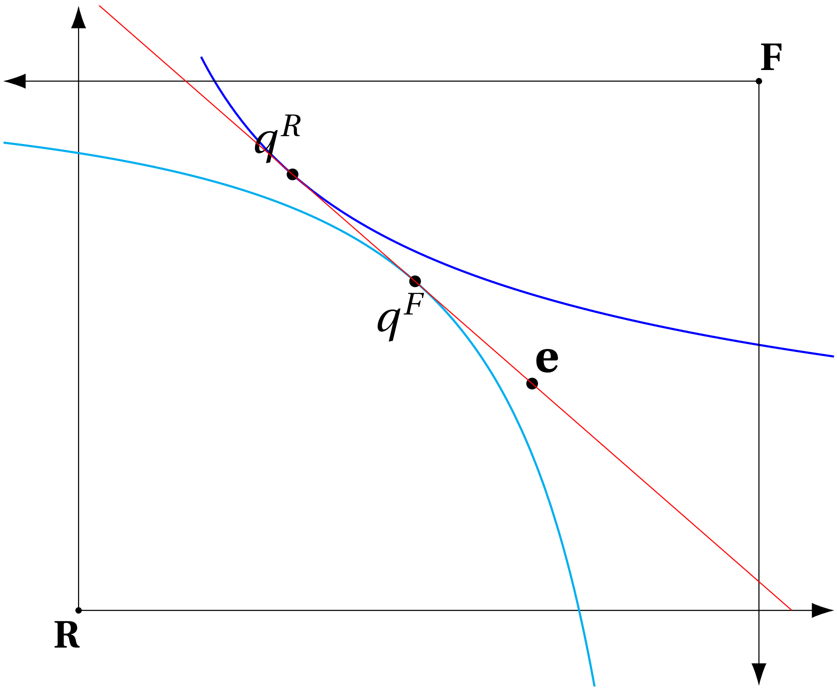
Once again, Friday has a budget line with the (absolute value of) slope $\frac{p_A}{p_B}$. If you rotate the figure upside down, that line still has a slope of $\frac{p_A}{p_B}$. The budget line again passes through Friday's endowment. Then, we can draw Friday's budget line: it passes through e and its slope has an absolute value of $\frac{p_A}{p_B}$. Below is a representative figure showing Friday's budget line in red and his optimal bundle q^F.



So far so good. We figured out how each agent behaves in a competitive market. As you recall from Chapter 1, an equilibrium requires that *each agent is optimizing*, so we are almost there. We need to take care of one final thing before we define what the equilibrium is.

Market Clearing

There is one final question we need to answer before we close the loop: how are the prices p_A and p_B determined? To make sense of this question, let's see the following figure.



In this figure, given prices p_A and p_B, Robinson and Friday are both optimizing. Robinson is consuming his optimal bundle $q^R = (q^R_A, q^R_B)$ and Friday is consuming his optimal bundle $q^F = (q^F_A, q^F_B)$. Still, my claim is that this cannot be an equilibrium. Why? Because in this figure, there is too much consumption of bananas:

$$q^R_B + q^F_B > e_B$$

So much so that the total consumption of bananas exceeds the total quantity of bananas in the island. Moreover, there is too little consumption of apples:

$$q^R_A + q^F_A < e_A$$

Clearly, this combination of optimal bundles is not sustainable – there are simply not enough bananas around, and some apples are just not consumed at all. An economist would say that there is **excess demand** for bananas and **excess supply** of apples. In this case, it is reasonable to imagine that the prices will adjust to reflect the excess demand and supply. That is, the price of apples will drop and the price of bananas will rise. This will result in $\frac{p_A}{p_B}$ decreasing, i.e., the budget line getting flatter. Because apples are cheaper, both agents will increase their consumption of apples and reduce their consumption of bananas. At the end, the prices will settle at a point where there is no excess supply or demand. This is the situation where the **markets clear**:

$$q^R_A + q^F_A = e_A, \quad \text{(the market for apples clears), and,}$$
$$q^R_B + q^F_B = e_B, \quad \text{(the market for bananas clears).}$$

This is the role prices play in a competitive market: **in equilibrium, prices clear the markets**. You may still ask "but who sets these prices?", which is a very reasonable question. However, at this point, we will be a little nontransparent about it. If you desire, you may imagine a social planner playing the role of the mediator between Robinson and Friday. The mediator gives hypothetical prices to Robinson and Friday, asks about their optimal bundles, adjusts the prices if there is excess demand/supply and asks again... Continuing until the markets clear. There are two things I want to emphasize: (i) the mediator in this scenario requires *much less* information than the social planner who needs to know the preferences. Here, the mediator only needs to know the excess demand and supply! (ii) Regardless of there is a mediator or not, the situation where the markets do not clear is unsustainable. Somehow, the prices will have to change – that situation cannot be an equilibrium. We just imagine there is a mediator to describe this adjustment process.

Competitive Equilibrium

All this work, and we are finally ready to define the notion of an **equilibrium** in the competitive market of an exchange economy. Simply put, the equilibrium is the situation where every agent optimizes given the prices, and the prices clear the markets. This is called the **competitive equilibrium**, or **market equilibrium**, or **Walrasian equilibrium** (named after economist Leon Walras, 1834-1910).

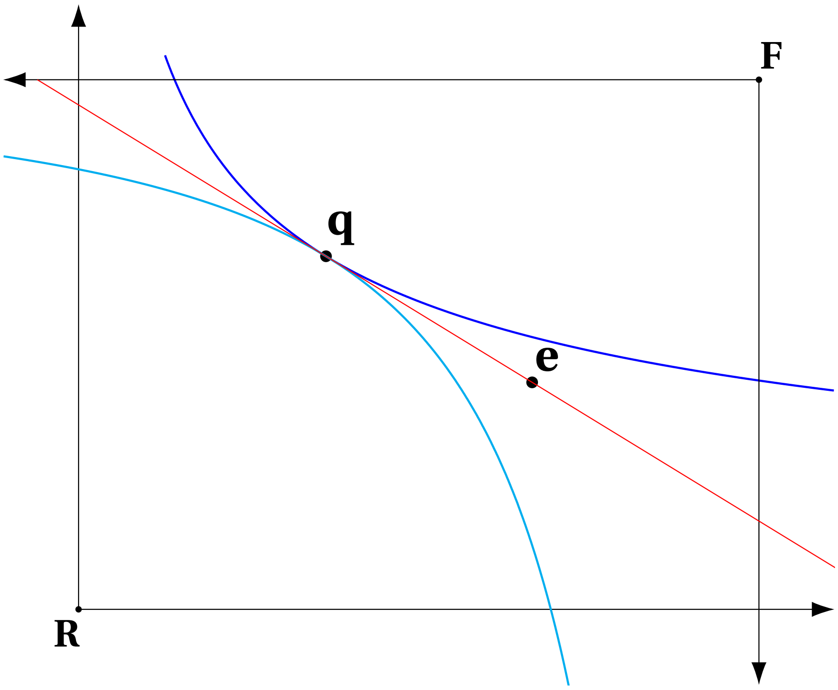
Definition 81.1 *A* **competitive equilibrium** *of an exchange economy is an allocation* $q = (q^R, q^F)$ *and prices* (p_A, P_B) *such that the following conditions hold:*

a. $q^R = (q_A^R, q_B^R)$ *is the optimal bundle for Robinson given prices* p_A, p_B*, and income* $I^R = p_A \cdot e_A^R + p_B \cdot e_B^R$.

b. $q^F = (q_A^F, q_B^F)$ *is the optimal bundle for Friday given prices* p_A, p_B*, and income* $I^F = p_A \cdot e_A^F + p_B \cdot e_B^F$.

c. *Markets clear:* $q_A^R + q_A^F = e_A$ *and* $q_B^R + q_B^F = e_B$.

Given our discussions so far, you may have imagined how the competitive equilibrium will look graphically. Below is an illustration of the competitive equilibrium in the Edgeworth Box.



3.4 Equilibrium Allocation is "Desirable"

Look at the figure illustrating Pareto efficient allocation: at a Pareto efficient allocation, the indifference curves of both agents are tangent to each other. Now, look at the competitive equilibrium allocation. The indifference curves are tangent to each other!

This is no surprise. We know that the indifference curves of each agent is tangent to the budget line, so they satisfy:

$$MRS^R_{B,A}(q^R) = \frac{p_A}{p_B} \text{ and } MRS^F_{B,A}(q^F) = \frac{p_A}{p_B}.$$

But then:

$$MRS^R_{B,A}(q^R) = MRS^F_{B,A}(q^F).$$

This is identical to condition (73.1), which is the condition for Pareto efficiency. Therefore, competitive equilibrium allocation is Pareto efficient!!!

This observation is obviously true for the indifference curves we drew here, but it is also fairly generalizable. It holds under very weak conditions —conditions we will not talk about in this class. What you need to remember is that this result is known as the **first fundamental theorem of welfare**, or **first welfare theorem**:

Theorem 83.1 [First Welfare Theorem] *Any competitive equilibrium allocation is Pareto efficient.*

Let us take a step back and remember what we have done. We started with a description of an environment with two agents, who have preferences over their allocations. We specified a "desirability" criterion for allocations, and argued that finding a desirable allocation may not be that easy. Then, we invented a mechanism that guarantees a desirable allocation! Note that the mechanism does not require coordination between the agents or anything: each agent can be quite self-interested, but from a societal point of view, we achieve a desirable allocation. This is the fundamental reason why a lot of economists love *market* as an allocation mechanism: it uses very little information, and it guarantees a desirable allocation.

A couple of notes before we leave you with your contemplation:

- First Welfare Theorem is a very strong result, and it has fundamentally shaped a lot of people's thinking. (You may argue that any political discussion about a "market economy", one way or another, is a discussion about First Welfare Theorem.) Due to this reason, it is a good idea to remind ourselves that the model contains a bunch of heroic assumptions: each agent has well-defined preferences, they face the same prices, agents act as price-takers, markets exist for all the relevant goods... Relaxing any of these assumptions may break the result.
- I know I am repeating myself, but this is a point that cannot be overemphasized: Pareto efficiency is an *efficiency* criterion, not a *fairness* criterion. Therefore, first welfare theorem says that a market mechanism achieves efficiency, but the resulting allocation may be really unfair.
- You may be wondering if there is a *Second Welfare Theorem*, and the answer is yes. Informally, it says that "Any Pareto efficient allocation can be achieved with a reallocation of endowments and a market mechanism." But we will leave this for Econ 203.

3.5 The "Cookbook"

Suppose someone gives you the description of an exchange economy (the endowments and preferences of Robinson and Friday) and asks you to characterize the competitive equilibrium. How to proceed?

First, revisit Definition 81.1 to remember that a competitive equilibrium is an allocation $q = (q^R, q^F) = ((q_A^R, q_B^R), (q_A^F, q_B^F))$ and prices (p_A, p_B). That is, you need to find a total of six numbers: $q_A^R, q_B^R, q_A^F, q_B^F, p_A, p_B$.

For these six numbers to be a competitive equilibrium, three conditions must be satisfied:

a. (q_A^R, q_B^R) is the optimal bundle for Robinson given prices p_A, p_B and income $I^R = p_A e_A^R + p_B e_B^R$.

   You can use Theorem 49.1 from Chapter 2 to express q_A^R and q_B^R as a function of p_A and p_B. This gives you two equations.

   For instance, if $q_A^R > 0$ and $q_B^R > 0$, then, by Theorem 49.1,

$$p_A q_A^R + p_B q_B^R = p_A e_A^R + p_B e_B^R \tag{84.1}$$

$$MRS_{B,A}^R(q^R) = \frac{p_A}{p_B} \tag{84.2}$$

b. (q_A^F, q_B^F) is the optimal bundle for Friday given prices p_A, p_B and income $I^F = p_A e_A^F + p_F e_B^R$.

   You can use Theorem 49.1 from Chapter 2 to express q_A^F and q_B^F as a function of p_A and p_B. This gives you another two equations.

   For instance, if $q_A^F > 0$ and $q_B^F > 0$, then, by Theorem 49.1,

$$p_A q_A^F + p_B q_B^F = p_A e_A^F + p_B e_B^F \tag{84.3}$$

$$MRS_{B,A}^F(q^F) = \frac{p_A}{p_B} \tag{84.4}$$

c. Market clearing gives you another two equations:

$$q_A^R + q_A^F = e_A \tag{84.5}$$

$$q_B^R + q_B^F = e_B \tag{84.6}$$

All in all, you end up with six equations (84.1)-(84.6) to find six unknowns $q_A^R, q_B^R, q_A^F, q_B^F, p_A, p_B$. This is, in principle, doable. It is up to you to decide how to proceed – you should arrive at the same numbers no matter how you do it.

My favorite way of proceeding is:

- First, play around with (84.1) and (84.2) to express q_A^R as a function of $\frac{p_A}{p_B}$.
- Then, play around with (84.3) and (84.4) to express q_A^F as a function of $\frac{p_A}{p_B}$.

- Then, use apple market clearing condition (84.5) to find $\frac{p_A}{p_B}$. Plug this into the equations above to find q_A^R and q_B^R.
- Repeat the process for q_B^R, q_B^F, and the banana market clearing condition.

You may have realized that we only find $\frac{p_A}{p_B}$ during this process. That is, we never find p_A and p_B separately; we only find the price ratio! (In other words, we never say how much an apple costs and how much a banana costs; we only say how much an apple costs in terms of a banana.) This is because the only thing that matters in a competitive equilibrium is the price ratio. If $p_A = 2$ and $p_B = 3$ are competitive equilibrium prices, $p_A = 4$ and $p_B = 6$ are **also** competitive equilibrium prices. Intuitively, if all the prices double, all the incomes will double and all the expenditures will double, but the consumption will **not** change. This is reminiscent of dropping some 0s from a country's currency. The prices and wages will change by the same ratio, but there should be no effect on consumption.[4] In practice, we normalize the price of one of the goods to 1, say $p_B = 1$, and find p_A.

The discussion in the above paragraph tells you that we do **not** find six unknowns. We only have five: $q_A^R, q_B^R, q_A^F, q_B^F, \frac{p_A}{p_B}$. You may ask:

> "Then why do we have six equations – isn't one of them redundant?"

The answer is:

> "Great observation – (have you ever though about doing a PhD in economics?) – and you are absolutely right. One of the equations is redundant. For instance, as long as (84.1)-(84.5) are satisfied, we can be certain that (84.6) is satisfied.
>
> This is known as the **Walras' Law**: as long as the apple market clears in a competitive equilibrium, the banana market also clears. (In general, when there are n goods, if $n-1$ markets clear then the remaining market also clears.)"

Let me also emphasize that just because (84.6) turns out to be redundant does not mean it is not useful. You can always use this to re-check your solution and make sure you did not make a math mistake.

So much for the detour with two agents, but we have learned some useful ideas about *efficiency, competitive markets, market clearing*, and *equilibrium*. They will reappear later this semester. For now, we will go back to the one agent model.

[4] At least according to this model – in real life, there are some psychological costs of dealing with too many 0s.

Extra Readings for Chapter 3

If you wonder what mechanism designers are up to, you should start by checking out the press release for the 2007 Nobel Prize. If you are further interested, you may read:

> Hurwicz, Leonid. "The Design of Mechanisms for Resource Allocation." *American Economic Review* 63, no. 2 (1973): 1-30.

This paper should be read in context: this is the height of the Cold War and people have been asking the question "Is it possible to have a central planning mechanism that achieves desirable allocations?". Within the context, this is a heavily political question. You can see the hints of Hurwicz's personal leanings, informed by growing up in Soviet Russia.

If you feel further intrigued, you may read:

> Hurwicz, Leonid, and Stanley Reiter. *Designing Economic Mechanisms.* Cambridge University Press, 2006.

and

> Milgrom, Paul. *Putting Auction Theory to Work.* Cambridge University Press, 2004.

On the notion of "markets", see the following article that investigates a particular market: the market for clinical psychologists in the United States.

> Roth, Alvin E., and Xiaolin Xing. "Turnaround time and bottlenecks in market clearing: Decentralized matching in the market for clinical psychologists." *Journal of Political Economy* 105.2 (1997): 284-329.

This is obviously too advanced reading for Econ 101. But when you have time, take a quick look at Section I of the paper. It defines a set of economic interactions among a set of agents (employers and psychologists) and gives an (almost anthropological) account of how the interactions evolve. It is useful to check the article to see what economics mean when they say "a market".

If you are further intrigued, check out:

> Roth, Alvin E. *Who Gets What – and Why: the New Economics of Matchmaking and Market Design.* Houghton Mifflin Harcourt, 2015.

and

> Sönmez, Tayfun. "Minimalist Market Design: A Framework for Economists with Policy Aspirations." (2024).

Exercises for Chapter 3

1) Consider an exchange economy with two consumers (Robinson and Friday) and two goods (apples and bananas). Suppose Robinson initially owns 200 apples and 100 bananas. That is, Robinson's endowment is

$$(e_A^R, e_B^R) = (200, 100).$$

On the other hand, Friday initially owns 100 Apples and 200 Bananas. That is, Friday's endowment is

$$(e_A^F, e_B^F) = (100, 200).$$

a. Draw the Edgeworth Box representing this economy and illustrate the point in the Edgeworth Box corresponding to the endowment allocation.

b. Suppose Robinson has Cobb-Douglas preferences where his marginal rate of substitution of Bananas for Apples at bundle $q^R = (q_A^R, q_B^R)$ is given by

$$MRS_{B,A}^R(q^R) = \frac{q_B^R}{q_A^R}.$$

Similarly, Friday has Cobb-Douglas preferences where his marginal rate of substitution of Bananas for Apples at bundle $q^F = (q_A^F, q_B^F)$ is given by

$$MRS_{B,A}^F(q^F) = \frac{q_B^F}{q_A^F}.$$

Is the endowment allocation Pareto efficient?

c. Identify all Pareto efficient allocations, i.e., the *Pareto set* or the *contract curve*.

d. Suppose that there is a competitive market for apples and bananas. If the market price for apples is p_A and the market price for bananas is p_B, what is Robinson's consumption of apples and bananas?

e. Suppose that there is a competitive market for apples and bananas. If the market price for apples is p_A and the market price for bananas is p_B, what is Friday's consumption of apples and bananas?

f. Identify the competitive equilibrium allocation of this exchange economy.

g. Revisiting your answer in part c, verify that the competitive equilibrium allocation is Pareto efficient.

Chapter 4

Towards the Demand Curve

After the detour with two agents, we now go back to the one-agent model. Now that we know how the consumer chooses her optimal bundle, we have the machinery to study how the optimal bundle changes with the parameters of the model (i.e., income of the consumer and prices of goods.) This is the fun stuff!

4.1 What If the Income Changes?

Let's start with a simple case. Consider an increase in a consumer's income (i.e., I goes up.) What happens?

Mathematically, the optimization problem changes because the *constraint set* changes. But that's fine – we did our analysis using a generic set of parameters, so the analysis still applies. In general, the optimal bundle $q^* = (q_1^*, q_2^*)$ satisfies

$$p_1 q_1^* + p_2 q_2^* = I$$

Moreover, if $q_1^* > 0$, $q_2^* > 0$, and preference is smooth, then

$$\mathrm{MRS}_{2,1}(q^*) = \frac{p_1}{p_2}$$

So far so good. Just solve this problem with a higher I. Geometrically, it corresponds to shifting the budget line higher, finding a new indifference curve tangent to it, and marking the point of tangency as the optimal bundle.

Let's do this graphically. Notation:

- Fix the prices at p_1 and p_2.

- Initial income: I^i.
- Optimal bundle under initial income: $q^i = (q_1^i, q_2^i)$.
- Final income: I^f.
- Optimal bundle under final income: $q^f = (q_1^f, q_2^f)$.

If $I^f > I^i$, it may look like Figure 90.1. The red line is the budget line under I^i. The dark red line is the budget line under I^f. Note that the two budget lines are parallel, because their slopes are the same: they are $-\frac{p_1}{p_2}$, which we keep fixed for this exercise. The black line is higher than the red line, because $I^f > I^i$.

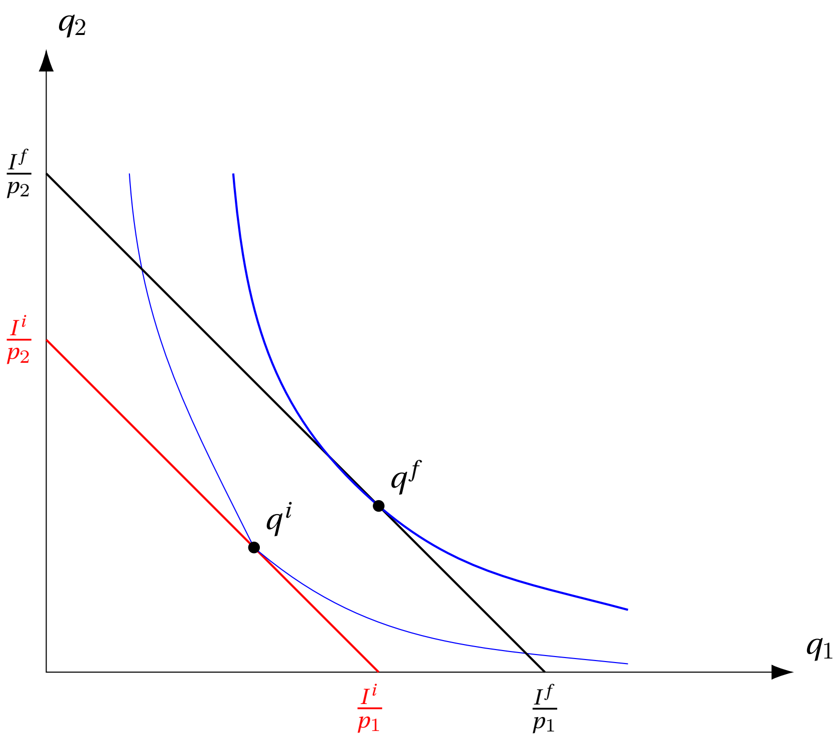


Figure 90.1: Optimal bundles under I^i and I^f.

The first thing that you should realize is that the consumer is *at least as happy as before* when she consumes q^f rather than q^i. There are two ways in which you can verify this.

a. The consumer is *richer* under income I^f compared to income I^i. This is because $I^f > I^i$, but you can verify this by looking at Figure 90.1. The budget set under I^f is *larger* than the budget set under I^i. This means that any feasible bundle under I^i is also feasible under I^f. Therefore, any bundle that the consumer can afford initially, she can also afford now. This means that the consumer **cannot be** worse off! In the worst case, she can consume the same bundle, q^i. This implies that q^f must be at least as good as q^i, i.e.

$$q^f \succsim q^i$$

b. Just eyeballing Figure 90.1, you can see that q^f is on a *higher* indifference curve than q^i. This is not surprising: because the consumer is richer, she cannot find her in a lower indifference curve. A higher indifference curve means that

$$q^f \succ q^i$$

You may be tempted to say "But isn't there a third way in which we can verify $q^f \succ q^i$? Monotonicity?" My answer is: yes for Figure 90.1, but not in general. Because one may have: $q_1^f < q_1^i$, but $q_2^f > q_2^i$. In such a case, monotonicity would not imply a preference between q^i and q^f. For instance, you may have a case like Figure 91.1. You can verify, using bullet points 1 and 2 above, that consumer is at least as happy as before when she consumes q^f rather than q^i. It is not due to monotonicity, though!

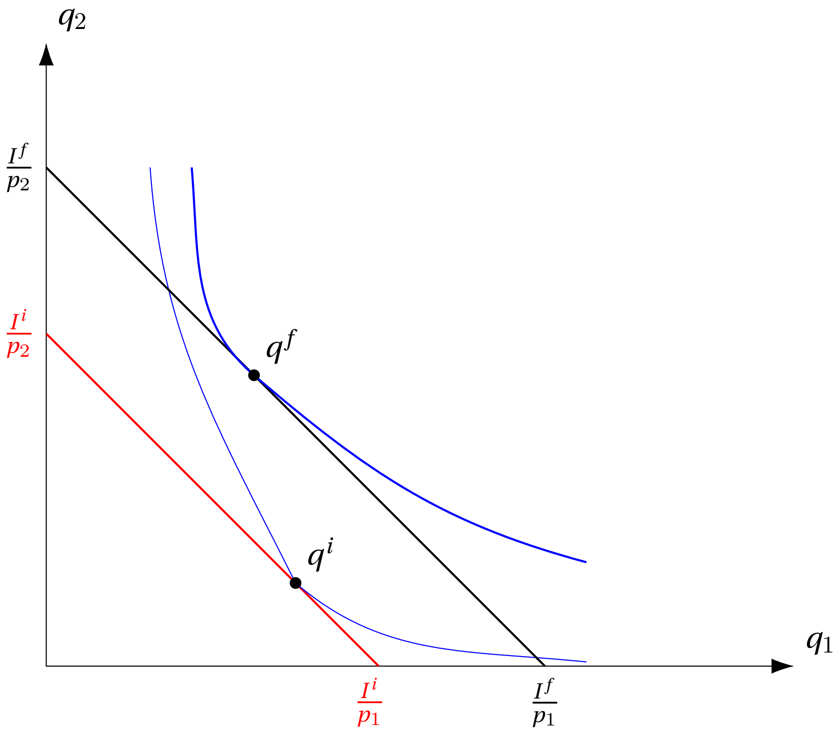


Figure 91.1: Optimal bundles under I^i and I^f, when good 1 is an inferior good.

This begs the question: what is the exact difference between Figure 90.1 and Figure 91.1? Here is the answer.

- If the indifference curves are as in Figure 90.1, the consumer consumes more of good 1 when she has higher income. We call goods like these **normal goods**.

  Definition 92.1 *Good i is a* **normal good** *if the consumer's consumption of good i increases with the consumer's income.*

  Examples of normal goods: goods that you consume more as you get richer. Cars, iPhones, sweaters, herbal teas, dishwashers...

- If the indifference curves are as in Figure 91.1, the consumer consumes *less* of good 1 when she has higher income. We call goods like these **inferior goods**.

  Definition 92.2 *Good i is an* **inferior good** *if the consumer's consumption of good i decreases with the consumer's income.*

  Examples of inferior goods: goods that you consume less as you get richer. Public transportation, rice, bulgur, instant noodle, instant coffee...

This classification of goods into two categories will be useful later.

You may have two questions at this point.

a. What if $I^f < I^i$? Just switch the labels of I^i and I^f. The budget line shifts inwards, and the consumer becomes worse off. If good 1 is a normal good, the consumer's consumption of good 1 decreases as the income decreases. If good 1 is an inferior good, the consumer's consumption of good 1 increases as the income decreases.

b. What if good 2 is an inferior good? Once again, just switch the labels of goods. If good 2 is a normal good, the consumer's consumption of good 2 increases as the income increases. If good 2 is an inferior good, the consumer's consumption of good 2 decreases as the income increases.

Below, I summarize what we have discussed so far. The relationship between the consumption of good i under optimal bundle (q_i^*) and income I is as follows.

	as $I \uparrow$...	as $I \downarrow$...
if i is a normal good, q_i^*...	$\uparrow$	$\downarrow$
if i is an inferior good, q_i^*...	$\downarrow$	$\uparrow$

4.2 What If the Price of a Good Changes?

Now, let's move on to a slightly more complicated case. Consider an increase in the price of good 1 (i.e., p_1 goes up.) Notation:

- Fix the income at I and the price of good 2 at p_2.
- Initial price of good 1: p_1^i.
- Optimal bundle under initial price of good 1: $q^i = (q_1^i, q_2^i)$.
- Final price of good 1: p_1^f.
- Optimal bundle under final price of good 2: $q^f = (q_1^f, q_2^f)$.

We can conduct a graphical analysis. If $p_1^f > p_1^i$, it may look like Figure 94.1. The red line is the budget line under p_1^i. The black line is the budget line under p_1^f. Note that the two budget lines are **not** parallel. The slope of the red line is $-\frac{p_1^i}{p_2}$, and the slope of the black line is $-\frac{p_1^f}{p_2}$. The budget set under the final price is smaller than the budget set under the initial price, because $p_1^f > p_1^i$.

Now, my claim is that the consumer is *at most as happy as before* when she consumes q^f rather than q^i. There are two ways in which you can verify this.

a. The consumer is *effectively poorer* under price p_1^f compared to price p_1^i. You can verify this by looking at Figure 94.1. The budget set under p_1^f is *smaller* than the budget set under p_1^i. This means that some feasible bundles under p_1^i are not feasible under p_1^f any more. The *purchasing power* of the consumer has decreased, even though she has the same income as before!

b. Just eyeballing Figure 90.1, you can see that q^f is on a *lower* indifference curve than q^i. This is because the consumer is effectively poorer.

What I am trying to say is: there is an **income effect** hidden in this graph. In Figure 94.1, the consumer reduces her consumption of good 1 from q_i^i to q_i^f due to two reasons.

a. Due to the **income effect**, the consumer is poorer. If good 1 is a normal good, the consumer reduces her consumption of good 1.

b. The relative price of good 1 in terms of good 2 is higher! Even if the consumer was not effectively poorer, she would choose to consume less of good 1 and more of good 2. Why?

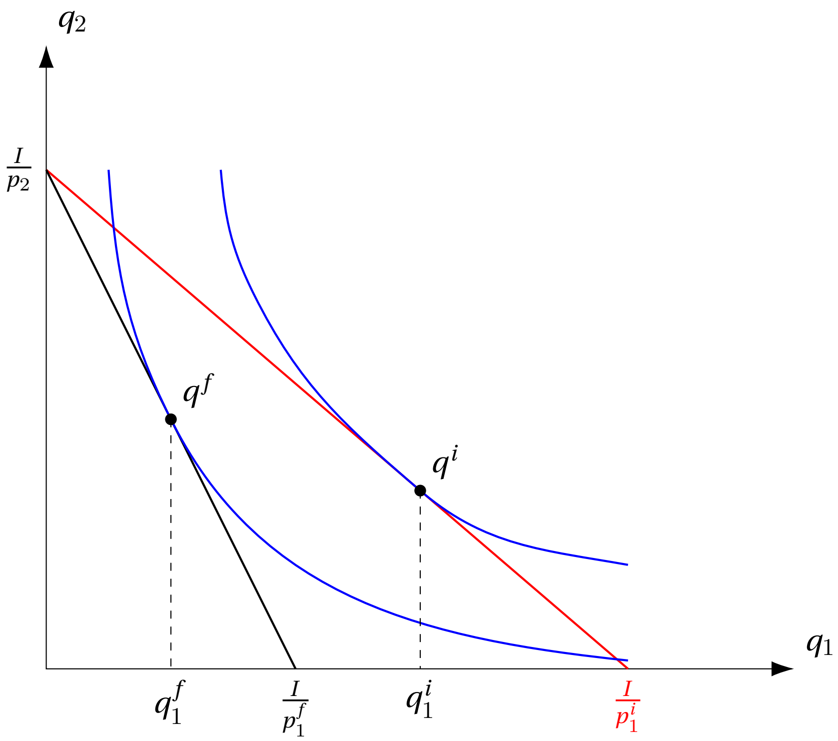


Figure 94.1: Optimal bundles under prices p_1^i and p_1^f.

Mathematically: Good 2 is now relatively cheaper, so that the consumer can reduce her consumption of good 1 a little bit and consume a lot of good 2 instead. Recall that the optimal bundle requires marginal rate of substitution of good 2 for good 1 to be equal to the price ratio. If price ratio is higher, the marginal rate of substitution is higher. But if the preferences satisfy diminishing marginal rate of substitution, this is achieved only when the quantity of good 1 is lower and the quantity of good 2 is higher.

Economically: The trade-off between good 1 and good 2 has changed. Now, in order to consume the same amount of good 1, the consumer needs to give up more of good 2. That is, the **cost** of good 1 in terms of good 2 is higher. Because of this, the consumer is less willing to consume good 1.

In any case, the consumer would *substitute* some of good 1 with good 2. This would happen *even if the consumer was not effectively poorer*. The consumer just finds it optimal to reduce her consumption of good 1 and increase her consumption of good 2. This is called the **substitution effect**.

So, in Figure 94.1, $q_1^f < q_1^i$ due to two effects. We want to decompose these two effects:

how much is the reduction in quantity of good 1 due to the consumer being poorer, and how much of it is due to good 1 being more expensive relative to good 2?

Recall what I said just above: the substitution effect would work towards the reduction in the quantity of good 1 "even if the consumer was not effectively poorer". This is the key: how can we think of a consumer who is not effectively poorer when the price of good 1 changes? The idea is: we will, hypothetically, **compensate** the consumer for the price change. That is, we will imagine we increase the consumer's income up to the point where, under the new prices, she is exactly as happy as she was before under the old prices.

More formally, we will find a level of income I^c such that the following holds. Suppose, under the prices p_1^f and p_2, if the consumer's income was I^c, her optimal bundle would be $q^c = (q_1^c, q_2^c)$. We want this optimal bundle to satisfy:

$$q^c \sim q^i$$

This construction makes sure that the consumer is exactly as happy as before, even though she is consuming a different bundle. After all, she is indifferent! This means that she is **compensated** for the increase in the price of good 1.

We will call I^c the **compensated income**, and q^c the **compensated demand**. The naming choice should be obvious by now.

Graphically, what we are doing is shifting the black line in Figure 94.1 until it is tangent to the indifference curve that contains q^i. The tangency point is q^c.

In Figure 96.1, the orange line is the budget line under compensated income I^c. If the consumer's income was I^c instead of I, she would consume q^c and be exactly as happy as if she was consuming q^i. Therefore,

- The move from q^i to q^c is due to the change in relative prices. It isolates the consumer's unhappiness due to being effectively poorer! She is as happy as before, she is just finding it optimal to consume less of good 1 and more of good 2 because good 1 is relatively more expensive.
- The move from q^c to q^f is due to the consumer being poorer. There is no effect of relative price, the consumer just changes her consumption because she is poorer.

Now, the move $q^c \rightarrow q^f$ should be familiar to you: this is the **income effect**. The comparison of q_1^c and q_1^f is the same as before. If good 1 is a normal good, $q_1^f < q_1^c$. If good 1 is an inferior good, $q_1^f > q_1^c$.

But what about the relationship between q_1^i and q_1^c? That is, what is the direction of

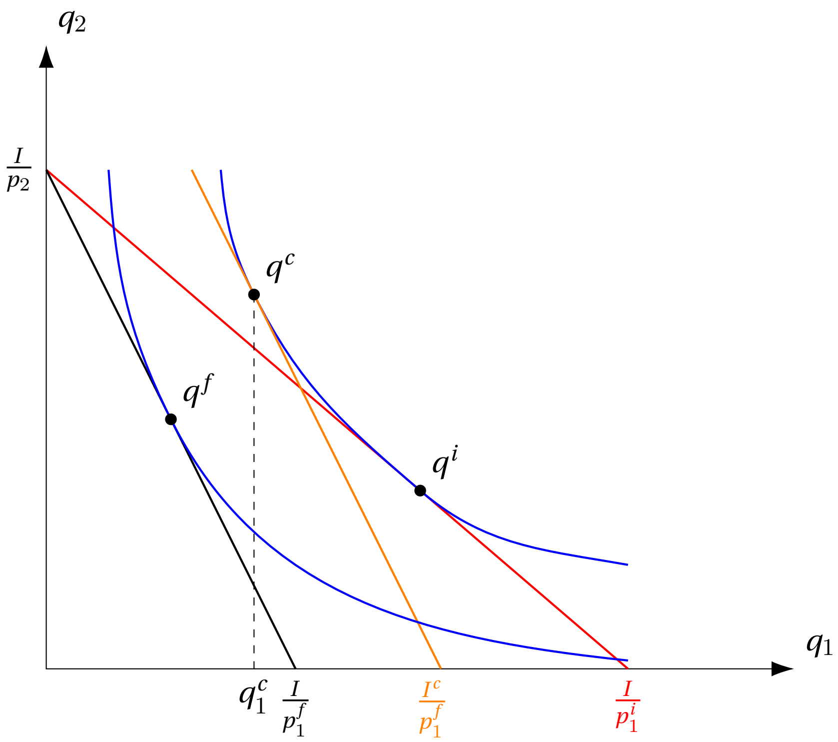


Figure 96.1: Compensated demand for good 1 (q_1^c) and compensated income I^c.

substitution effect? My claim is that, as long as diminishing marginal rate of substitution is satisfied, we must have $q_i^c < q_1^i$. Why?

- **Intuitively**, the move from q_1^i captures the effect of good 1 being relatively more expensive in terms of good 2. When something is more expensive, you consume less of it!
- **Mathematically**, q^i in Figure 96.1 satisfies:

$$MRS_{2,1}(q^i) = \frac{p_1^i}{p_2}$$

and, by construction, q^c satisfies:

$$MRS_{2,1}(q^c) = \frac{p_1^f}{p_2}$$

But since $p_1^f > p_1^i$, $\frac{p_1^f}{p_2} > \frac{p_1^i}{p_2}$. Therefore,

$$MRS_{2,1}(q^c) > MRS_{2,1}(q^i)$$

But recall that q^c and q^i are on the same indifference curve by construction! Since the preferences satisfy diminishing MRS, $MRS_{2,1}(q^c) > MRS_{2,1}(q^i)$ is satisfied only when q^c is to the northwest of q^i. Then, we must have $q_1^c < q_1^i$.

This is what I am saying: as long as the diminishing marginal rate of substitution is satisfied, for any good i:

	as $p_i \uparrow$...	as $p_i \downarrow$...
the substitution effect is such that q_i^*...	$\downarrow$	$\uparrow$

But recall that the total effect is a combination of substitution effect and income effect. For a normal good, let's put them together.

	as $p_1 \uparrow$...		
	substitution effect	income effect ($I \downarrow$)	total effect
if i is a normal good, q_i^*...	$\downarrow$	$\downarrow$	$\downarrow$

Both effects work in the same direction! The total effect is a decrease in the quantity of good 1 consumed. Graphically, this looks like Figure 96.1. In Figure 98.1 below, I demonstrate the two effects. Recall that $q_1^i \to q_1^c$ is the substitution effect, and $q_1^c \to q_1^f$ is the income effect. As long as diminishing MRS is satisfied, $q_1^c < q_1^i$. As long as good 1 is a normal good, $q_1^f < q_1^c$.

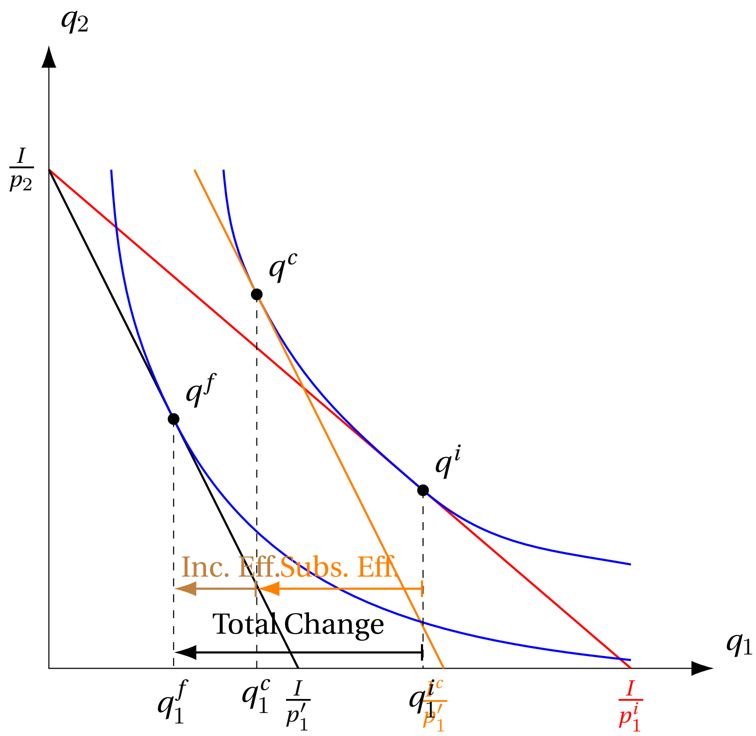


Figure 98.1: Effect of an increase in the price of good 1, when good 1 is a normal good.

What is good 1 is still a normal good, but it price decreases? You can just imitate the same analysis. All the effects will be reversed.

	as $p_1 \downarrow$...		
	substitution effect	income effect ($I \uparrow$)	total effect
if i is a normal good...	$q_i^* \uparrow$	$q_i^* \uparrow$	$q_i^* \uparrow$

See Figure 99.1 below.

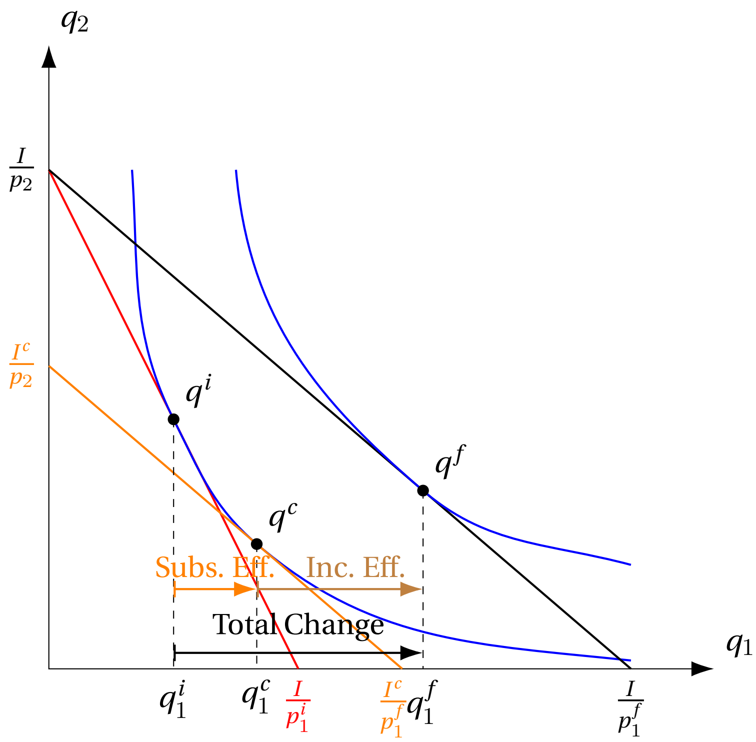


Figure 99.1: Effect of a decrease in the price of good 1, when good 1 is a normal good.

The next question is: what if good 1 is an inferior good and the price of good 1 increases? The substitution effect would still work in the direction of reducing the consumption of good 1. But now, income effect pulls in the opposite direction.

	as $p_1 \uparrow$...		
	substitution effect	income effect ($I \downarrow$)	total effect
if i is an inferior good, q_i^*...	$\downarrow$	$\uparrow$	?

Hm, this looks like a tricky case. If the income effect dominates, the consumer consumes more of good 1 when it is more expensive! What is happening? The consumer is so poor as a result of the price change that she moves away from higher quality consumption options and starts consuming good 1 even more.

We economists call such good **Giffen goods**, named after Robert Giffen. In a letter written to his friend Alfred Marshall, Giffen suggested the following phenomenon: in the late 19th century, as the price of bread increased, very poor individuals in Britain consumed more bread:

As Mr. Giffen has pointed out, a rise in the price of bread makes so large a drain on the resources of the poorer labouring families [...] that they are forced to curtail their consumption of meat and the more expensive farinaceous foods: and, bread being still the cheapest food which they can get and will take, they consume more, and not less of it.

-Alfred Marshall, 1895

Formally,

Definition 100.1 *Good i is a* **Giffen good** *if the consumer's consumption of good i increases as the price of good i increases.*

Note that for a good to be a Giffen good, it has to be an inferior good: the income effect should pull towards an increase in the quantity consumed. But being an inferior good is not enough in itself! The good has to be **so inferior** that the income effect must dominate the substitution effect! Graphically, it looks like Figure 100.1.

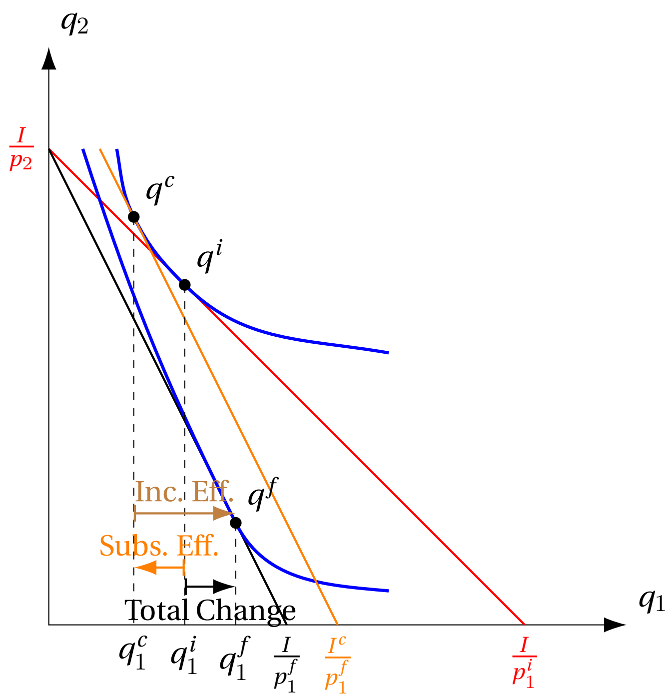


Figure 100.1: Effect of an increase in the price of good 1, when good 1 is a Giffen good.

I would claim that having a Giffen good is a mathematical possibility, but economically it is so unlikely that we can just assume it away. A good such that as it becomes more expensive, you buy more of it... I don't find this possibility very compelling. Giffen's observation about the bread in late 19th century Britain is controversial: we are not sure it empirically holds. Some claim that potatoes during the great Irish famine may be considered a Giffen good. Well, maybe, but even if that's true, that is a very particular time and location in history. In virtually any economic scenario we consider, the likelihood of having a Giffen good is so small that we can just discard that possibility. From now on, we will assume that a good is not a Giffen good. It may still be an inferior good, but even then we will assume that the income effect does not dominate the substitution effect. Those goods are sometimes called **ordinary goods**.

Definition 101.1 *Good i is an* **ordinary good** *if the consumer's consumption of good i decreases as the price of good i increases.*

From now on, let's agree that a good is not a Giffen good.

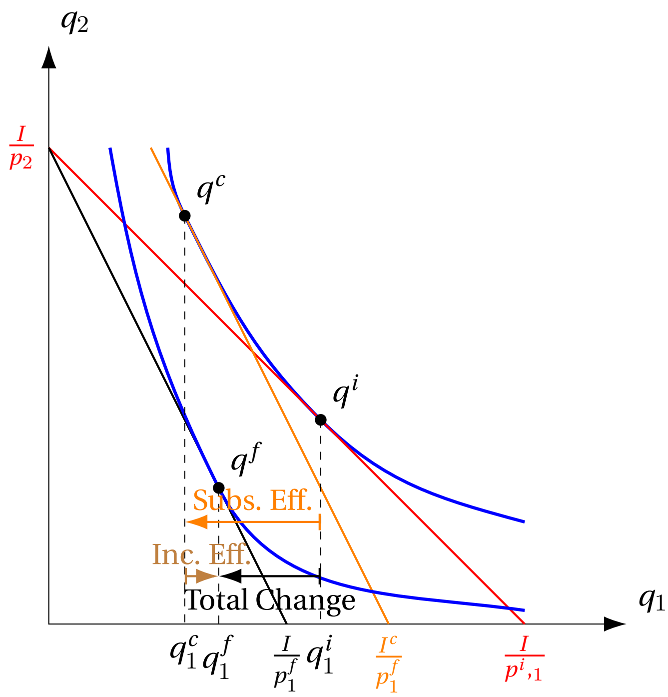


Figure 101.1: Effect of an increase in the price of good 1, when good 1 is an inferior but not a Giffen good.

4.3 Application: Income and Consumption Taxes

Consider the following scenario: the consumer's income is I TL, the price of good 1 is p_1 TL/unit and the price of good 2 is p_2 TL/unit. Under these prices and income, the consumer consumes bundle q.

Now suppose the state comes in and decides to impose a *tax* on the consumer. The state does so by imposing a **sales tax** on good 1. This works in the following manner: for each unit that the consumer consumes, the state charges an additional t TLon the consumer. Effectively, then, the price of good 1 increases to $p_1 + t$ TL/unit.

Based on what we covered in this section, you should be able to figure out what happens to the consumer's consumption. There will be a new budget set and a new optimal bundle q^t, and we will have: $q \succsim q^t$. See Figure 102.1.

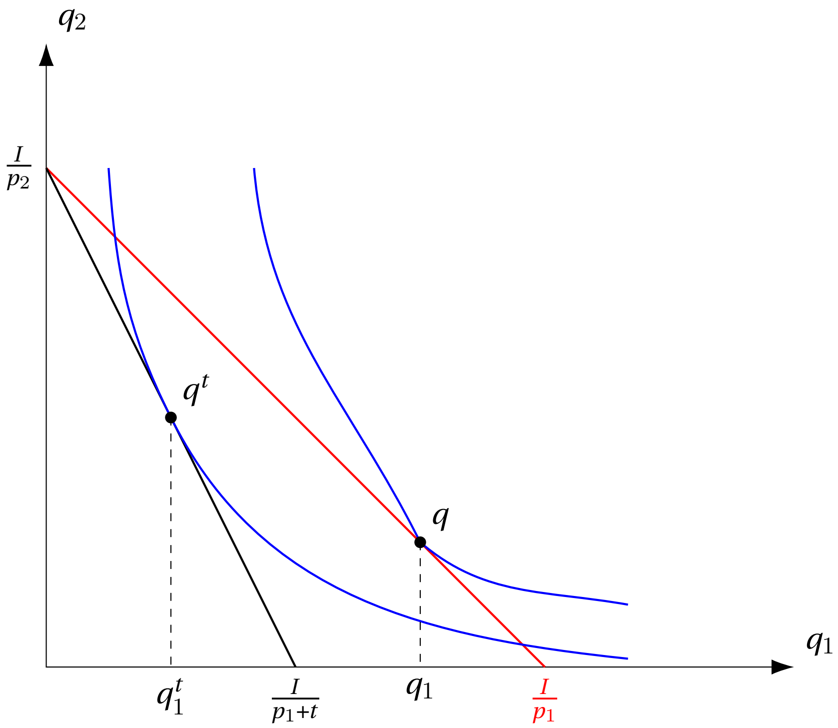


Figure 102.1: Optimal bundles with and without the sales tax.

Therefore, by imposing a sales tax of t TL/unit on good 1, the state obtains a tax revenue of $t \cdot q_1^t$ TL's. Could the state have obtained this tax revenue through other means? Yes; the state could have imposed an **income tax** of $t \cdot q_1^t$ TL's on the consumer. This is the money that will be taken away from the consumer's pocket, regardless of her consumption.

Under the income tax instead of the sales tax, the consumer would have an income of $I - (t \cdot q_1^t)$ TL's and the prices would remain p_1 and p_2. The optimal bundle would be q'. You know how to illustrate it, see Figure 103.1.

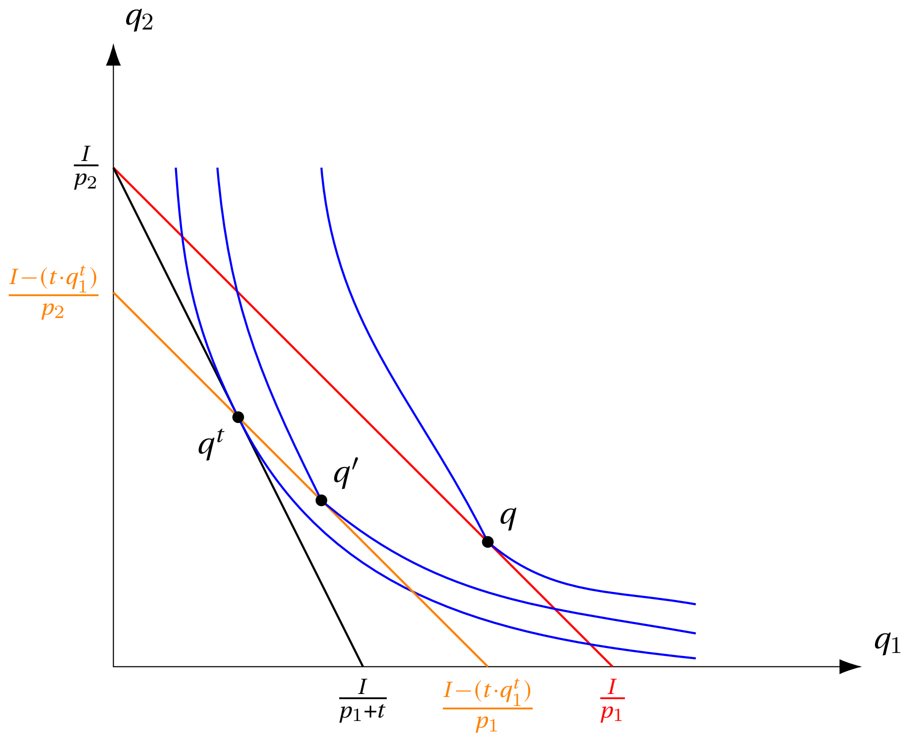


Figure 103.1: Optimal bundles with the income tax, with the sales tax, and without any tax.

As you can see, $q' \succsim q^t$. Take a moment to convince yourself that this is a general phenomenon! This is a valuable insight we gain thanks to our model of consumer's choice. **The consumer would rather be taxed through income tax, than the sales tax, if it generates the same tax revenue.** Then, a state whose objective is to generate some tax revenue should use income taxes rather than consumption taxes. This is because the consumption taxes are *distortionary*: they distort the consumer's choice by changing the consumer's trade-off between the goods. Income taxes have no such distortion.

What a powerful insight!

I should add that when states impose taxes, they have other concerns beyond generating tax revenue. We will talk about such concerns later in the semester.

Extra Readings for Chapter 4

Basically the only empirical evidence of the existence of a Giffen good is:

> Jensen, Robert T., and Nolan H. Miller. "Giffen behavior and subsistence consumption." *American Economic Review* 98, no. 4 (2008): 1553-1577.

The authors demonsrate that in very poor parts of China, rice is a Giffen good.

Also see:

> Heutel, Garth. "Theme park rides are Giffen goods." *Southern Economic Journal* 91, no. 3 (2025): 1107-1139.

The author shows that some rides in theme parks are Giffen goods (if we consider their wait times as their price), but this may be slightly outside our framework.

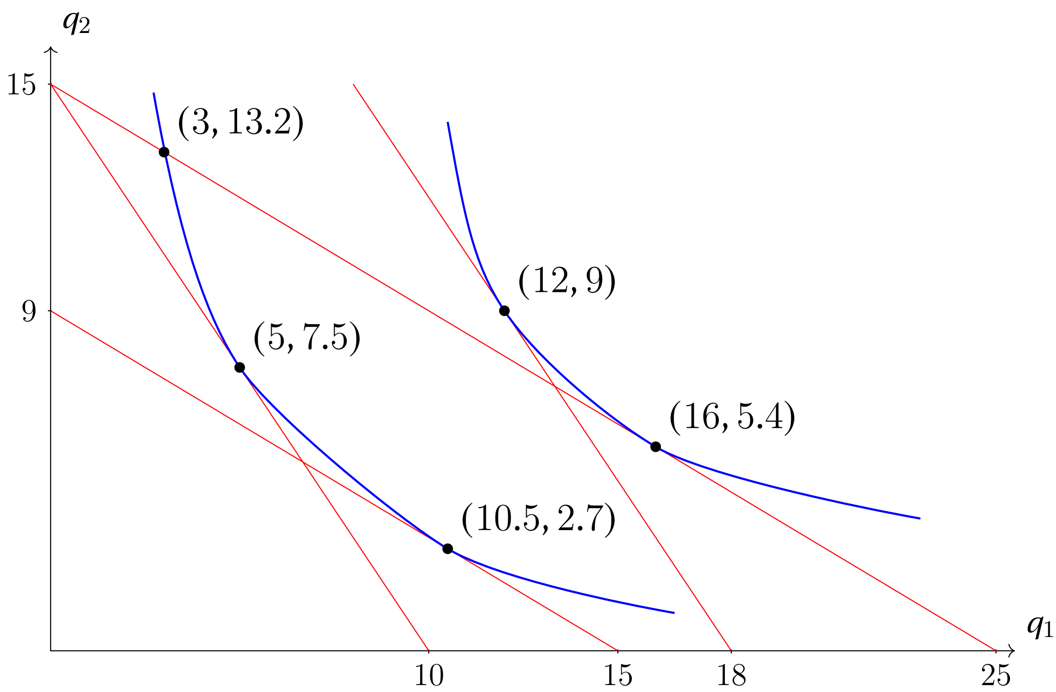
Our students sometimes point out that "consuming a good more when it is more expensive" occurs in the case of very high-end luxury goods as well. (Think of Louis Vitton purses.) While this behavior is observational the *Giffen behavior*, the reason for such behavior is fundamentally different. Rather than consuming these items more due to substinance effects, people consuming these goods more because they would like to *signal* to others that they can afford them. This is called *conspicuous consumption* – to analyze it, we need to develop a machinery of asymmetric information so that we can talk about signaling. You need to wait until Econ 204. Still, it is worth mentioning that the term "conspicuous consumption" was coined by Thorstein Veblen (1857-1929), a famed economist and sociologist. If interested, check out his magnum opus:

> Veblen, Thorstein. *The Theory of the Leisure Class: An Economic Study of Institutions.*

Exercises for Chapter 4

1) Suppose that you only consume two goods, say good 1 and good 2. If the price of good 1 increases by 10%, the price of good 2 does not change, and your income increases by 10%, will you become better off, worse off or not affected (be indifferent) as a result of the change?

2) Two indifference curves of a consumer and some possible budget lines are shown in the picture below:



a. Given that the income of the consumer is 75 TL, the price of good 1 is 3 TL/unit, and the price of good 2 is 5 TL/unit, what is optimal bundle of the consumer?

b. If the price of good 1 increased to 7.50 TL/unit (income and price of good 2 does not change), how much would the consumption of good 1 change?

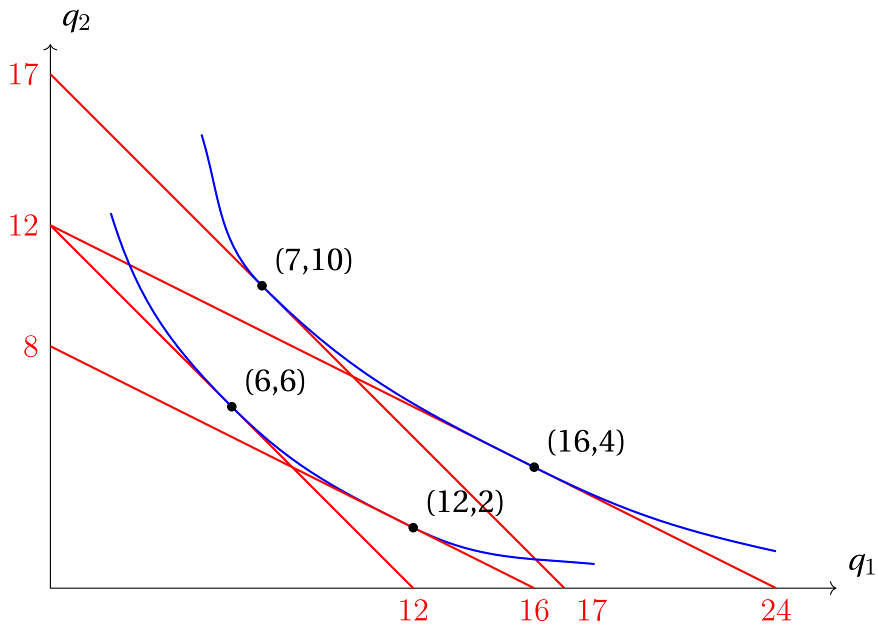
c. How much of this change is due to income effect and how much of it is due to substitution effect?

d. If we wanted to compensate the consumer for the increase in the price of good 1, how much extra income would we need to provide?

3) A consumer is considering buying two items: hand sanitizer and cologne. Let good 1 be the hand sanitizer, and good 2 be cologne. Let q_1 denote the amount of hand

sanitizer the consumer buys (in centiliters) and q_2 denote the amount of cologne the consumer buys (in centiliters). That is, if $(q_1, q_2) = (3, 5)$, the consumer consumes 3 cl of hand sanitizer and 5 cl of cologne.

The figure below illustrates two indifference curves of the consumer's preferences and several budget lines. The points marked by a circle are the points where the indifference curves are tangent to the lines that pass through those points.



The income of the consumer is 48 TL and the price of cologne is 4 TL/cl.

a. Initially, the price of hand sanitizer is 2 TL/cl. What is the optimal bundle? How much hand sanitizer will the consumer consume at the optimal bundle?

b. Now assume that the price of hand sanitizer increased to 4 TL/cl. How much does the consumption of hand sanitizer change? How much of this change is due to substitution effect and how much of it is due to income effect?

c. Suppose we want to compensate the consumer for the increase in the price of hand sanitizer by giving her some extra income (the price of hand sanitizer is still 4 TL/cl.) How much extra income should we give to the consumer that she is exactly as happy as before?

Now assume that the income of the consumer is 24 TL and the price of cologne is 2 TL/cl.

a. Initially the price of hand sanitizer is 2 TL/cl. What is the optimal bundle? How much hand sanitizer will the consumer consume at the optimal bundle?

b. Now assume that the price of hand sanitizer decreases to 1 TL/cl. How much does the consumption of hand sanitizer change? How much of this change is due to substitution effect and how much of it is due to income effect?

Chapter 5

Demand

In this chapter, we will draw a **demand curve** for a particular good and discuss its properties. A demand curve is the first important element of a **market for goods and services**. (The second important element of a market is the **supply curve**, which we will get to soon). Basically, we are very close to wrapping up our discussion on the **consumer side** of the market.

There are two important insights I'd like you to keep in mind from consumer theory.

a. When a consumer chooses how much to consume among several goods, her decision process resembles the following. **The consumer keeps buying a good until her relative valuation for that good is equal to the relative price of that good.**

   To digest this idea better, let me now focus on the consumption of a single good. Let good 1 be the good that we are focusing on, and let good 2 be the "composite good" that includes every other good and service the consumer can buy in the world. Well, to buy those other goods and services, the consumer will need to use money. Then, let good 2 be the money that the consumer keeps in her pocket to buy all the other goods and services. We will use the framework we developed in the previous chapter to analyze this scenario.[1]

   Now, q_1 is the amount of good that the consumer buys (denominated in kg's, lt's, lb's...) and let q_2 denote the amount of money the consumer "buys" (denominated in TL). Of course, the consumer does not actually buy money with money, but there is nothing wrong with imagining that the consumer can go ahead and buy 1 TL by paying 1 TL. Then,

[1]Hopefully, this will also convince you on how useful and generalizable this framework is.

- I is the income of the consumer (in TL's),
- p_1 is the price of good 1 per unit (in TL/kg, TL/lt...)
- p_2 is the price of good 2 per unit (in TL/TL). By construction, $p_2 = 1$ TL/TL. ("The price of one lira is one lira.")

The rest is the same. We can just imitate the analysis we made in Chapter 2 and find the optimal amount of good 1 for the consumer. Here, the consumer has preferences between bundles, where a bundle $q = (q_1, q_2)$ consists of q_1 units of the good and q_2 TL's. Given the consumer's preferences, at a bundle q, one can still define $MRS_{2,1}(q)$. This is the answer to the following question:

> **"Suppose the consumer is endowed with q_1 units of the good. If I take away one unit of the good away from the consumer, how many extra TL's should I give to the consumer, so that she is left indifferent?"**

If you think a little bit about it, this is a measure of how much the consumer values the marginal good when she already has q_1 units of the good. Let's give this a name:

Definition 110.1 *The* **marginal benefit** *(or* **marginal valuation***) of the consumer for the q-th unit is how much money the consumer is willing to pay for the last unit of the good at the margin, when she has q units of the good. This is given by $MB(q)$.*

To reiterate what I said before: marginal benefit for the q-th unit $\neq$ the benefit for the first q units of the good. The consumer may find her first t-shirt very valuable (i.e., the marginal benefit of the first t-shirt may be very high), but she may not care about the 100-th t-shirt if she already has 99 t-shirts (i.e., the marginal benefit of the 100-th t-shirt may be very low).

Now, recall that if we have an interior solution ($q_1^* > 0, q_2^* > 0$), the optimal bundle $q^* = (q_1^*, q_2^*)$ satisfies:

$$MRS_{2,1}(q^*) = \frac{p_1}{p_2}$$

In this setup where good 2 is money, we can replace $MRS_{2,1}(q^*)$ with $MB(q_1^*)$. Moreover, $p_2 = 1$. Therefore, the **quantity of good 1 consumed by the consumer when the price is** p_1 satisfies:

$$MB(q_1^*) = p_1$$

Under the optimal quantity, the marginal benefit is equal to the price!

In my experience, imagining the following process is a good way to think about the optimal quantity. The consumer starts by buying small quantities of good 1. At this quantity, if the consumer (marginally) values good 1 more than its price, she buys some more good 1. This will reduce the marginal benefit of good 1 due to the diminishing marginal rate of substitution. If, at the new bundle, she still values good 1 more than its price, she again buys some more good 1. The process goes on like this until the consumer does not want to buy any more good 1. But this is exactly the point where the marginal benefit is equal to the price.

(This process is just a product of our imagination. In this model, the consumer buys the quantity *at once*: the process does not move sequentially. It is not an incremental process. But this is a useful visualization. Moreover, there is nothing wrong with thinking that the consumer "imagines" this process as well.)

Another point: please note the crucial role played by the diminishing marginal rate of substitution in this argument. It ensures that as the consumer buys one more unit of a good, her valuation for the next unit of the same good is lower. This gives the decision process a certain **regularity**. We also believe that this is a reasonable assumption for many goods: most consumers really value the first unit of a good a lot, whereas they do not value 100th unit that much.

b. **If the price of an ordinary good increases, the consumer buys it less.** A reminder: all goods we will consider from now on will be ordinary goods.

   As you recall from Chapter 2, diminishing marginal rate of substitution also plays a very crucial role in this argument. Why? Diminishing marginal rate of substitution ensures that substitution effect works in the "proper" way. That is, it ensures that the substitution effect is such that: if p_1 increases, q_1^* decreases. And as you also recall from Chapter 4, for a good to be an ordinary good, the substitution effect must dominate.[2]

   Now, go back and check the heuristic we developed in the point above. What happens if the price of a good increases? The consumer stops buying the good earlier. Therefore, once we check the quantity consumed by the consumer, we will realize that it is lower. This is consistent with everything we said so far!

[2]Good 1 may be a normal good, in which case the substitution and income effects works in the same direction. Or, good 1 may be an inferior good but not a Giffen good, in which case substitution effect is stronger than income effect.

5.1 Demand Schedules

Let us now focus on a single good. Fix the income of the consumer and prices of other goods, and consider an ordinary good 1. We will study the possible prices of good (P) and the quantities of good 1 consumer buys at these prices (q_1^*). That is, we will study the **demand** of the consumer for good 1 at various prices.

Based on the things I reiterated above, let me now construct a **demand schedule**. It is basically an excel sheet of possible prices P and quantities demanded at these prices q_1^*. I construct this by going to the consumer and asking the following question repeatedly:

> "If the price of good 1 per unit is P, what is the quantity you demand q_1^*?"

(This is a hypothetical exercise. I ask this question for different values of P and record the answer on an excel sheet. I am not worried about the consumer lying to me. In real life, we can construct this excel sheet by looking at the data. Suppose, over time, the price of good 1 varies. We can record the amount the consumer buys at different prices. This would construct a demand schedule.)

To fix ideas, suppose good 1 is the cups of tea the consumer drinks per day. Let P be the price of a cup of tea for every cup of tea the consumer drinks. Let q_1^* denote the number of cups of tea the consumer drinks per day.

- I may go ahead and ask: "If the price of tea is 7 liras per cup, how many cups of tea would you consume a day?" Suppose the consumer says: "Zero. I am not willing to buy even a single cup of tea if it was 7 liras."

  What does it mean? Based on the process I described in Section 1, what the consumer says means the following. "The value of the first cup of tea is less than 7 liras."

- I then ask: "What if the price is 6 liras per cup?" Suppose the consumer says: "I am willing to buy one cup of tea."

  What does it mean? Based on the process I described in Section 1, what the consumer says means the following. "The valuation of the first cup of tea I consume is more than 6 liras. The valuation of the second cup is less than 6 liras." (This is because the consumer stops before buying the second cup.)

- I then ask: "What if the price is 5 liras per cup?" Suppose the consumer says: "I am still willing to buy one cup of tea."

  What does it mean? Based on the process I described in Section 1, what the consumer says means the following. "The valuation of the second cup is less than 5 liras."

- I then ask: "What if the price is 4 liras per cup?" Suppose the consumer says: "I am willing to buy two cups of tea."

  What does it mean? Based on the process I described in Section 1, what the consumer says means the following. "The valuation of the second cup of tea I consume is more than 4 liras. The valuation of the third cup is less than 4 liras." (This is because the consumer stops before buying the third cup.)

- ...

- I then ask: "What if the price is 0 liras per cup?" (i.e., what if tea was free?) Suppose the consumer says: "I am willing to buy five cups of tea."

  What does it mean? Based on the process I described in Section 1, what the consumer says means the following. "The valuation of the fifth cup of tea I consume is more than 0 liras. The valuation of the sixth cup is less than 0 liras." (This is because the consumer stops before consuming the sixth cup.)

Based on this survey, I can construct a demand schedule (an excel sheet). It looks like this:

P (price per unit)	q_1^* (no. of units bought)
7	0
6	1
5	1
4	2
3	3
2	3
1	4
0	5

Table 113.1: An Example of a Demand Schedule.

To reiterate:

The **demand schedule** of a consumer for a good is a relation between the possible prices of the good and the quantities the consumer would like to consume at these prices.

5.1.1 Another Way of Constructing a Demand Schedule

The demand schedule is simply an excel sheet. I can easily rearrange the columns of this excel sheet. This would correspond to asking the following question repeatedly:

> "If I want you demand q_1^* units of good 1, what should the maximum price P of good 1 per unit be?"

(Once again, this is a hypothetical exercise. I am not worried about the consumer lying to me. In real life, this is coming from data.)

I am not doing much indeed, just reordering the columns. It now looks like this.

q_1^* (no. of units bought)	P (price per unit)
1	6
2	4
3	3
4	1
5	0

Table 114.1: An Example of the Same Demand Schedule as in Table 113.1.

Even though I am not doing much, the interpretation of the table now differs. If you are following closely, what I am doing corresponds to the following procedure.

- I go ahead and ask: "If I want you to consume one cup of tea per day, what is the price I should charge per cup of tea?" The consumer says: "Six liras. I am not willing to consume even one cup of coffee if the price exceeds six liras."

  What does it mean? "My valuation of the first cup of tea is six liras." / "The marginal benefit of the first cup of tea is six liras."

- I then ask: "If I want you to consume two cups of tea per day, what is the price I should charge per cup of tea?" The consumer says: "Four liras. I am not willing to consume the second cup of tea if the price per cup of tea exceeds four liras."

  What does it mean? "My valuation of the second cup of tea is four liras." / "The marginal benefit of the second cup of tea is four liras."

- ...

- I then ask: "If I want you to consume five cups of tea per day, what is the price I should charge per cup of tea?" The consumer says: "Zero liras. I am not willing to consume the fifth cup of tea unless tea is free."

  What does it mean? "My valuation of the fifth cup of tea is zero liras."/"The marginal benefit of the fifth cup of tea is zero liras."

To reiterate:

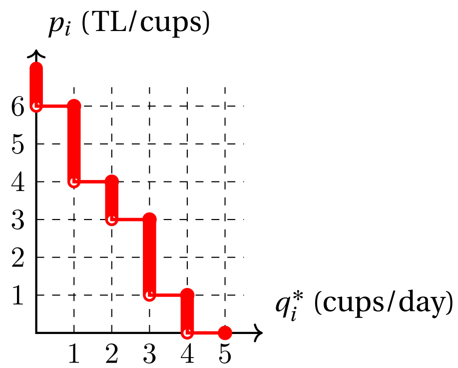
> The *demand schedule* of a consumer for a good is a relation between the quantities of the good and the marginal benefit that the consumer gets from consuming the last unit of the quantity.

5.2 Individual Demand Curves

As it happens, displaying information in an excel sheet is not the optimal way of illustrating it. We, as economists, prefer graphs! So let me summarize the information I obtained throughout this survey in a graph. This is called an **(individual) demand curve.**

As a convention, I will put the price (P) in the y-axis and the quantity (q_1^*) in the x-axis. NOTE THAT THIS IS A DIFFERENT GRAPH THAN THOSE WE WERE USING EARLIER. The previous one was showing the constrained optimization problem the consumer faces, for a given P. This one shows the outcome of the optimization problem for different values of P.

We end up with a graph looking like this:



Let me just re-emphasize one thing. Because a demand curve contains the same information as in a demand schedule, it also has two interpretations.

a. (From P to q_1^*) It shows, at each price, the quantity demanded by consumer.

b. (From q_1^* to P) It shows, at each quantity q, the marginal benefit of consumer for the q-th unit of the good.

It is important to keep both interpretations in mind, as there are cases when either is useful.

5.3 Market Demand Curves

Individual analysis is all nice and good, but sometimes we want to analyze economic forces on the aggregate level. To do this, we will go from individual level to **market level**. From Section 3.3.1:

> A **market** is an infrastructure that facilitates interactions among economic agents.

So we need to define what the market under consideration is. As long as we define it nicely, we can analyze a market. For instance:

- The market may be the Starbucks in the ground floor of Faculty of Business Administration. There is a well defined set of consumers in this market (Bilkent affiliates). We can go ahead and ask every single Bilkent affiliate about their individual demands of cups of tea they demand per day from that Starbucks.
- The market can be the whole Bilkent campus. Once again, there is a well defined set of consumers (Bilkent affiliates). We can go and ask every single consumer: "How many cups of tea you demand per week at the following prices?"
- The market can be the Migros in the Bilkent Center. There is a well defined set of consumers (people who shop from that Migros). We can go and ask: "How many kilograms of tea you demand per year at the following prices?"
- The market can be the whole country. The consumers are the citizens. We go and ask: "How many tons of tea you demand per year at the following prices?"
- The market can be the whole world. The consumers are everyone in the world. "How many tons of tea you demand per year at the following prices?"
- This can be the Shell gas station in the Bilkent road, or the market for gasoline in the whole country...

Regardless of what the market is, as long as we have a clear definition of it, we can obtain the market demand curve. To obtain the market demand curve, we add up the individual demands of every consumer in the market. That is, at every single price, we add up the individual demands of the consumers at the said price. The total we obtain is the **quantity demanded** by the consumers in the market. We will denote this quantity by Q.

Figure 117.1 is a representative figure where we add up the individual demand curves of two consumers. For more than two consumers, the process is the same.

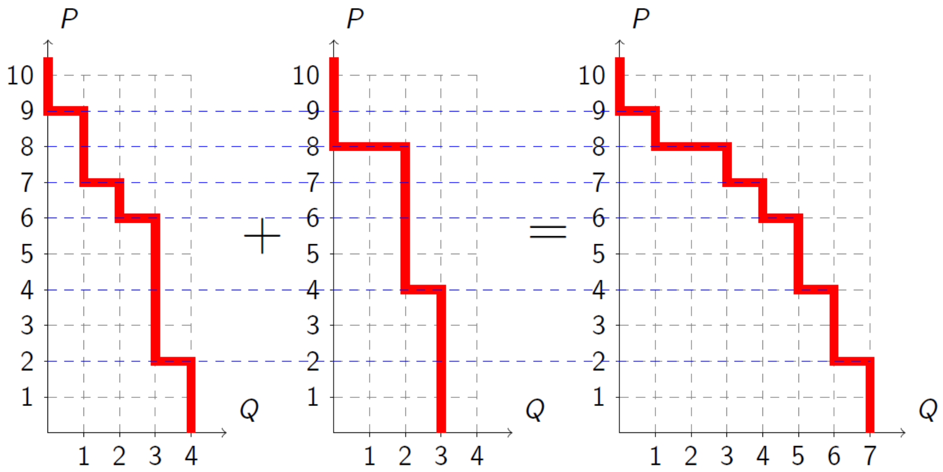


Figure 117.1: Addition of two individual demand curves.

Let me make one more modification. The demand curves we have so far have a lot of kinks. This is fine as long as you know how they are derived. But in the future, we will give some equations of the demand curves and conduct some mathematical analysis. Giving the equation for a curve with so many kinks is very difficult! To circumvent this problem, I will draw "smoother" individual demand curves. Of course, the market demand curve (which is merely an addition of individual demand curves) will be smooth as well. So it will look like Figure 118.1.

From now on, we will draw "smooth" demand curves. There are at least three ways to defend smooth demand curves.

a. As discussed above, it is easier to write down equations for smooth curves.

b. You can imagine us having finer and finer increments in quantities and prices. Instead of asking for the quantity demanded at each lira, we ask for quantity demanded at each kuruş. We may also have finer increments in quantities: instead of asking in terms of kilograms, we may ask in terms of miligrams etc. Because the increments are finer, the jumps in the demand curve will also be smaller. It will look much more like a smooth curve!

c. You may imagine a smooth curve as an "approximation" to a curve with kinks. As long as we understand what happens in the benchmark case (i.e., the case

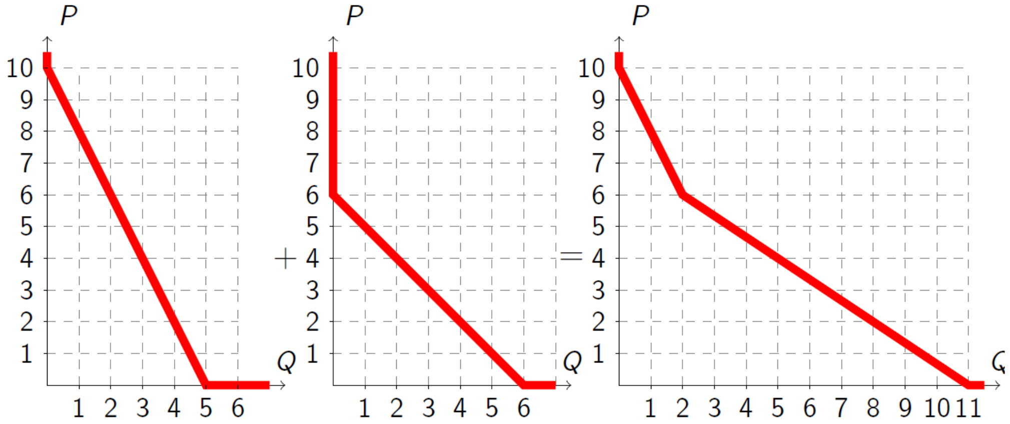


Figure 118.1: Addition of two (smooth) individual demand curves.

with smooth curves), the general insights will go through.

From now on, we will draw a demand curve for a market as a smooth one. Figure 118.2 illustrates a representative demand curve. We will use the letter D to label a demand curve, which stands for "Demand".

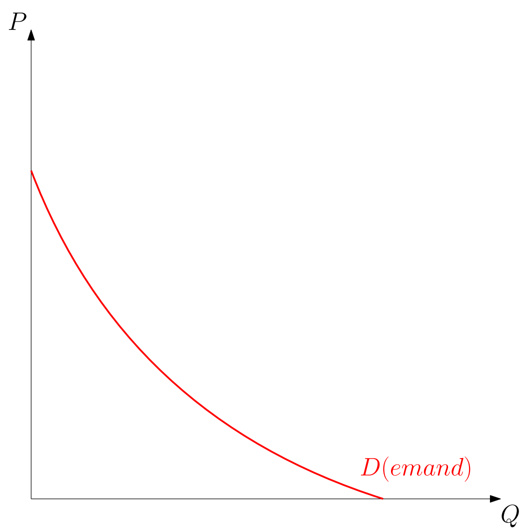


Figure 118.2: A representative demand curve.

A couple of general points about demand curves follow.

First, realize the convention of labeling the axises. We are using P for the market price and Q for the market quantity of a good. We will keep this notation for the rest of this class (and, to be frank, for the rest of your economics education).

Second, note that the market demand curve is decreasing. This is not surprising: a market demand curve is the summation of individual demand curves. Because we are considering ordinary goods, each individual demand curve is decreasing. Of course, if you add up several decreasing curves, the end result will be a decreasing curve!

The idea that a market demand curve is decreasing (also known as "downward-sloping") will be carried for the rest of this class (and, to be frank, for the rest of your economics education). This is such a widely accepted fact that economists call it a law.

Definition 119.1 [Law of Demand] *Holding everything else constant, when the price of a good rises, the quantity demanded falls.*

Third, a simple remark. Most economics textbooks (including ours) draw demand curves as lines, not curves. That is made for the sake of convenience, but that may be misleading sometimes. **A demand curve can be a line, but it does not have to be.** The only requirement we are imposing on the demand curve is being downward-sloping.

Finally, just like an individual demand curve, a market demand curve also has a dual interpretation.

a. (From P to Q) It shows, at each price, the total quantity demanded by the consumers in the market.

b. (From Q to P) It shows, at each quantity Q, the marginal benefit of the marginal consumer.

   This can sometimes be confusing. By the marginal consumer, I mean the following. At the quantity Q and price P, there is a consumer who is at the edge of buying the last unit of the good or dropping her consumption by one unit. If the price increases a tiny bit, this consumer would reduce her consumption by a tiny bit. P, therefore, is exactly this consumer's valuation for that last unit of good.

   Perhaps it is easier to understand through the following example (which is not a general example, but it is illustrating). Consider a good that a consumer buys at most one unit of. For instance, the Econ 101 textbook. Consider the demand for Econ 101 textbooks in Meteksan bookstore in a semester. This is a downward-sloping curve: if the price P is lower, more students will but the textbook, resulting in a higher Q. For the sake of the argument, let $P = 150$ liras and $Q = 147$ textbooks be on this curve. This has two meanings: (i) when the price of textbook is 150 liras, Meteksan will sell 147 textbooks. (ii) If we order the students by their valuation of the textbook, the 147th student has a valuation of 150 liras. Why? At any price higher than 150 liras, Meteksan sells less than 147 textbooks, which means this particular student stops buying the textbook.

5.4 Structural Changes in the Economy

Recall what we said when we defined law of demand: "Holding everything else constant..."[3] What is "everything else"? It includes income of the consumers, prices of other goods, consumers' tastes etc. Basically, they include structural changes in the economy, other than the price of the good.

Now, as you can imagine, if we change "other things", quantity demanded will change as well. For instance, at a given price, if the income of the consumers increase, and if the good under consideration is a normal good, the consumer will demand a higher quantity due to income effect. Moreover, because such an increase happens at any given price, quantity demanded Q will increase **at every price** P. We will denote it with a **shift in the northeastern direction** in the demand curve. That is, we draw a new demand curve that has a higher Q at every P. Consider Figure 121.1. The new demand curve (under higher income) is D'. Note that D' has a higher quantity demanded at each price. For instance, at price P, the quantity demanded is Q', which is higher than Q.

What if the income of the consumers decrease, and the good under consideration is a normal good? Then, the demand curve will **shift in the southwestern direction**, representing a decrease in the quantity demanded at each price. Consider Figure 122.1. The new demand curve (under lower income) is D'. Note that D' has a lower quantity demanded at each price. For instance, at price P, the quantity demanded is Q', which is lower than Q.

One thing I want to emphasize: most textbooks illustrate a shift in the demand curve by *shifting it by the same amount in every price.* That is, *they draw a parallel curve.* That is made for convenience, and it does not have to be that way. For instance, the quantity demanded may increase more in lower quantities (e.g., the income effect may be high in lower quantities), but it may increase less in higher quantities (e.g., the income effect may be low in higher quantities).

Below is a discussion of several variables that may shift the demand curve.

5.4.1 Income of the Consumers

If the good is a normal good,

[3] If you enjoy plugging in Latin phrases, you can also say *ceteris paribus.* It literally translates as *all else constant.*

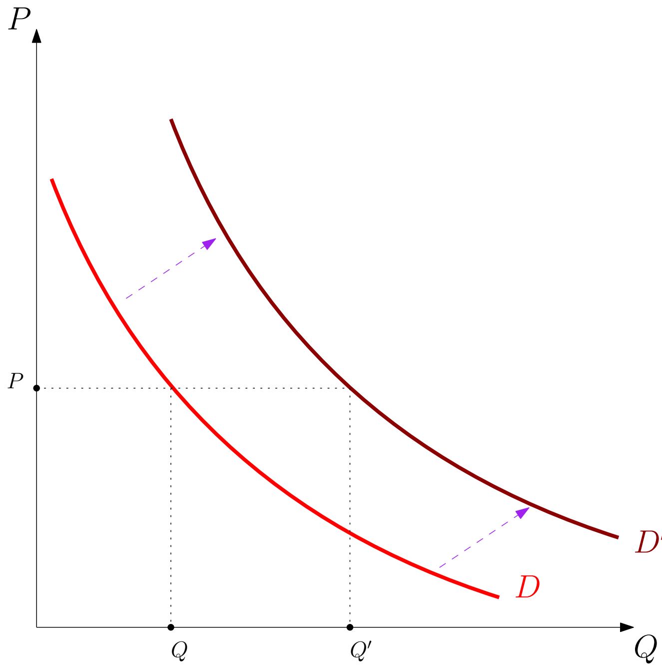


Figure 121.1: A shift in the northeastern direction of the demand curve. Some resources also call this a “shift upwards” or an “outward shift”.

- An increase in income will cause a shift of the demand curve in the northeastern direction.
- A decrease in income will cause a shift of the demand curve in the southwestern direction.

If the good is an inferior good,

- An increase in income will cause a shift of the demand curve in the southwestern direction.
- A decrease in income will cause a shift of the demand curve in the northeastern direction.

5.4.2 Prices of Other Goods

We say that two goods are **substitutes** if they can be used for the same purpose, i.e. if they can substitute each other easily. Examples of substitutes: tea and coffee, rice and bulgur, hand sanitizer and cologne, Samsung smart phones and iPhones, PlayStation and XBox, blue shirts and white shirts. (Cologne can be used instead of hand sanitizer, I can wear a white shirt instead of a blue shirt.)

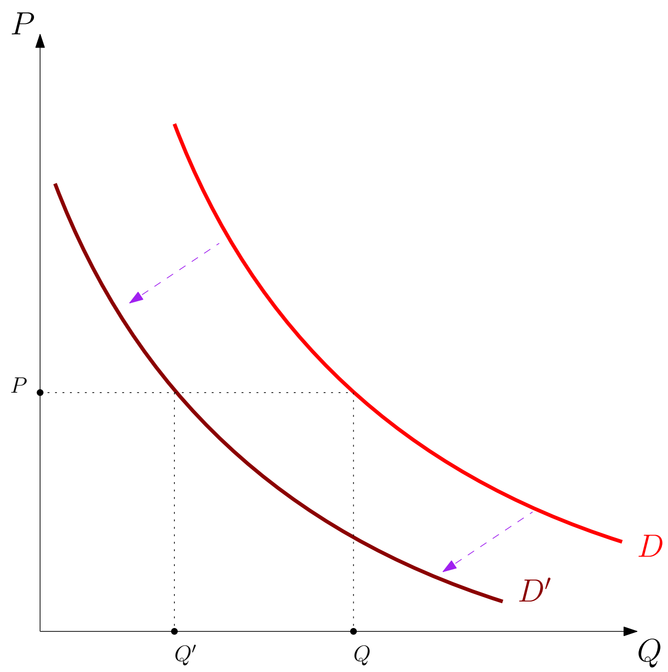


Figure 122.1: A shift in the southwestern direction of the demand curve. Some resources also call this a "shift downwards" or an "inward shift".

Consider the demand curve for good X, and consider another good Y which is a substitute of good X.

- An increase in the price of good Y will cause a shift in the northeastern direction of the demand curve for good X.
- A decrease in the price of good Y will cause a shift in the southwestern direction of the demand curve for good X.

Why? When good Y is more expensive, the consumers will start substituting good Y with good X. This will result in the consumers demanding a higher quantity of good X at every price. For instance, if blue shirts are more expensive, people will start buying white shirts instead, which will result in an shift in the demand curve for white shirts in the northeastern direction.

We say that two goods are **complements** if they are used together, i.e. if they complement each other easily. Examples of complements: coffee and sugar, Chai Tea Latte and cinnamon, cars and gasoline, smart phones and accompanying apps, a game console and accompanying games, tuxedos and bowties. (People consume PlayStation and Uncharted together, most people wear a bowtie with a tuxedo.)

Consider the demand curve for good X, and consider another good Y which is a complement of good X.

- An increase in the price of good Y will cause a shift in the southwestern direction of the demand curve for good X.
- A decrease in the price of good Y will cause a shift in the northeastern direction of the demand curve for good X.

Why? When good Y is more expensive, the consumers will start buying it less ("the law of demand"). But since good X and good Y are used together, then the consumer will buy less of good X as well. This will result in the consumers demanding a lower quantity of good X at every price. For instance, if cars are more expensive, people will start buying fewer cars, and as a result they will consume less gasoline as well. This will result in an shift in the demand curve for gasoline in the southwestern direction.

5.4.3 Changes in Tastes and Preferences

Sometimes people will start preferring a good more due to an exogenous change in tastes and preferences. This will cause an shift in the demand of the said good in the northeastern direction. Advertising is a classical example: it can shift a demand curve in the northeastern direction. (When Emma Chamberlain drinks 17 cups of iced coffee in her 30-minute videos, a lot of people develop an interest in consuming iced coffee.) On the contrary, a lawsuit against a firm may make people move away from the products produced by that firm, causing a shift in the demand in the southwestern direction. (When Harvey Weinstein scandal blew up, people -rightfully- reduced their consumption of movies produced by the Weinstein Company.)

This does not have to be just tastes as well: the external conditions may lead to consumers preferring a good more. In 2020, a pandemic was going around. (You may have heard about it.) Early on, the pandemic it caused a **tremendous** northeastern shift in the demand curve for hand sanitizers, face masks, bread machines, and toilet papers. On the other hand, it caused in southwestern shift in the demand curve for plane tickets, restaurants, haircuts, and souvenirs. See Figure 124.1.

5.4.4 Expected Future Prices

If the consumers will expect the price of a good to increase in the future, they will want to buy it now rather than later. This will result in an northeastern shift in the demand curve. The opposite case happens if the price is expected to decrease.

A classical example is electronic items. If the consumers expect the government to impose a tax on certain electronic items in three months, the quantity demanded for

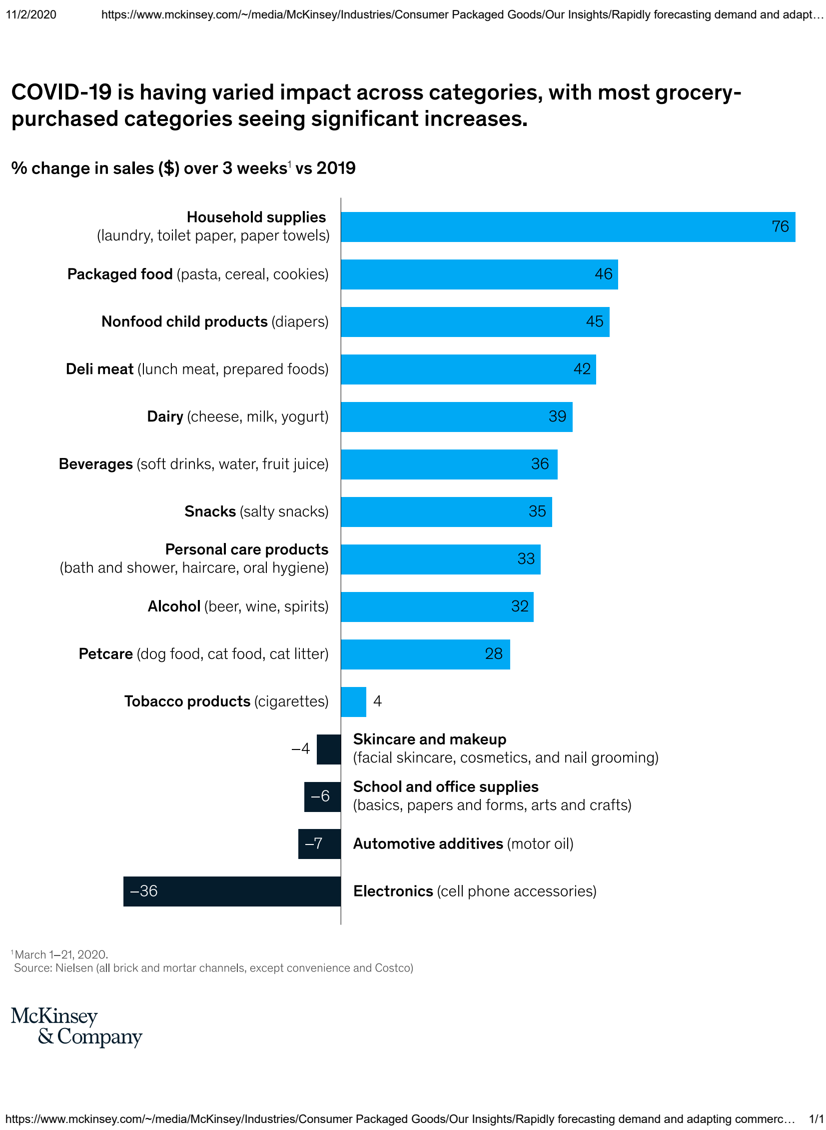


Figure 124.1: The pandemic caused an outward shift in the demand curves of some goods and services, and an inward shift in others. Source: McKinsey.

those items right now will increase. If, on the other hand, people expect Apple to reduce the price of iPhone 11 in three months, the quantity demanded for iPhone 11 right now will decrease.

5.4.5 Population and Demographics

The definition of a "market" contains a well-defined group of consumers, but that group may change as well. If Bilkent admits more students, the demand for tea at Starbucks in Business Administration building will increase, and the demand curve will shift in the northeastern direction. If there is a lot of immigration towards a city, the demand for houses will increase, and the demand curve will shift northeast.

Similarly, the characteristics of the population may change over time. If the population gets older, the quantity demanded for adult diapers will increase at every price, causing a northeastern shift in the demand curve. If the population gets younger, the quantity demand for K-Pop albums will increase at every price, causing a northeastern shift in the demand curve.

5.5 Elasticity of Demand

For our next exercise, we will fix a market demand curve (we will not shift it!) and study its properties. Perhaps the most important information a demand curve is the responsiveness of quantity demanded to the price. If the price increases a little bit, we know that the quantity demanded will decrease (this is the *law of demand*.) But how much will it change? By a little, or a lot? How will it compare the change in price? To answer these questions, we will introduce the notion of **elasticity of demand**.

- Informally: elasticity of demand (to be more precise, **own price elasticity of demand**) is a measure of responsiveness of quantity demanded to changes in price.
- A bit more formally: (own price) elasticity of demand is a measure of percentage change in quantity demanded in response to a percentage change in the price.
- Most formally: (own price) elasticity of demand is the rate at which the percentage change in quantity demanded changes in response to a percentage change in the price of the good resulting from a "small" change in the price of the good.

So we are looking for an answer to the following question: "If the price of a good increases by one percent, by what percent the quantity demanded will decrease?"

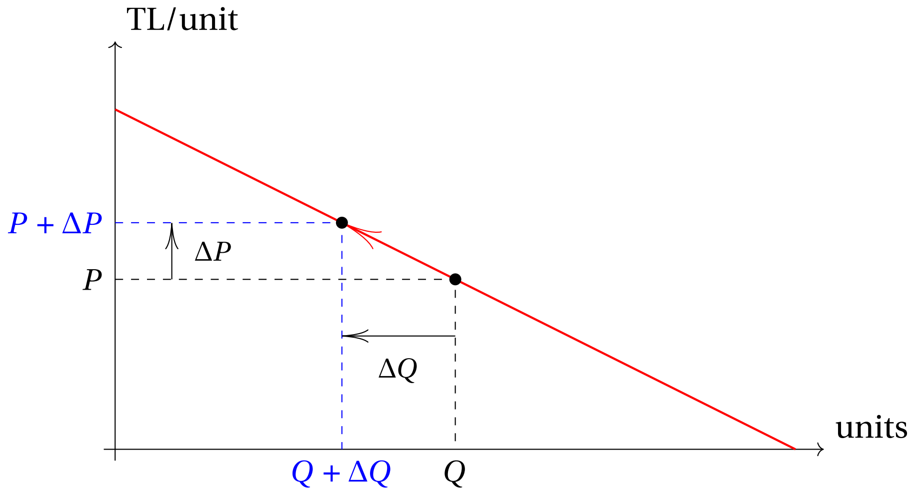
Here is some notation to get us going:

- P: the price of the good at which we would like to find the elasticity.
- Q: the quantity demanded at the price P.
- ΔP: the change in price.

- ΔQ: change in quantity demanded in response to change in the price.

  Therefore, $Q + \Delta Q$ is the quantity demanded at price $P + \Delta P$. Note that when $\Delta P > 0$, we have $\Delta Q < 0$ (by the law of demand).

The figure below illustrates:



Percentage change in price is

$$\frac{\Delta P}{P} 100$$

Percentage change in quantity demanded is

$$\frac{\Delta Q}{Q} 100$$

The measure of the responsiveness of quantity demanded to a change in price is simply the ratio of these two:

$$\begin{aligned}
\text{(own price) elasticity of demand} &= \frac{\text{percentage change in quantity demanded}}{\text{percentage change in price}} \\
&= \frac{\frac{\Delta Q}{Q} 100}{\frac{\Delta P}{P} 100} \\
&= \frac{P}{Q} \frac{\Delta Q}{\Delta P}
\end{aligned}$$

Let me make one more modification to this formula. I want to ensure that I consider "small" changes in price. Therefore, I will consider small ΔP's (which implies small ΔQ's as well).

Definition 126.1 (Own price) elasticity of demand at price P*, denoted* $\epsilon(P)$*, for a good is therefore defined as:*

$$\epsilon(P) = \lim_{\Delta P \to 0} \frac{P}{Q} \frac{\Delta Q}{\Delta P}$$

A couple of notes:

- By the law of demand, $\epsilon(P)$ is always negative. This is because P and Q are positive, and ΔP and ΔQ have the opposite sign.
- However, when we talk about about whether an elasticity is large or small, we typically talk about its absolute value. A large absolute value of an elasticity means that quantity demanded is more responsive to changes in price. A small absolute value means that the quantity demanded is less responsive to changes in price.

  Indeed, we classify elasticities based on their absolute value as follows.

If $\lvert\epsilon\rvert$ ...	then we say demand is...
< 1	inelastic
> 1	elastic
$= 1$	unit elastic
$= 0$	perfect inelastic
$= \infty$	perfectly elastic

- What is the interpretation of this number? An elasticity of $\epsilon(P) = -3$ means that if the price increases by one percent, the quantity demanded will decrease by approximately three percent. Similarly, if the price decreases by one percent, the quantity demanded will increase by approximately three percent.

  This is extremely useful information for business owners. If you are operating a hot dog stand, you want to know the own price elasticity of demand for your hot dogs. Why? Because it tells you how many customers you will gain if you cut your prices a little bit. Of course, it also tells you how many customers you will lose if you increase your prices a little bit. If you care about maximizing your revenue (quantity demanded times price), you should follow this rule of thumb:

  > "When the absolute value of own price elasticity is less than one, increase the price. When the absolute value of own price elasticity is larger than one, decrease the price."

This is because if $|\epsilon(P)| = 3$, a one percent reduction in price leads to a three percent increase in quantity demanded. You can sell much more hot dogs by cutting your price, and the net effect on revenue is positive! You should reduce your price. On the other hand, if $|\epsilon(P)| = 0.2$, a one percent increase in price leads to a 0.2 percent reduction in quantity demanded. You can charge a higher price for your hot dogs, and it is true that the quantity demanded is lower, but it is lower by a small amount! You should charge a higher price for your hot dogs.

Let me just repeat the rule of thumb using the terminology.

> "If you want to increase your revenue: When the demand is inelastic, increase the price. When demand is elastic, decrease the price."

- You can go one step further and calculate approximately how much you need to increase (or decrease) your price. Suppose you are selling your hot dogs at 9TL per hot dog, and you are selling 50 hot dogs per day. Suppose you hire an economist to study the market you operate in. The economist runs some calculations and tells you that the own price elasticity of hot dogs at the price of 9TL is -5.

  Given this information, if you want to sell 60 hot dogs per day instead of 50, what should be the price approximately be?

  Let's express this in our notation. We have:

$$\begin{aligned} P &= 9 \\ Q &= 50 \\ \epsilon(P) &= -5 \\ Q + \Delta Q &= 60 \end{aligned}$$

  First, note that $\Delta Q = 60 - 50 = 10$. Then, using the formula for elasticity,

$$\begin{aligned} \epsilon(P) \approx \frac{P}{Q}\frac{\Delta Q}{\Delta P} \quad &\implies \quad -5 \approx \frac{9}{50}\frac{10}{\Delta P} \\ &\implies \quad \Delta P \approx \frac{1}{-5}\frac{9}{50}10 = -0.36 \end{aligned}$$

  Therefore, if you reduce your price by 0.36 TL (i.e., sell you hot dogs at $9 - 0.36 = 8.64$TL per hot dog), you will approximately sell 60 hot dogs per day! $P + \Delta P = 9 - 0.36 = 8.64$.

  Intuitively, you want to increase quantity demanded from 50 to 60, which is a 20 percent increase. Because the quantity demanded is five times as responsive to price changes, only a four percent reduction in price is sufficient. This corresponds to a reduction of 0.36TL in price.

5.5.1 Geometric Interpretation of Elasticity

Let's rearrange the formula:

$$\epsilon(P) = \lim_{\Delta P \to 0} \frac{P}{Q} \frac{1}{\Delta P / \Delta Q}$$

But $\lim_{\Delta P \to 0} \Delta P / \Delta Q$ is the slope of demand curve at P. Therefore,

$$\epsilon(P) = \frac{P}{Q} \frac{1}{\text{slope of demand curve at } P}$$

What does it mean, geometrically?

- If the demand curve is *steeper*, the absolute value of its slope is higher. Thus, the absolute value of elasticity is lower. The demand is *more inelastic!*
- If the demand curve is *flatter*, the absolute value of its slope is lower. Thus, the absolute value of elasticity is higher. The demand is *more elastic!*

Indeed, you can compare the elasticities of two demand curves that pass through the same point just by looking at them. The steeper one is more inelastic. For instance, consider Figure 129.1. Here, D_1 (the red demand curve) pass through the same point as D_2 (the dark red demand curve).

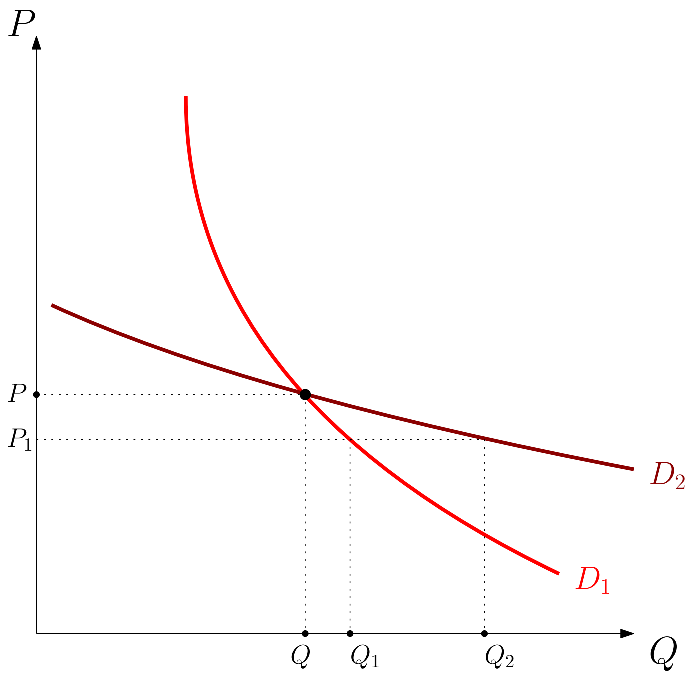


Figure 129.1: Two demand curves, D_1 and D_2, pass through the same point at price P. Because D_1 is steeper than D_2 at this point, it is more inelastic (less elastic) than D_2 at price P.

Let s_1 denote the slope of the red demand curve at P, and let s_2 denote the slope of the dark red demand curve at P. Because the red demand curve is steeper, $|s_1| > |s_2|$.

For the red curve, the own price elasticity of demand at price P is:

$$\epsilon_1(P) = \frac{P}{Q}\frac{1}{s_1}$$

and for the dark red curve, the own price elasticity of demand at price P is:

$$\epsilon_2(P) = \frac{P}{Q}\frac{1}{s_2}$$

Because $|s_1| > |s_2|$, $1/|s_1| < 1/|s_2|$ and

$$|\epsilon_1(P)| < |\epsilon_2(P)|$$

so the red curve is more inelastic (or less elastic, or have lower elasticity, or its elasticity is low, depending on how you want to call it) at price P. It actually makes a lot of sense: the dark red curve is more responsive to changes in price than the red curve. For instance, consider a reduction in price from P to P_1. If the demand curve is D_1, the quantity demanded increases from Q to Q_1: a modest increase. But if the demand curve is D_2, the quantity demanded increases from Q to Q_2: a larger response to the same reduction in price.

NOTE: this comparison is only about the relative magnitudes. For instance, both D_1 and D_2 may be elastic at P (i.e., we may have $|\epsilon_1(P)| > 1$ and $|\epsilon_2(P)| > 1$). Still, D_1 is more inelastic than D_2 at price P.

So, overall, flatter curves are more elastic. Two extreme cases:

- If the demand is perfectly elastic, the slope of demand curve is zero: it is a horizontal line. In this case, if the price increases by a tiny bit, the quantity demanded reduces all the way to zero.
- If the demand is perfectly inelastic, the slope of demand curve is infinity: it is a vertical line. In this case, no matter what the price is, the quantity demanded is the same. (No responsiveness at all.)

5.5.2 Determinants of the Price Elasticity of Demand

What makes a demand more elastic or less elastic? In other words, under what conditions are quantities demanded more responsive to price changes? Here are some characteristics of the good and the market that play a role in the determInation of elasticity.

a. **Availability of Close Substitutes:** If a good has many substitutes available, the demand for the good will be more elastic.

   Intuitively, when there are many substitutes available, the consumer would responds to a small increase in prices by switching to alternatives, and reducing the quantity demanded a lot.

   Example: consider the demand for regular car tires manufactured by Pirelli. There are many alternative brands of car tires, so the demand is very elastic. In contrast, Pirelli is the sole producer of Formula 1 car tires. Therefore, the demand for Formula 1 car tires manufactured by Pirelli is very inelastic.

b. **Time:** If the time frame is longer, the demand will be more elastic.

   This is because over time, people can adjust their purchasing habits more easily. This results in a higher ability to respond to price changes over the long run.

   Example: the demand for gasoline is quite inelastic in the short run. If the price for gasoline decreases, people will start driving their cars more, but not that much. Yet, over a longer time frame, consumers can buy their own cars etc. so the response to a decrease in gasoline price will be stronger.

c. **Luxuries versus Necessities:** If a good is a necessity, the demand will be more inelastic.

   Intuitively, if the good is a necessity, consumers have to buy it no matter what the price is. If it is a luxury, we can live without buying it. This gives the ability to respond to price changes.

   Example: the demand for certain medicines is quite inelastic (a diabetes patient has to use insulin no matter what the price is). In contrast, the demand for daily vitamin supplements is more elastic.

d. **Definition of the Market:** If the market is more narrowly defined, the demand will be more elastic.

   Narrowly-defined markets include particular brands, particular locations etc. so that consumer have alternatives to switch.

   Example: the demand for Recep Ivedik movies in a particular theater is quite elastic, because consumer can just go to the next theater to watch the same movie. The demand for Recep Ivedik movies overall in the country is more inelastic. The demand for movie tickets overall in the country is even more inelastic.

   Example: The demand for Komili Olive Oil is quite elastic. The demand for Olive Oil is more inelastic.

e. **Share of the Good on a Consumer's Budget:** If the expenditure on a good constitutes a small portion of the consumers' income, the demand will be more inelastic.

   This is because when the consumer is spending only a small amount of money on a good, she will not feel the difference much and will not respond to price changes.

   Example: demand for table salt is inelastic compared to the demand for furniture.

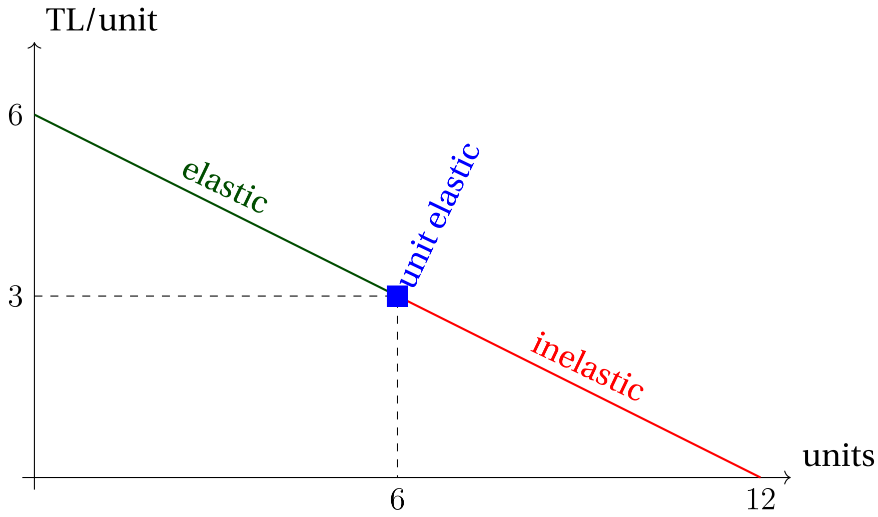
Figure 132.1 is a table of estimated own price elasticities of some goods, taken from a blog post. You should visit the source at http://econbeh.blogspot.com/2019/05/some-estimates-of-price-elasticity-of.html, and the references there, if you are interested in finding out how these numbers are calculated.

Google Play apps	3.7
Cinema	2.5
Apple App Stpre	2
Organic milk	1.8
Air travel for non business	1.5
Vacations	0.8
Beef	0.8
Premier League football	0.7
Fruit	0.7
Air travel business	0.7
Opera	0.7
Milk	0.6
Public transport	0.4
Broadband	0.4
Wine	0.3
Theatre	0.3
Beer	0.2
Car fuel	0.2

Figure 132.1: (Absolute values of) own price elasticities of some goods. Source: http://econbeh.blogspot.com/2019/05/some-estimates-of-price-elasticity-of.html

5.5.3 Changes in Elasticity Along a Demand Curve

As it turns out, the elasticity along a curve is **not** constant even along the same demand curve. This may be because the slope of the curve changes as we move along the curve. But even when the slope is constant (i.e., even when the curve is linear), the elasticity changes because $\frac{P}{Q}$ changes. For higher values of P (and, due to the law of demand, for lower values of Q), the elasticity is higher. As P increases (and, due to the law of demand, Q decreases) along a linear demand curve, the elasticity decreases. The figure below illustrates the elasticities at different points at the same demand curve, whose equation is $Q = 12 - 2P$.



Now, combine what you see in this figure with the rule of thumb for the revenue maximization we derived before. If $P > 3$, demand is elastic: decrease the price. If $P < 3$, demand is inelastic: increase the price. It therefore turns out that the price that maximizes revenue is $P = 3$. At this price, $Q = 6$ units are sold and the total revenue is $3 \times 6 = 18$.[4]

Now, this analysis assumes that the seller is interested in maximizing the revenue. Most of the time, the sellers are interested in maximizing **profit** (revenue minus costs), not profit. We will get to profit maximization in the next chapter.

[4]You could have found this answer by writing down the revenue as $P \cdot Q = P \cdot (12 - 2P)$ and finding the P that maximizes this. But isn't the heuristic more insightful?

Appendix to Chapter 5

Recall that the own price elasticity of demand is defined as "percentage change in quantity demanded divided by percentage change in income". We can use the "percentage change divided by percentage change" idea to define other measures of responsiveness. Here are some examples.

Cross Price Elasticity

The **cross-price elasticity** of demand for good i with respect to (the price of) good j is a measure of the responsiveness of the demand for good i to changes in the price of good j.

More formally, the **cross price elasticity** of demand for good i with respect to (the price of) good j, denoted $\epsilon_{i,j}$, is the rate at which the *percentage change in quantity of good i demanded* changes in response to a *percentage change in the price of good j* resulting from a small change in the price of good j.

$$\epsilon_{i,j} = \lim_{\Delta P_j \to 0} \frac{\frac{\Delta Q_i}{Q_i} \cdot 100}{\frac{\Delta P_j}{P_j} \cdot 100} \ .$$

- If good i is a substitute for good j (at the current prices and income level), then the cross price elasticity of good i with respect to good j is positive, i.e.,

$$\epsilon_{i,j} > 0 \ .$$

- If good i is a complement for good j (at the current prices and income level), then the cross price elasticity of good i with respect to good j is negative, i.e.,

$$\epsilon_{i,j} < 0 \ .$$

Income Elasticity

Income elasticity of demand is a measure of responsiveness of demand to changes in income.

More formally, the **income elasticity of demand**, denoted ϵ_I, is the rate at which the *percentage change in quantity of the good demanded* changes in response to a *percentage change in the income* of the consumer resulting from a "small" change her income.

$$\epsilon_I = \lim_{\Delta I \to 0} \frac{\frac{\Delta Q}{Q} \cdot 100}{\frac{\Delta I}{I} \cdot 100} \quad .$$

- If the good is an **inferior** good (at the current prices and income level), then the income elasticity of the good is negative, i.e.,

$$\epsilon_I < 0 \quad .$$

- If the good is a **normal** good (at the current prices and income level), then the income elasticity of the good is positive, i.e.,

$$\epsilon_I > 0 \quad .$$

- If a normal good is a **luxury** good, then the income elasticity of the good is greater than 1, i.e.,

$$\epsilon_I > 1 \quad .$$

- If a normal good is **necessity** good, then the income elasticity of the good is less than 1, i.e.,

$$0 < \epsilon_I < 1 \quad .$$

Extra Readings for Chapter 5

When we draw our demand curves, we keep the *income* of the consumer fixed. An alternative approach keeps the consumers' *purchasing power* fixed, rather than income.

The mental exercise is the following: imagine that as the price of a good P increases, we keep the consumer's purchasing power fixed by giving her the *compensated income* from Chapter 4. By definition, this would keep the consumer on the same indifference curve. Hence, we would keep track of the *substitution effect* only (as opposed to a combination of substitution and income effects, as we do here).

The idea of tracing the effect of price while keeping the purchasing power fixed yields to a different type of demand curve, called **Hicksian demand curve** (named after prominent British economist John Hicks, 1904-1989). In contrast, the demand curves we draw here as **Marshallian demand curves** (named after prominent British economist Alfred Marshall, 1842-1924).

Some people still argue the merits of adopting the Hicksian approach. For an example, see:

> Friedman, Milton. "The Marshallian Demand Curve." *Journal of Political Economy* 57, no. 6 (1949): 463-495.

(You can kinda sorta infer that Friedman maybe isn't the biggest fan of Marshall...)

The Hicksian demand curve is, of course, different than the Marshallian demand curve. If the good is a normal good, the Hicksian demand curve is less elastic than Masrhallian demand (this is because Hicksian demand cancels out the income effect, which reinforces the substitution effect). If the good is an inferior good, Hicksian demand curve is more elastic than the Marshallian demand curve.

While drawing the Hicksian demand curve, there is the question whether one should keep the consumer at the initial indifference curve or the final indifference curve. This leads to various approaches such as using **Compensating Variation** versus **Equivalent Variation**. You should wait until Econ 203 to learn about Hicksian demand.

Exercises for Chapter 5

1) Discuss the effects of the following on the market demand curve for oranges in Turkey:

   a. There is an influenza epidemic and the newspapers report the tremendous curing effect of orange juice.

   b. The price of tangerines, a substitute to oranges in consumption, decreases.

   c. The orange juice producing technology is improved, the machines allow for more orange juice production using the same amount of oranges.

   d. Tourists from all over the world travel to Turkey, significantly increasing the population during this period.

2) Explain why the own price elasticity of demand for cigarettes is more inelastic in the short run than in the long run.

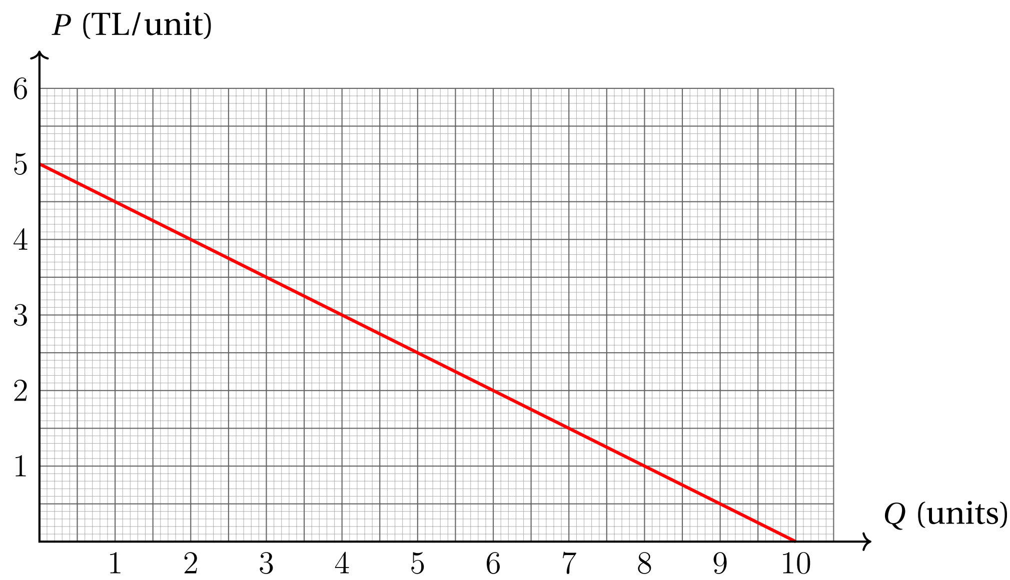
3) For each pair of goods given below, which good has a larger own price elasticity of demand (in absolute value)? Explain your answers.

   a. Ayran and Özer Hisar brand of ayran.

   b. Water and Sprite.

   c. Ties and suits.

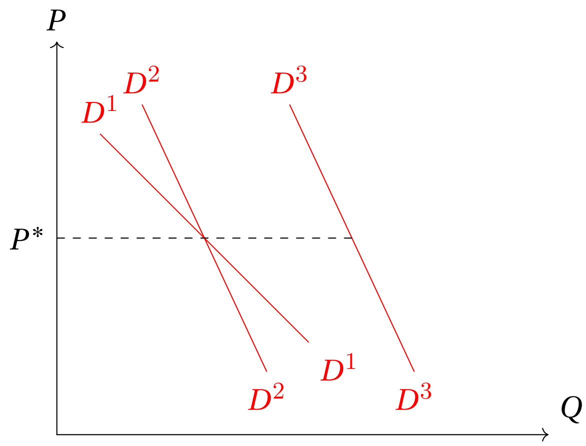
4) The following graph gives the demand for a certain good:



a. Find the own price elasticity of demand at the price of 4 TL/unit.

b. Find the own price elasticity of demand at the price of 2 TL/unit.

5) Consider the three linear demand curves shown in the figure below:



Note that demand D^3 is parallel to demand D^2. Let ϵ^1, ϵ^2, and ϵ^3 denote the own-price elasticities of the demands D^1, D^2, and D^3, respectively, at the price $P*$. What is the relationship between (magnitude-wise, i.e., greater, less, equal etc.) the absolute values of the three elasticities? Explain.

Chapter 6

Producer Theory

With this chapter, we start a brand new topic – even though the concepts and analysis methods are very similar with the ones used in consumer theory. We are introducing the formal model of another economic agent: a **firm**. We consider the firm's decision to produce and sell goods and services.

Some motivation: from Section 3.3.1, we remember that a market is an infrastructure that facilitates **interactions** among **economic agents**. One of the most widespread economic interactions are the buying and selling of services. We discussed the decision to buy, and finally now it is time to consider selling! To this end, after spending several chapters on economic agents who buy (consumers), now we are making our transition to the economic agents who sell (firms). After we are done with the theory of the firm, we will bring the consumers and firms together and analyze a **market**.

6.1 What is a Firm?

The question of what makes a firm a firm is one of the most important questions of 20th century economics. There has been at least four Economics Nobel prizes related to defining what "a firm" is (Herbert Simon, Ronald Coase, Oliver Williamson, Oliver Hart) and we still do not have a clear answer. Some of the historically popular answers are:

- "The firm is a mechanism to circumvent the **price discovery**, which is a costly activity. By setting up a firm, an entrepreneur can procure essential materials by itself without going to the market and working around the price mechanism." (early 20th century)
- "The firm is a way of ensuring **adaptation** through giving authority to a boss."

(Simon)

- "The firm is a device to minimize **transaction costs** among producers who interact frequently." (Coase, Williamson)
- "The firm is a mechanism for **monitoring** performance when there is a need for joint production." (Alchian, Demsetz)
- "The firm is a way to ensure proper investments through giving **property rights** to a boss." (Hart)

But... This is Econ 101, and are keeping things simple. We will adopt a much, much simpler (and admittedly more boring) definition of a firm.

6.1.1 The Firm as a Technology

Here is the definition of the firm for the purposes of this class.

Definition 140.1 *A* **firm** *is an entity which uses inputs in order to produce outputs.*

Here, **inputs** include natural resources, labor, machinery etc. and **outputs** include goods and services (the things that consumers buy).

So, a firm is not an organization according to our definition. It is just a technology that takes some inputs and transforms it into outputs. Just like a consumer is defined by her preferences, a firm is defined by its technology. This is a tremendous simplification, or as we call it, an *abstraction of reality*.

Speaking of abstractions, here are a couple of simplifications we assume throughout this chapter:

- The firm produces a single output. In reality, Bilkent (as a firm) produces many outputs: diplomas, research papers, boarding, parking spaces... We are abstracting away from that: in this model, Bilkent only produces teaching. A pizza parlor in this model produces only a single type of pizza: medium pepperoni pizzas.

  When the firm is able to produce multiple goods at once, its decisions become more complicated to analyze but the basic insights of this simple model remain.
- The firm has already chosen which product to produce. The decision of what to produce is also extremely important, but that is not what we are focusing here. For models of product choice, you should take a class on industrial organization.
- The firm can buy as many inputs as it wants, as long as they are profitable. Effectively, we are assuming that the firm is not cash-constrained. What we have

in mind is a financial market (multiple investors) willing to lend money to the firm as long as the investment is profitable.

6.2 The Firm as an Economic Agent

As every economic agent, a firm is also defined by its **constraints** and **preferences**. Before we define whet these are, let us introduce some notation.

A firm uses several **inputs** to produce an **output**. The output is just the good or service the firm has chosen to produce. Because there is no question about what to produce, the question is **how much to produce**. To denote the quantity of good produced by the firm, we use the letter Q. This should be familiar from consumer theory. (Some textbooks denote it with letter y.)

The next question is **how to produce** Q units of output. What inputs should the firm use? Broadly speaking, we can categorize inputs into three groups. These are also called **factors of production**:

a. **Land (or natural resources):** goods which are not created by human beings. Examples: Air, forests, minerals, soil, etc. Example: Bilkent uses the land it is on to produce teaching.

b. **(Physical) Capital:** Human-made goods used to produce other goods. Examples: machinery, computers, factories, roads, etc. Example: Bilkent uses the buildings, projectors, computers, software... for teaching.

   Note: it is common to think of *financial capital* (i.e. the money you invest and receive return on) when someone says "capital". What we mean by "capital" is different. We mean physical capital, i.e. machines of one sort or another.

c. **Labor:** Human effort used in production. Example: Bilkent uses the effort provided by faculty members, TAs, administrative assistants... for teaching.

Typically,

- the quantity of land used in production is denoted by D,
- the quantity of capital used in production is denoted by K, and,
- the quantity of labor used in production is denoted by L.

At this point, we will forget about land/natural resources. (They are trickier to introduce to this model because they are, by definition, in limited supply. This does not bode well with the assumption we made: "The firm can buy as many inputs as it wants.") Just assume a firm uses two inputs: machines and labor. If you are operating a pizza parlor, you use labor (workers) and capital (pizza ovens) to produce output (medium pepperoni pizzas).

It is not crazy to think that more or less all production activities require inputs of both forms. They may be at different ratios, of course. Producing tea requires human effort and machines (tractors), but famously the human effort is a very large component. Producing Bitcoins requires a lot of capital (processors to mine Bitcoins), but also a human has to occasionally stand in front of a computer.

Note: in this model, all the inputs and outputs are measured in **flows**, i.e. their quantities are given **per unit of time**. For instance, if the unit of time is week, we will say it takes *a total of 42 labor hours per week* and *a total of 16 machine hours per week* to produce *23 hours of teaching per week*. Or we may say it takes *a total of 3 workers per day* and *3 pizza ovens per day* to produce *1100 pizzas per day*.

6.2.1 The Constraint

A firm faces many constraints imposed by consumers, its competitors, government, and nature. Here, we will focus on the constraints imposed by **nature**. These are some **technological constraints** that capture the following idea: there are only certain ways to produce Q units of output from labor and capital.

Given the quantities of inputs L, K, the quantity of output Q is **(technologically) feasible** if and only if

$$Q \leqslant f(L, K)$$

The set of feasible output quantities given (L, K) pairs is also called the **production set**. The function that specifies the maximum amount of output that can be produced using L, K, i.e., $f(L, K)$, is the **production function**.

Graphically, the production function is just the boundary of the production set. It is difficult to visually represent a function with two inputs,[1] so we will typically keep one input fixed and plot the production set by varying the other input. For instance, keep capital fixed at $\bar{K}$. Then the production set and the production function $f(L, \bar{K})$ are illustrated as in Figure 143.1.

At this point, it should be clear that a production set plays the role of a budget set in the consumer theory (defining what is feasible). Similarly, the production function is like the budget line: the maximum feasible thing given the constraints. Of course, you will see that the firm's optimal choice always lies on this curve, just like the consumer's optimal bundle always lies in the curve given by the production function.

[1]But not impossible! You can illustrate them through what people call **isoquants**. You have to wait until Econ 203 to see them.

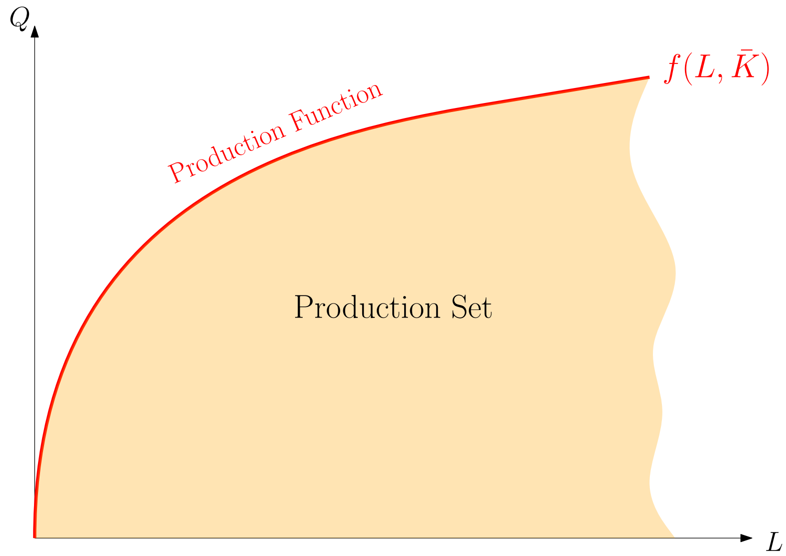


Figure 143.1: An example of a production set.

As usual, we will impose some assumptions on the production function. The following is the first property we impose.

- The production function is **monotonic**: if you increase the amount of at least one input, you can produce at least as much output as you were producing before.

  Formally, for any $L' \geqslant L$ and $K' \geqslant K$,

  $$f(L', K) \geqslant f(L, K) \quad \text{and} \quad f(L, K') \geqslant f(L, K)$$

  This makes a lot of sense: if you put more inputs you should be able to produce at least as many outputs. If you need a further defense of monotonicity, what is hidden under this property is the assumption of **free disposal of inputs**. The firm may always hire the extra worker. Even if that extra worker reduces the total output, the firm can just tell that worker to stay behind and not do anything, effectively not using that worker. In that case, the firm has disposed this particular worker and managed to keep the same amount of output as before.

  Geometrically, monotonicity imposes the production function to be **(weakly) increasing** in both L and K, just like we drawn in Figure 143.1.

  As you probably see, this is analogous to the monotonicity of preferences ("more is better") in the consumer theory. The only difference here is that we impose this on the constraint, not on the preferences.

Marginal Product of Inputs

The next property we will impose on the production function requires us to define a new object.

The **marginal product of an input** is the rate at which output increases when the "last" unit of the given input is used in production. A little less formally, this is the additional output produced as a result of employing the last "unit" of input.

Suppose K units of capital are hired, and we are considering the the marginal product of L-th unit of labor. The additional output produced as employing L units of labor rather than $L - \Delta L$ units of labor is:

$$f(L, K) - f(L - \Delta L, K)$$

Therefore, additional output *per units of labor hired* is:

$$\frac{f(L, K) - f(L - \Delta L, K)}{L - (L - \Delta L)} = \frac{f(L, K) - f(L - \Delta L, K)}{\Delta L}$$

To express the marginal product of L-th unit of labor when K units of capital hired, $MPL(L, K)$, we take the smallest increments possible. See Figure 144.1.

$$MPL(L, K) = \lim_{\Delta L \to 0} \frac{f(L, K) - f(L - \Delta L, K)}{L - (L - \Delta L)} = \lim_{\Delta L \to 0} \frac{f(L, K) - f(L - \Delta L, K)}{\Delta L}$$

This is just the slope of $f(L, \bar{K})$ at L. Thus, when $MPL(L, K)$ is larger, $f(K, L)$ is steeper. This means that adding an extra unit of labor increases productivity a lot.

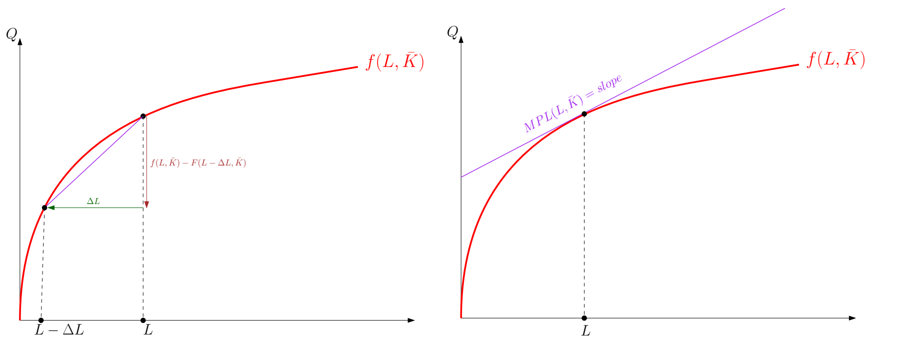


Figure 144.1: Marginal product of labor: Taking the limit as ΔL goes to zero in the figure on left, we obtain the figure on right.

When the units are finer, we assume that the production function is "smooth" so that its slope exists and is well-defined.

The marginal product of capital is defined analogously:

$$MPK(L,K) = \lim_{\Delta K \to 0} \frac{f(L,K) - f(L,K-\Delta K)}{\Delta K}$$

Now, we are ready to define the second property we impose on the production function.

- The production function satisfies **diminishing marginal product**: the marginal product of each input is decreasing in that input.

  This is sometimes called **the law of diminishing returns**. This is not really a law, it is just a common feature of many production functions. Why? Typically the first unit you use is very effective. But as you keep adding more and more units of the same input, their effectiveness is less prevalent. Ex: The first faculty member hired by Bilkent Economic department is very crucial: she will teach Econ 101, Econ 102, Econ 203... But the 24th faculty member will only teach some elective class if there is demand for such a class.

  Geometrically, diminishing marginal product states that the production function gets flatter as L increases. Figure 143.1 is not a coincidence!

  This is analogous to the diminishing MRS ("if you have more of it, the extra units you get is less valuable") in the consumer theory. Once again, the difference here is that we impose this on the constraint, not on the preferences.

  Note: Some textbooks define diminishing marginal product as the requirement that "the marginal product is diminishing *at some point*". That is, some textbooks allows for MPL to be increasing for small values of L.

To wrap up: firm's constraints are given by a production set, whose boundaries is given by the production function. We assume that the production function satisfies monotonicity and diminishing marginal product of inputs.

6.2.2 The Preferences

We will assume that a typical firm maximizes its **profit**. This is a reasonable case for many firms in the economy. Formally,

$$profit = revenue - costs$$

The Greek letter π is typically used to denote **profit**.

If the firm produces and sells Q units of output in a given time period at a price of P per unit, its **revenue** is given by $P \cdot Q$.

If the firm uses L units of labor to produce such output at a cost of w per unit of labor per time, its **labor costs** are $w \cdot L$. Similarly, if the firm uses K units of capital to produce such output at a cost of r per unit of capital per time, its **capital costs** are $r \cdot K$. All in all, the firm's **costs** from using inputs (L, K) is: $w \cdot L + r \cdot K$.

Therefore, if the firm uses a pair of inputs (L, K) and produces Q units of outputs, its profit is:

$$\pi(Q, L, K) = P \cdot Q - (w \cdot L + r \cdot K)$$

The firm's preferences are very simple: for any two combinations (Q, L, K) and (Q', L', K'), the firm (weakly) prefers the former over the latter when $\pi(Q, L, K) \geqslant \pi(Q', L', K')$. That is, the firm prefers to choose combinations of inputs and outputs that yield higher profits.

Costs of Inputs

We have extensively talked about P when discussing consumer theory. How about w and r?

- w is somewhat easier to interpret: it is the **wage per time** paid on labor hired. If the pizza parlor hires 3 workers in a given day ($L = 3$), and if the daily wage is 98.10 TL/day ($w = 98.10$), then the labor cost is $w \cdot L = 294.30$ TL/day.
- r is a little bit more involved: it is the **rental rate of capital per time** that needs to be paid on capital used in production. The easiest way to think about this is imagining that the firm rents (does not own) the capital it uses. For instance, imagine that the pizza parlor does not own its pizza ovens, but rents it out from a wholesale pizza oven operator. If the pizza parlor rents two pizza ovens in a given day ($K = 2$) and if the **rental rate of a pizza oven per day** is 150 TL/day, then the capital cost is $r \cdot K = 300$ TL/day.

Opportunity Costs One thing that I want to emphasize is: **all the costs considered here include opportunity costs.** For a refresher on the concept, you should revisit Section 1.2. To reiterate what we had there: the wage rate includes the best alternative you give up by employing the extra unit of labor. For instance, imagine the owner of the firm is self-employed: he does not pay any salary to herself. An accountant would say that the wage paid for the labor supplied by this worker is zero. Yet, the economic logic suggests that it is not zero, because it involves the opportunity cost. Suppose, if the firm did not exist, the owner would join the labor force and find a job paying a salary of 15000 TL/month. So, by employing herself, she forgoes a salary of 15000

TL/month. Therefore, the opportunity cost is 15000 TL/month, and that should be the labor cost associated with that employee.

A similar reasoning goes for capital: if the firm owns the pizza ovens and can operate them freely, the explicit cost (i.e., the accounting cost) is zero but the opportunity cost not. This is because if the firm did not operate the ovens, it would be able to rent out these ovens and obtain a return of r. Thus the opportunity cost of using a pizza oven is still r, even though the firm does not make any explicit payments on it. Therefore, the rental rate of capital enters into the calculus even when the firm owns the capital.

Long story short: due to the costs being opportunity costs, the ownership of labor or capital (whether they are owned by the firm or merely hired/rented by the firm) does not really matter.

Sunk Costs What do these costs **not** include? They don't include the **sunk costs**. These are the costs that are already paid before the production decisions and cannot be recovered. Suppose, for instance, the firm paid some installation fee for the ovens: it hired some contractors to transport and install the ovens months ago. Because those costs are already paid, they are not included in the calculation of the cost of capital.

The concept of **sunk cost** is an important economic concept: it is useful to know that a rational economic agent would never take those costs into account. For instance, if you already paid for your gym membership fee, that amount you already paid should not affect your decision to go to the gym or not. Similarly, if you already bought a concert ticket, but a better alternative came up, you should not say "But I already paid for this concert ticket so I should go to the concert." Still, many economic agents are not that rational and cannot stop themselves from taking sunk costs into account when deciding. This phenomenon is aptly called the **sunk cost fallacy**. Some examples:

- A lot of people hesitate to finish their relationships because "They invested a lot in that relationship."
- Sometimes governments stick with a policy that took a lot of effort into passing, even though that policy turns out to be not very desirable.

Long story short: the costs a firm consider include opportunity costs, and exclude sunk costs.

Perfectly Competitive Markets

Okay, so there are prices of output and inputs in the markets the firm operate in. But what are these prices (P, w and r) determined? We will discuss this in more detail in the upcoming chapters, but for now, what you need to know is that **the firm takes these prices as given**.

The running assumption behind this statement is the following: the firm operates in a **perfectly competitive market**. We discussed this concept in Section 3.3.1. Now we are revisiting this notion for a market with firms.

To reiterate, I know that the term "competitive" has a negative connotation: it suggests the existence of some very aggressive economic agents. Nevertheless, an economist's definition of "competitive" is much more innocuous. It merely means that there are many, many firms available in the market, and none of them are large enough to affect the prices. Formally:

Definition 148.1 *A* **perfectly competitive market** *for firms is a market where: (i) there are many firms selling an identical good or service to consumers, and (ii) an individual firm or consumer is not powerful enough to affect the price.*

So, following the pizza parlor example: consider a large town with many pizza parlors which produce and sell identical medium pepperoni pizzas. Our pizza parlor is a small one, so it cannot unilaterally say "I decided to sell my medium pepperoni pizzas at a different price than the other pizza parlors." Effectively, the pizza parlor is a price-taker: it cannot control the price of pizza, the only thing it can control is how many pizzas to produce.

This is the most important aspect of perfectly competitive markets: the economic agents in perfectly competitive markets are **price-takers**. We had already assumed this for consumers when we were covering consumer theory and exchange economies: we assumed that a consumer cannot affect the prices. Now we are making the same assumption for firms: a firm merely takes P, w and r as given.

Are markets perfectly competitive in real life? Most of the time, no. The closest we probably get to a competitive market is an agriculture market with many small farmers. The small apple farms in rural Ohio (i) all produce (more or less) identical products, and (ii) do not have the power to unilaterally affect the price of apple. But beyond those rare cases, most of the time the producers have some leeway in setting their prices. Yet, this is a benchmark we want to analyze first.

To be honest, most of economics deals with studying deviations from the perfectly competitive markets benchmark. The deviations are both more realistic and more interesting! But to study deviations, we need to understand the benchmark first. Hence the study of perfectly competitive markets.[2]

[2] It also deserves some emphasis that we are not making any value judgments at this stage: I am not

Isoprofit Curves

To summarize what we said so far: the firm takes P, w and r as given, and chooses (Q, L, K) to maximize its profit.

As you can imagine by now, there is a nice geometric illustration of the firm's preferences. These are illustrated through what we call **isoprofit line**. A small reminder: in Greek, *iso* means *equal*, so an isoprofit line is the set of points (Q, L, K) that all yield the same profit. Formally, for any two points (Q, L, K) and (Q', L', K') on an isoprofit line, we have: $\pi(Q, L, K) = \pi(Q', L', K')$.

A couple of points worth emphasizing:

- Recall that the firm wants to maximize its profit. Therefore, if two input-output combinations yield the same profit, the firm is indifferent between them. Therefore, the firm is indifferent between any two points on an isoprofit line. **Isoprofit lines are just the indifference curves for a firm.**

- Because illustrating things in three dimensional graphs are difficult, let us fix capital at a level $\bar{K}$ again. In this case, the set of points $(Q, L, \bar{K})$ that constitute an isoprofit line is the set of points where the profit $\pi(Q, L, \bar{K})$ is equal to a certain number π:

$$\begin{aligned} \pi(Q, L, \bar{K}) = \pi &\iff P \cdot Q - (w \cdot L + r \cdot \bar{K}) = \pi \\ &\iff Q = \frac{\pi}{P} + \frac{w}{P}L + \frac{r}{P}\bar{K} \end{aligned}$$

  Therefore, in a two-dimensional graph with L in the x-axis and Q in the y-axis, and isoprofit line is a line with **intercept** $\frac{\pi}{P} + \frac{r}{P}\bar{K}$ and **slope** $\frac{w}{P}$. It is positively-sloped, because a higher L is more costly. For the profit to remain constant, the revenue must be higher as well, i.e. Q must be higher if L is higher.

  Figure 150.1 illustrates an isoprofit line.

- Just like we plot multiple indifference curves, we can also plot multiple isoprofit lines. See Figure 151.1. It plots three different isoprofit lines, which correspond to different profit levels with $\pi_3 > \pi_2 > \pi_1$. Note that they are parallel to each other (after all, the slope of an isoprofit line is $\frac{w}{P}$, which the firm takes as given). They only differ in their intercepts, where a higher π corresponds to a higher intercept. Recall that the firm prefers to have higher profit, and therefore "higher" isoprofit lines correspond to input-output combinations that are more preferable for the firm. This is just like how "higher" indifference curves are more preferable to the consumer.

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saying "a perfectly competitive market is an ideal we want to achieve." A lot of times, it is not. We are just saying that this is a benchmark.

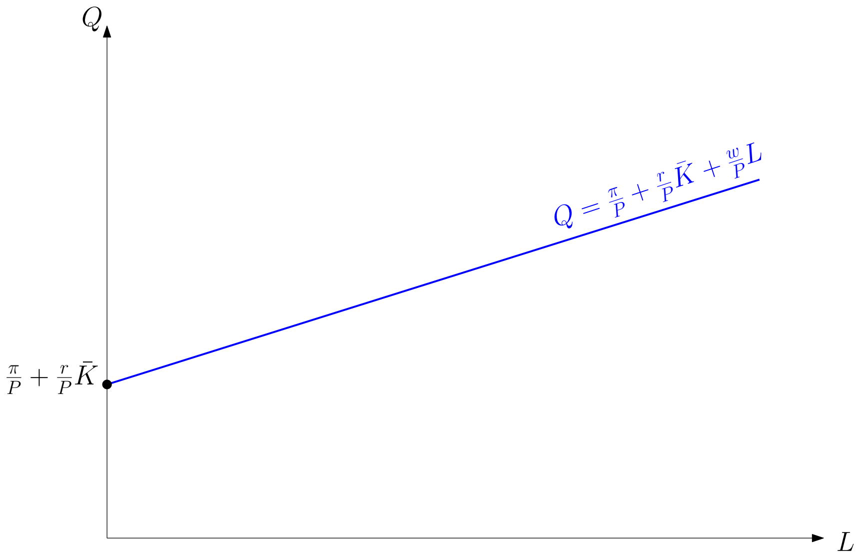


Figure 150.1: An isoprofit line is a line with an intercept of $\frac{\pi}{P} + \frac{r}{P}\bar{K}$ and a slope of $\frac{w}{P}$.

- To sum up, the firm just wants to find the "highest" isoprofit line it can find (subject to technological constraints), just like the consumer wanted to find the "highest" indifference curve she could afford (subject to budget constraints).

6.3 Profit Maximization

Okay, now we have a definition of firm's constraints and preferences. We even have a graphical representation of them! The next step is finding the input-output combinations which yield the highest profit. Before we do that, we will take a small detour.

6.3.1 Short-Run versus Long-Run

So far in our discussions, we usually kept capital fixed at $\bar{K}$ and allowed L to vary. One reason behind this choice is the easiness of illustration. But there is a deeper reason: there is indeed a substantial difference between capital and labor. It is the following: **the firm can easily change the quantity of labor it uses, but adjusting the quantity of capital takes time**. This is because hiring/firing labor, asking the workers to work extra shifts etc. is much easier to do than buying/selling/building/destroying machines and buildings. Therefore, the time frame used in the analysis matters.

- **Short run:** The time frame where the quantity of capital used in production cannot change.

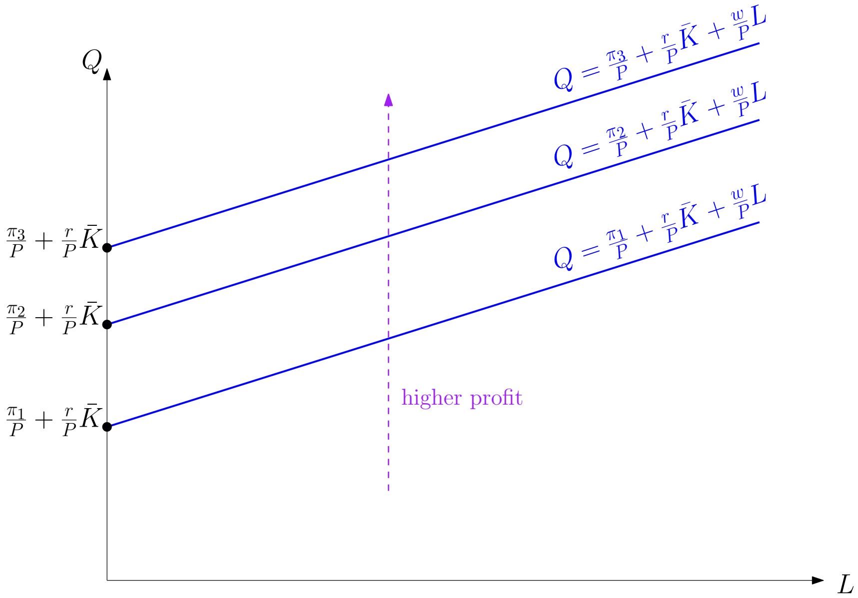


Figure 151.1: Multiple isoprofit lines corresponding to multiple profit levels, with $\pi_3 > \pi_2 > \pi_1$. The "higher" isoprofit lines correspond to higher profits, and therefore are more preferable to the firm. (Just like how "higher" indifference curves are more preferable to the consumer.)

- **Long run:** The time frame where all the inputs used in production can change.

The factors of production that cannot be adjusted in the short run are sometimes referred to as **fixed factors**. The factors of production that can be adjusted in the short run are referred to as **variable factors**. The terminology of *fixed versus variable* will be important in a few pages.

6.3.2 Short-Run Profit Maximization

In the short-run, the firm keeps capital fixed at $\bar{K}$ and only chooses (Q, L).

Given the output price P, input prices w, r, and a capital level $\bar{K}$, an input-output combination (Q^*, L^*) profit-maximizing in the short-run if and only if

- $Q \leqslant f(L^*, \bar{K})$ (i.e., (Q^*, L^*) is feasible), and
- for any input-output combination (Q, L), if $Q \leqslant f(L, K)$ (i.e., if (Q, L) is feasible), then $\pi(Q^*, L^*, \bar{K}) \geqslant \pi(Q, L, \bar{K})$ (i.e., (Q^*, L^*) yields at least as high a profit as any feasible input-output combination).

There is actually a mathematical derivation of the profit-maximizing input-output combination (just like the derivation of the optimal bundle in consumer theory), but we will skip it because we will cover a similar argument in a few pages. Instead, we will only give a geometrical argument.

Geometrically, the firm is trying to find the highest isoprofit line it can find, but has to adhere by the production set (i.e., (Q^*, L^*) has to satisfy $Q \leqslant f(L^*, \bar{K})$). **First** of all, a visual inspection suffices to argue that $Q = f(L^*, \bar{K})$, i.e.,the firm must be choosing an input-output combination in the boundary of the production set.

Next, imagine the (Q, L) combination on the production function that lies on the highest isoprofit line. You will see that at that point, the isoprofit line barely touches the production function, i.e., it is tangent to the production function. See Figure 152.1 below.

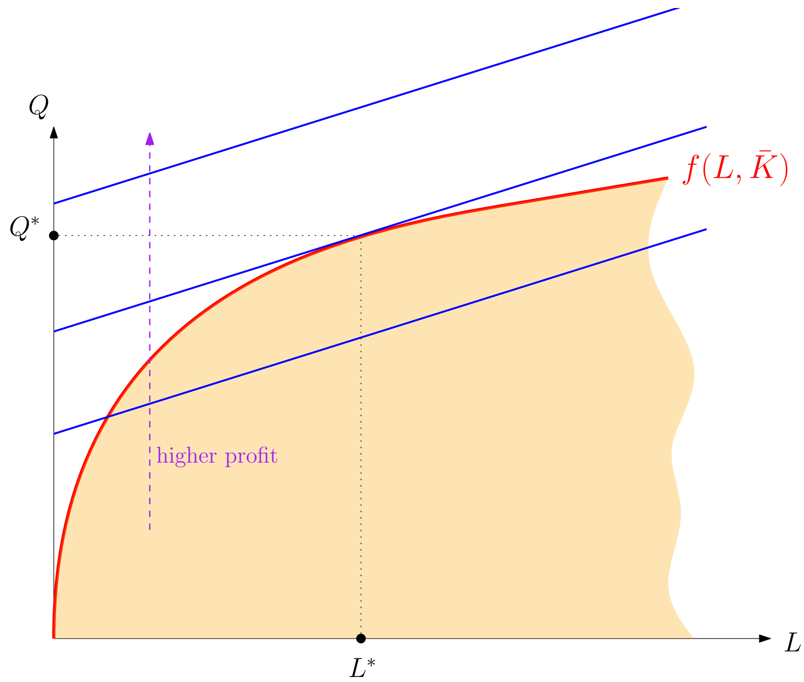


Figure 152.1: The profit-maximizing input-output combination (Q^*, L^*) is on the production function, and the isoprofit line passing through it is tangent to the production function.

Here is the interesting thing about this figure: at (Q^*, L^*), the isoprofit line is tangent to the production function. Therefore, slope of production function = slope of the isoprofit line.

- The slope of the production function at L^* is: $MPL(L^*, \bar{K})$.
- The slope of isoprofit line is: $\frac{w}{P}$.

Therefore, the profit-maximizing input-output combination satisfies:

$$MPL(L^*, \bar{K}) = \frac{w}{P}$$

Rearrange this to get:

$$MPL(L^*, \bar{K}) \cdot P = w \tag{153.1}$$

The left-hand side of this equation is "the revenue brought by the last unit of labor hired" (the output produced by the last unit times the output price). It is sometimes referred to as the **marginal revenue product of labor** or **value of the marginal product of labor (VMPL)**.

The right-hand side is simply the cost of the last unit of labor hired.

Therefore, the profit-maximizing input-output combination has the following feature: **the revenue brought by the last unit of labor hired is equal to the cost of the last unit of labor hired**. Intuitively: the firm starts by hiring some small amount of labor. As long as the next unit of labor hired brings higher revenue than its cost, the firm keeps hiring. But remember: the marginal product of labor decreases as more labor is hired. Therefore, the revenue brought by the last unit of labor keeps decreasing, and the firm stops at the point where it is no longer profitable to hire the next unit of labor.

What is nice about this graphical interpretation is that: you can also change the parameters of the model to see how (Q^*, L^*) changes!

- Suppose, for instance, w is higher. You can easily see that this means: isoprofit lines are steeper. Just play around with Figure 152.1 or Equation (153.1) so see that: L^* is lower when w is higher! This makes perfect sense: labor is more costly, so the firm hires less labor (or, stops earlier when hiring labor).

- Suppose, for instance, P is higher. You can easily see that this means: isoprofit lines are flatter. Just play around with Figure 152.1 or Equation (153.1) so see that: L^* is higher when P is higher! Since $Q^* = f(L^*, \bar{K})$, Q^* is also higher when P is higher. This also makes perfect sense: output is more expensive, so the firm hires produces more (or, stops later when hiring labor) and sells more.

  Indeed, following this argument, one can construct a **firm supply schedule** which gives a list of profit-maximizing P, Q^* pairs. One can then draw a **firm supply curve**. This is eventually what we will do, but let's not get ahead of ourselves.

6.3.3 Long-Run Profit Maximization

Profit maximization in the long-run is very similar to the profit maximization in the short-run, except that the firm also chooses K in the long-run. But the basic idea remains the same.

The profit-maximizing input-output combination (Q^*, L^*, K^*) now satisfies two equalities:

$$
\begin{aligned}
MPL(L^*, K^*) \cdot P &= w \\
MPK(L^*, K^*) \cdot P &= r
\end{aligned}
$$

Thus, the marginal revenue brought by each input must be equal to the cost of the respective input. Hopefully, this convinced you a little bit more towards how valuable *thinking at the margin* is. It is a powerful tool of analysis!

It is worth noting that the firm chooses Q^*, L^* and K^* simultaneously: after all, this is an optimization problem with three variables. Yet, these equations carry a sense of *the firm choosing* (L^*, K^*) *first and then producing the maximum output* Q^* *that can be produced with* (L^*, K^*)*, i.e.,* $Q^* = f(L^*, K^*)$. Let me reiterate: this is just an illusion, this is not what these equations mean, but this is typically how people think about these optimality conditions.

Next, we will turn this around and imagine that the firm chooses Q^*. We will essentially be writing down the same optimization problem, but there will be a single decision variable Q. To this end, we will define the cost of producing Q. This cost will include the amount of (L, K) necessary to produce Q, and the costs of those inputs (w, r). But we will sideline this at this stage, and just take the cost function as given.

Extra Readings for Chapter 6

Here is an article that discusses some of the theories of the firm, if you are interested. You can check out Section 1 to read the "informal discussion".

> Gibbons, Robert. "Four Formal(izable) Theories of the Firm?" *Journal of Economic Behavior & Organization* 58 (2005): 200–245.

If you are further intrigued, check out:

> Williamson, Oliver. *The Economic Institutions of Capitalism : Firms, Markets, Relational Contracting*. 1985

and

> Hart, Oliver. *Firms, Contracts, and Financial Structure*. 1995.

Exercises for Chapter 6

1) Consider a firm in a perfectly competitive market, with a production function defined as:

$$f(L,K) = 2 \cdot \sqrt{L} \cdot K \text{ kg/month} \quad .$$

a. Find the marginal product of labor and the marginal product of capital.

b. Does this production function satisfy diminishing marginal product of labor?

For the remaining parts of the question, suppose:

- The market price of the good that the firm produces is $P = 8$ TL/kg.
- The capital used in production is a building. The rental price of a building is: $r = 16$ TL/month.
- The wage rate is $w = 4$ TL/month.
- In the short run, the capital is fixed at $\overline{K} = 1$.

c. If the firm uses 16 workers to produce 8 kg's of output in a month, what is the firm's monthly profit in the short run?

d. What is the profit-maximizing quantity of labor in the short run?

e. What is the profit-maximizing quantity of output in the short run?

f. What is the maximum profit the firm can attain in the short run?

Chapter 7

Towards the Supply Curve

As the chapter title suggests, we are building towards drawing a supply curve. That is, we are trying to understand how *quantity supplied* changes with *price*. To this end, we first need to understand how an individual firm's quantity Q^* changes with price P.

The good news is that we already covered how a firm chooses Q^*. To reiterate what we learned in Chapter 6, when capital is fixed at $\bar{K}$,

1. the firm chooses L^* to satisfy $MPL(L^*, \bar{K}) \cdot P = w$.

2. the firm chooses Q^* to satisfy $Q^* = f(L^*, \bar{K})$.

Based on this, we can argue how Q^* changes with P. When P increases,

1. the firm chooses a higher L^*. This is because $MPL(L^*, K) \cdot P = w$ implies that $MPL(L^*, \bar{K})$ must be smaller when P is larger. Since the production function satisfies diminishing returns, $MPL(L, \bar{K})$ is decreasing in L, thus L^* must increase.

2. the firm chooses a higher Q^*. This is because $Q^* = f(L^*, \bar{K})$, L^* increases, and the production function is monotonic.

So... We can just say "When P increases, Q^* increases" and skip this section. But a part of this reasoning goes through how L^* changes, and we do not necessarily want to think about how L^* changes every time we consider the price changes. In other words, rather than considering a chain like

$$P \rightarrow L^* \rightarrow Q^*$$

we want to work with:

$$P \rightarrow Q^*$$

This chapter will allow us to establish a direct relationship between P and Q^*.

7.1 The "Cost Approach" to Production

The key object of analysis in this chapter will be the cost of producing Q. The cost is implicitly derived through production function, and the firm's choice of inputs to produce the necessary quantity. Formally:

Definition 158.1 *The* **cost function** *is the relation between the quantity produced Q and the cost of the optimal combination of inputs needed to produce the given quantity.*

A brief reminder that the costs that the firm considers are *opportunity costs*: they include the best alternative use of inputs (working elsewhere, renting away capital etc.) Moreover, they don't include *sunk costs*.

7.1.1 Fixed and Variable Costs

The total cost of producing Q units of output with the optimal combination of inputs is the **total cost**, denoted by $TC(Q)$.

We can decompose $TC(Q)$ into two components:

a. **Fixed Cost:** This is the component of total cost that does not change with the quantity produced. The fixed cost is due to the **fixed factors of production** (i.e., capital in the short-run).

   Because the fixed cost, by definition, does not depend on Q, we will denote it with FC.

b. **Variable Cost:** This is the component of the total costs that changes as the quantity produced changes. The variable cost is due to the **variable factors of production** (i.e., labor in the short-run).

   We will denote the variable cost with $VC(Q)$.

All in all, we have:

$$\text{Total Cost} = \text{Fixed Cost} + \text{Variable Cost}$$

or, to use the notation,

$$TC(Q) = FC + VC(Q)$$

A brief remark: As we discussed before, in the long run, all factors of production are variable. Thus, in the long-run, all costs are variable costs, i.e., $FC = 0$ in the long run.

7.1.2 Where the Total Cost Comes From

I have mentioned above that "the cost is implicitly derived through production function". This is another way of saying that, behind the scenes, there is some choices made by the firm (how much labor to hire, etc.). The cost approach allows us to circumvent those choices and work directly with $TC(Q)$. Still, it is useful to keep in mind that $TC(Q)$ is the cost of inputs that the firm is choosing, in order to produce Q. Below is a brief discussion on what exactly goes on behind the scenes.

Consider a firm with a production function $f(L,K)$. For the sake of the argument, consider the problem faced by the firm in the short-run, where the capital is fixed at $\bar{K}$. What is $TC(Q)$?

The first thing you need to realize is that, since the capital stock is fixed at $\bar{K}$, the firm has to pay $r \cdot \bar{K}$ no matter what it does. In other words, the fixed cost is $r \cdot \bar{K}$:

$$FC = r \cdot \bar{K} \ .$$

Next, realize that: in order to produce Q units of output, the firm needs to use L units of labor such that:

$$Q = f(L, \bar{K}) \ .$$

In other words, defining $g(L) = f(L, \bar{K})$, the firm's choice of L must satisfy:

$$Q = g(L) \iff L = g^{-1}(Q) \ .$$

(It is useful to remember that as long as $f(L,K)$ is monotonic, $g(L)$ is an increasing function of L. Then, $g^{-1}(Q)$ is an increasing function of Q.[1])

Therefore, in order to produce Q units of output, the firm needs to use $g^{-1}(Q)$ units of labor, which will cost $w \cdot g^{-1}(Q)$. Recalling that the variable cost is due to the variable factors of production (i.e., labor in the short-run), we conclude that:

$$VC(Q) = w \cdot g^{-1}(Q) \ .$$

Therefore,

$$\begin{aligned} TC(Q) &= FC + VC(Q) \\ &= r \cdot \bar{K} + w \cdot g^{-1}(Q) \ . \end{aligned}$$

Long story short: $TC(Q)$ does not just fall from the sky; it is derived from the firm's choices based on the production function.

[1] Also, if $f(L,K)$ satisfies diminishing marginal product, $g(L)$ is concave in L. This implies $g^{-1}(Q)$ is convex in Q. This is somewhat advanced reasoning, but it is good to know because it will be relevant in a minute.

7.1.3 Marginal Cost

As I have hopefully convinced you by now, it is useful to think at the margin! To incorporate marginal thinking into firm's problem, we define **marginal cost**.

Formally, **Marginal Cost** of producing the Q-th unit is the rate at which cost changes when output increases by a "small" amount so that the final output is Q units. We denote the marginal cost function with $MC(Q)$.

The increase in total cost when Q units of output rather than $Q-\Delta Q$ units is produced is:

$$TC(Q) - TC(Q - \Delta Q)$$

Therefore, the increase in total cost *per unit* is:

$$\frac{TC(Q) - TC(Q - \Delta Q)}{Q - (Q - \Delta Q)} = \frac{TC(Q) - TC(Q - \Delta Q)}{\Delta Q}$$

To express the marginal cost of producing Q-th unit, we take the smallest increments possible.

$$MC(Q) = \lim_{\Delta Q \to 0} \frac{TC(Q) - TC(Q - \Delta Q)}{\Delta Q} = \frac{dTC(Q)}{dQ}$$

This is literally the derivative of $TC(Q)$. Because FC is a constant, we also have:

$$MC(Q) = \frac{dTC(Q)}{dQ} = \frac{d(FC + VC(Q))}{dQ} = \frac{dFC}{dQ} + \frac{dVC(Q)}{dQ} = \frac{dVC(Q)}{dQ}$$

So, $MC(Q)$ is also the derivative of $VC(Q)$.

Below is a very crucial observation about the marginal cost:

- If the production function satisfies diminishing marginal product, then $MC(Q)$ is increasing. So it looks like the one in Figure 161.1.

  Intuitively, this is because each additional input is less productive. Therefore, it takes more inputs to produce the marginal output. Thus, the cost of producing the last unit of output is higher.

  Note: Many textbooks assume "diminishing marginal product **at some point**" (this is a natural implication of assuming "diminishing marginal product *at some point*. According to that definition, $MC(Q)$ is increasing **after some value of** Q, i.e., it may be decreasing at first. In these lecture notes, I will adopt a simpler requirement and assume that $MC(Q)$ is always increasing. But the overall conclusions would not change.

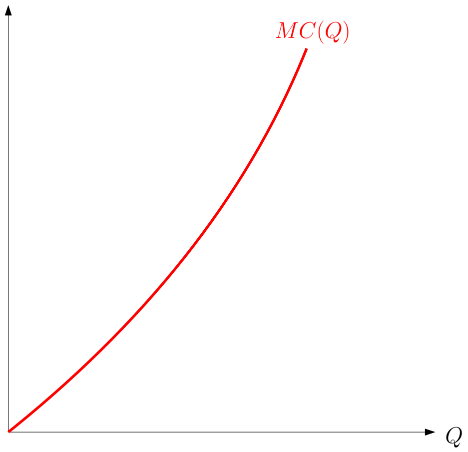


Figure 161.1: An increasing marginal cost function.

7.1.4 Average Costs

Beyond marginal cost, it turns out it is also useful to think about "average costs", i.e. costs per unit produced. Please note that average is different than marginal: this is **not** the cost of **last** unit produced, but the average of **all** units produced!

To calculate the average costs, we take the equation:

$$TC(Q) = FC + VC(Q)$$

and divide it by Q to calculate average costs:

$$\frac{TC(Q)}{Q} = \frac{FC}{Q} + \frac{VC(Q)}{Q}$$

We now define:

- **Average Total Cost** of producing Q units: total cost of producing Q units divided by Q.
$$ATC(Q) = \frac{TC(Q)}{Q}$$
- **Average Fixed Cost** of producing Q units: the fixed cost divided by Q.
$$AFC(Q) = \frac{FC}{Q}$$
- **Average Variable Cost** of producing Q units:variable cost of producing Q units dived by Q.
$$AVC(Q) = \frac{VC(Q)}{Q} \quad .$$

Therefore, we have:

$$ATC(Q) = AFC(Q) + AVC(Q)$$

A couple of notes about average costs are in order.

- By construction, $AFC(Q) = \frac{FC}{Q}$ is decreasing in Q.
- $AVC(Q)$ is the average of $MC(Q')$ of all $Q' \leqslant Q$.
  - To see this, first realize that: $VC(Q)$ is the sum of marginal costs of first Q units.
  - Then, $AVC(Q)$ is the average of the marginal costs of first Q units.

  Since $MC(Q)$ is increasing, $AVC(Q)$ is increasing in Q. (When calculating $AVC(Q)$, for larger values of Q, the firm is taking average of more items, where the items added later are larger quantities.)
- $AC(Q)$ is the sum of $AFC(Q)$ and $AVC(Q)$. Thus, it is first decreasing and then increasing in Q.

In light of these discussions, we end up with a figure like Figure 162.1.

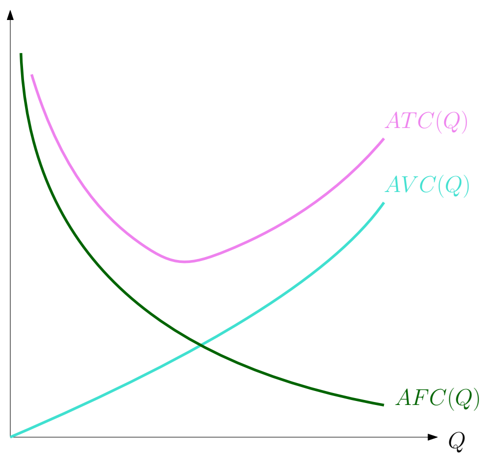


Figure 162.1: Average cost functions when $MC(Q)$ is increasing.

7.1.5 Putting Marginal and Averages Together

To sum up, here is what he have:

$$
\begin{aligned}
MC(Q) &= \frac{d}{dQ}TC(Q) = \frac{d}{dQ}\left(FC + VC(Q)\right) = \frac{d}{dQ}VC(Q) \\
AFC(Q) &= \frac{FC}{Q} \\
AVC(Q) &= \frac{VC(Q)}{Q} \\
ATC(Q) &= \frac{TC(Q)}{Q} = \frac{FC + VC(Q)}{Q} = \frac{FC}{Q} + \frac{VC(Q)}{Q} = AFC(Q) + AVC(Q)
\end{aligned}
$$

where $MC(Q)$ and $AVC(Q)$ are increasing, $AFC(Q)$ is decreasing, and $ATC(Q)$ is first decreasing and then increasing.

Here is another set of critical observations:

- $AVC(Q) \leqslant MC(Q)$ for every Q.
  - This is because, when calculating $AVC(Q)$, the firm is taking average of $MC(Q')$ for all $Q' \leqslant Q$.
  - Since marginal cost is increasing, $MC(Q') \leqslant MC(Q)$ for all $Q' \leqslant Q$. Therefore, the firm is taking average of a bunch of items, each less than $MC(Q)$.
- $MC(Q)$ intersects $ATC(Q)$ at the point where $ATC(Q)$ is minimized.

  We can prove this by taking derivatives and using simple calculus. Let Q^* be the point where $ATC(Q)$ reaches its minimum value. If the cost functions are "smooth",

  $$\frac{d}{dQ}ATC(Q^*) = 0$$

  By definition of $ATC(Q)$, this is equivalent to:

  $$\frac{d}{dQ}\left(\frac{TC(Q^*)}{Q^*}\right) = 0$$

  which is equivalent to:

  $$\frac{\frac{d}{dQ}TC(Q^*)Q^* - TC(Q^*)}{(Q^*)^2} = 0$$

which implies:

$$\frac{d}{dQ}TC(Q^*)Q^* - TC(Q^*) = 0$$

By rearranging,

$$\frac{d}{dQ}TC(Q^*) = \frac{TC(Q^*)}{Q^*}$$

But recall that $\frac{d}{dQ}TC(Q^*) = MC(Q^*)$ and $\frac{TC(Q^*)}{Q^*} = ATC(Q^*)$. Therefore, at the quantity Q^* where $ATC(Q)$ attains its minimum value,

$$MC(Q^*) = ATC(Q^*)$$

Pretty cool, huh? Not really surprising, though: for $Q \leqslant Q^*$, $MC(Q)$ is small so it is pulling $ATC(Q)$ downwards. For $Q \geqslant Q^*$, $MC(Q)$ becomes large enough so that it pulls $ATC(Q)$ upwards. At $Q = Q^*$, $MC(Q)$ pulls $ATC(Q)$ neither downwards nor upwards.

All in all, the cost curves look like the ones in Figure 164.1.

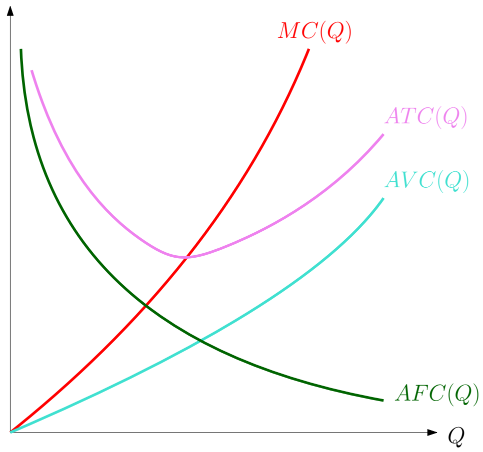


Figure 164.1: Cost functions when $MC(Q)$ is increasing.

7.2 Profit Maximization Using Cost Functions

Okay, now let's put what we have to work. Recall that the firm chooses its quantity Q to maximize its profit, where:

$$\text{Profit} = \text{Revenue} - \text{Cost} \ .$$

and

$$\text{Revenue} = \text{Price} \times \text{Quantity} \ .$$

We know the "cost" and "quantity", but what about the price? If the firm would like to sell Q units, it must set its price according to *demand*. Recall that the demand curve gives the relationship between quantity demanded (Q) and prices (P).

Consider the demand curve for a firm (which may be different from the demand curve for an entire market). This curve gives a the price of the good as a function of the quantity produced. Let $P(Q)$ denote the price level at which Q units of the good will be demanded.

Now,

- The **profit** from producing and selling Q units of output is: $\pi(Q)$
- The **total revenue** from selling Q units of outputs is: $TR(Q) = P(Q) \cdot Q$.
- The **total cost** of producing Q units of output is: $TC(Q)$.

Then,

$$\pi(Q) = TR(Q) - TC(Q)$$

7.2.1 Marginal Revenue

Should I remind you about the value of thinking at the margin?

So profit has two components: revenue and cost. We know what marginal cost is. How about we define **marginal revenue**?

Formally, **Marginal Revenue** brought by the Q-th unit is the rate at which revenue changes when output increases by a "small" amount so that the final output is Q units. We denote marginal revenue function with $MR(Q)$.

The increase in total revenue when Q units of output rather than $Q - \Delta Q$ units is produced is:

$$TR(Q) - TR(Q - \Delta Q)$$

Therefore, the increase in total revenue *per unit* is:

$$\frac{TR(Q) - TR(Q - \Delta Q)}{Q - (Q - \Delta Q)} = \frac{TR(Q) - TR(Q - \Delta Q)}{\Delta Q}$$

To express the marginal revenue of producing Q-th unit, we take the smallest increments possible.

$$MR(Q) = \lim_{\Delta Q \to 0} \frac{TR(Q) - TR(Q - \Delta Q)}{\Delta Q} = \frac{dTR(Q)}{dQ}$$

7.2.2 Profit-Maximizing Quantity

The firm is choosing Q to maximize:

$$\pi(Q) = TR(Q) - TC(Q)$$

How do we go around solving this problem? The classical method is taking the derivative and identifying the first-order condition. I will skip the steps of the argument here, but you should realize that if $Q^* > 0$ is a profit-maximizing quantity, then it should satisfy:

$$\begin{aligned} \frac{d\pi(Q^*)}{dQ} = 0 &\implies \frac{d(TR(Q^*) - TC(Q^*))}{dQ} = 0 \\ &\implies \frac{dTR(Q^*)}{dQ} - \frac{TC(Q^*)}{dQ} = 0 \\ &\implies MR(Q^*) - MC(Q^*) = 0 \\ &\implies MR(Q^*) = MC(Q^*) \end{aligned}$$

Let's express this as a theorem. (I relegate the proof to the Appendix in case you are interested.)

Theorem 166.1 *If Q^* is a profit-maximizing quantity and $Q^* > 0$, then*

$$MR(Q^*) = MC(Q^*) \tag{166.1}$$

Equation (166.1) is a very famous equality. It is colloquially called the **golden rule of profit maximization**!

The intuitive way to understand Theorem 166.1 is to visualize the following process. The firm keeps producing and selling output as long as the revenue brought by the last unit is higher than its cost. The firm stops at the point where the marginal revenue is not higher any more. Under reasonable conditions (such as increasing marginal cost and decreasing marginal revenue), this process stops at some point.

7.3 Firms in a Perfectly Competitive Market

Suppose the firm operates in a perfectly competitive market. This means that the firm is a **price-taker**: it takes P as given and cannot change it. More formally, the firm's demand curve is a perfectly elastic curve, which is a horizontal line at P.

In this case, since $P(Q)$ does not depend on Q, we have:

$$TR(Q) = P \cdot Q$$

and

$$MR(Q) = P$$

Therefore, by Theorem 166.1 the profit-maximizing quantity $Q^* > 0$ satisfies:

$$P = MC(Q^*) \tag{167.1}$$

What does Equation (167.1) mean? Basically it says: "In a competitive market, if the firm is producing anything at all, it produces the quantity such that the price equals marginal cost. (We will take care of the "if the firm is producing anything at all..." part in the next subsection.)

In a competitive market, the firm keeps producing and selling output as long as the price is higher than the marginal cost. The firm stops at the point where the price equals the marginal cost. If marginal cost of increasing, this process stops at some point.

7.3.1 Shut Down Criterion

Let's go back to the annoying clause we had above: "if the firm is producing anything at all..."

Note that the firm always has the option of shutting down and not producing anything ($Q = 0$ is always an option for the firm). Clearly, this would keep $VC(0) = 0$: if the firm does not produce anything, it does not need to hire any variable factors of production. Still, in the short run, FC is still positive: even when the firm does not produce anything it needs to pay for the fixed factors of production. Thus, $TC(0) = VC(0) + FC = FC$. Therefore, shutting down yields a profit of $\pi(0) = P \cdot 0 - TC(0) = P \cdot 0 - FC = -FC$ in the short run.

When does the firm produce anything at all? Recall that the maximum profit the firm receives from producing anything at all is attained when the firm produces Q^*. This gives a profit of:

$$\pi(Q^*) = P \cdot Q^* - TC(Q^*) = P \cdot Q^* - \big(VC(Q^*) - FC\big)$$

The firm shuts down in the short run if not producing anything yields a higher profit than producing Q^*, i.e., if $\pi(0) > \pi(Q^*)$. This is the case when:

$$-FC > P \cdot Q^* - \big(VC(Q^*) - FC\big)$$

which is equivalent to:

$$0 > P \cdot Q^* - VC(Q^*)$$

which is equivalent to:

$$P < \frac{VC(Q^*)}{Q^*} = AVC(Q^*)$$

Therefore, **the firm chooses to shut down in the short run if the price is so low that it doesn't even cover the average variable costs**. The firm is getting a revenue of P per unit produced, and paying a variable cost of $AVC(Q^*)$ per unit! If $P < AVC(Q^*)$, there is no reason for the firm to produce anything: the revenue does not even cover the cost of variable inputs!

The flip side of this is: **the firm operates in the short run if** $P \geqslant AVC(Q^*)$.[2]

The condition of $P < AVC(Q^*)$ is sometimes referred as the **shut down criterion**: this is the condition for the firm to shut down in the short run. Note that the fixed costs are not included in this calculation. This is because they are like **sunk costs** in the short run: the firm has to pay them no matter what it produces, and therefore they should not be taken into account in the decision to produce or not in the short run.

Reassuringly, when $MC(Q)$ is increasing, the shut down criterion is never satisfied. This is because Q^* satisfies $P = MC(Q^*)$. But recall that $AVC(Q) \leqslant MC(Q)$ for all Q. Therefore,

$$AVC(Q^*) \leqslant MC(Q^*) = P$$

So, if $MC(Q)$ is increasing for all Q, we do not need to worry about the firm shutting down in the short run.

The shut down criterion becomes a larger concern if we have "eventually increasing marginal cost" as your textbook assumes. Then, $MC(Q)$ may not always be increasing, and we may have $AVC(Q) > MC(Q)$ for some values of Q. I am adding a discussion of eventually increasing marginal cost to the Appendix, if you are interested.

[2]When $P = AVC(Q^*)$, the firm is indifferent between producing or not. I am breaking the tie in favor of producing. Because this is a knife-edge case, it doesn't really matter.

Appendix to Chapter 7

Proof of Theorem 166.1

This proof assumes that total revenue and cost are "smooth" functions, and thus their derivatives exist.

Begin by noting:

$$\begin{aligned}\pi(Q) - \pi(Q - \Delta Q) &= TR(Q) - TC(Q) - (TR(Q - \Delta Q) - TC(Q - \Delta Q)) \\ &= TR(Q) - TR(Q - \Delta Q) - (TC(Q) - TC(Q - \Delta Q))\end{aligned}$$

Then, dividing both sides by ΔQ,

$$\frac{\pi(Q) - \pi(Q - \Delta Q)}{\Delta Q} = \frac{TR(Q) - TR(Q - \Delta Q)}{\Delta Q} - \frac{TC(Q) - TC(Q - \Delta Q)}{\Delta Q} \tag{169.1}$$

Suppose Q^* is a profit-maximizing quantity and $Q^* > 0$.

- Consider any $\Delta Q > 0$ such that $Q^* - \Delta Q > 0$. Q^* being the profit-maximizing quantity implies: $\pi(Q^*) \geqslant \pi(Q^* - \Delta Q)$. Rearranging:

  $$\pi(Q^*) - \pi(Q^* - \Delta Q) \geqslant 0$$

  Dividing both sides by $\Delta Q > 0$ gives:

  $$\frac{\pi(Q^*) - \pi(Q^* - \Delta Q)}{\Delta Q} \geqslant 0$$

  Substituting (169.1),

  $$\frac{TR(Q^*) - TR(Q^* - \Delta Q)}{\Delta Q} - \frac{TC(Q^*) - TC(Q^* - \Delta Q)}{\Delta Q} \geqslant 0$$

  Since the above inequality is true for any positive $\Delta Q < Q^*$, it is also true for "small" ΔQ. But this implies

  $$MR(Q^*) - MC(Q^*) \geqslant 0$$

  or

  $$MR(Q^*) \geqslant MC(Q^*) \tag{169.2}$$

- Now, consider $\Delta Q < 0$. Once again, Q^* being the profit-maximizing quantity implies: $\pi(Q^*) \geqslant \pi(Q^* - \Delta Q)$. Rearranging:

  $$\pi(Q^*) - \pi(Q^* - \Delta Q) \geqslant 0$$

  Dividing both sides by $\Delta Q < 0$ gives:

  $$\frac{\pi(Q^*) - \pi(Q^* - \Delta Q)}{\Delta Q} \leqslant 0$$

  Substituting (169.1),

  $$\frac{TR(Q^*) - TR(Q^* - \Delta Q)}{\Delta Q} - \frac{TC(Q^*) - TC(Q^* - \Delta Q)}{\Delta Q} \leqslant 0$$

  Since the above inequality is true for any $\Delta Q < 0$, it is also true for "small" ΔQ. But this implies

  $$MR(Q^*) - MC(Q^*) \leqslant 0$$

  or

  $$MR(Q^*) \leqslant MC(Q^*) \tag{170.1}$$

Combining (169.2) and (170.1) gives the following statement. If Q^* is a profit-maximizing quantity of output and $Q^* > 0$, then

$$MR(Q^*) \leqslant MC(Q^*) \text{ and } MR(Q^*) \geqslant MC(Q^*) \implies MR(Q^*) = MC(Q^*)$$

"Eventually" Increasing Marginal Cost

A reminder that many textbooks merely imposes "diminishing marginal product **at some point**" on the production function. This translates into "increasing marginal cost **at some point**", i.e., $MC(Q)$ only has to be increasing after some value of Q. That is, $MC(Q)$ only needs to be **eventually** increasing. This is a weaker requirement than increasing $MC(Q)$: it is sufficient if $MC(Q)$ is *eventually* increasing. (Indeed, "increasing marginal cost" is just a special case of "eventually increasing marginal cost" – it amounts to saying that "eventually" hits very early.)

From a broad point of view, not much changes by adopting the requirement of "eventually increasing $MC(Q)$".

- $AVC(Q)$ is *eventually* increasing in Q.

- Moreover, $AVC(Q) \geqslant MC(Q)$ for small values of Q and $AVC(Q) \leqslant MC(Q)$ for larger values of Q. In other words, $AVC(Q) \leqslant MC(Q)$ for sufficiently large values of Q.

All in all, when $MC(Q)$ is eventually increasing, the cost curves look like the ones in Figure 171.1.

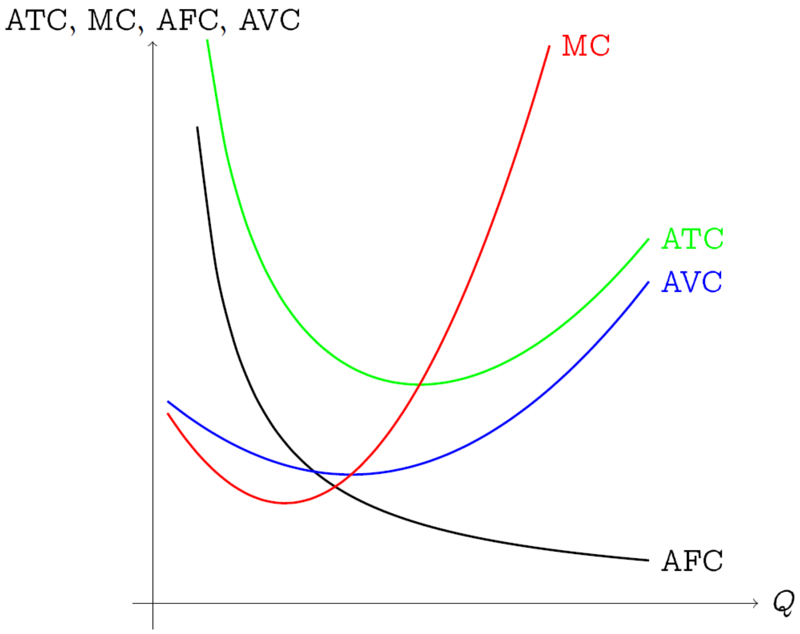


Figure 171.1: Cost functions when $MC(Q)$ is eventually increasing.

Another observation: $MC(Q) = AVC(Q)$ at the point where $AVC(Q)$ is minimized, just like $MC(Q) = ATC(Q)$ at the point where $ATC(Q)$ is minimized. Once again, not a coincidence! We can indeed also prove this by taking derivatives and using simple calculus. I'm leaving it as an exercise.

Shut Down Criterion and "Eventually" Increasing Marginal Cost

When $MC(Q)$ is eventually increasing, we actually need to worry about the shut down criterion. Therefore, for P small enough so that $P < AVC(Q^*)$, the firm actually shuts down: the firm chooses $Q = 0$ instead of Q^*. This means, for small enough P, the firm produces nothing. Once P exceeds $AVC(Q^*)$, the firm chooses Q^* such that $P = MC(Q^*)$.

But when is $P < AVC(Q^*)$? Recall that $MC(Q^*) = P$, so that the shut down criterion is:

$MC(Q^*) < AVC(Q^*)$. Now, recall that $MC(Q) = AVC(Q)$ at the point where $AVC(Q)$ is minimized. Therefore, $MC(Q) < AVC(Q)$ at the quantities smaller than the quantity that minimizes $AVC(Q)$. We end up with the following observation:

> In the short run, the firm shuts down if P is less than the minimum value of $AVC(Q)$.

Exercises for Chapter 7

1) Consider a firm in a perfectly competitive market, with a production function defined as:

$$f(L, K) = 2 \cdot \sqrt{L} \cdot K \text{ kg/month} \quad .$$

Suppose:

- The market price of the good that the firm produces is $P = 8$ TL/kg.
- The capital used in production is a building. The rental price of a building is: $r = 16$ TL/month.
- The wage rate is $w = 4$ TL/month.
- In the short run, the capital is fixed at $K = 1$.

a. Find the total cost, fixed cost, variable cost, average total cost, average fixed cost, average variable cost, and marginal cost functions of this firm in the short run.

b. Based on the cost functions, what is the profit-maximizing quantity of output in the short run? What is the firm's profit?

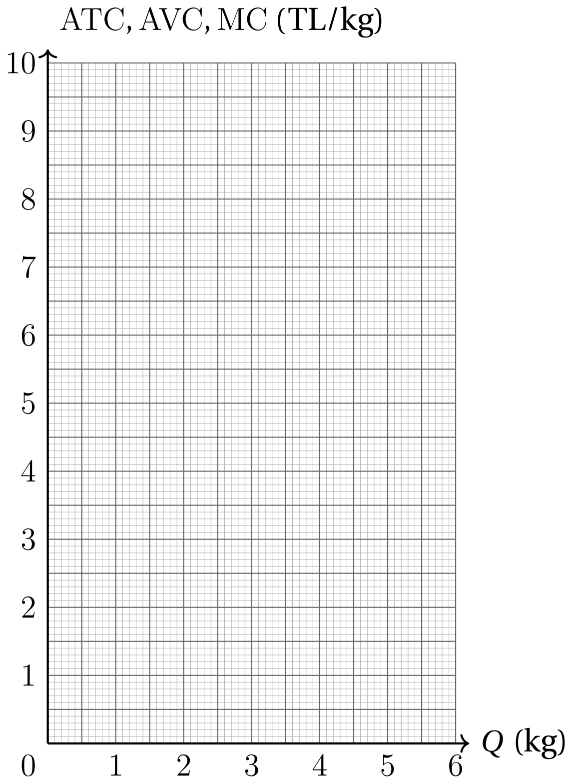
2) Consider a competitive market where the short run total cost function of a typical firm is given by:

$$\text{TC}(Q) = 4 + 2Q + Q^2 \quad ,$$

where the quantity Q is measured in kg.

a. Find the fixed cost, average total cost, average variable cost, and marginal cost of this firm.

b. Draw the marginal cost, average variable cost, and the average total cost on the graph below:



c. When the price is P, what is the quantity supplied by this firm?

Chapter 8

Supply

In this chapter, we draw a **supply curve**. A supply curve is the second important element of a **market**. This chapter is the counterpart of Chapter 5 for the producer side, and we will be wrapping up our discussion on the **producer side** of the market with this chapter.

8.1 Supply Schedules and Firm Supply Curves

Just like a demand schedule, a **supply schedule** is an excel sheet of possible prices P and quantities supplied at these prices Q^*. Just like an individual demand curve, a **firm supply curve** is just the plotted version of this information, where P is on the y-axis and Q is on the x-axis. We construct graph by hypothetically going to the firm and asking the following question repeatedly, and plotting it:

> "If the price of good you are producing is P per unit, what is the quantity you supply Q^*?"

From the previous chapter, we know that the firm's answer is the following:

> "I supply the quantity Q^* such that $P = MC(Q^*)$."

What does it mean, graphically? To get an intuition, consider Figure 176.1 which plots $MC(Q)$. If the price is P_1, the firm chooses Q_1^*. If the price is P_2, the firm chooses Q_2^*. If the price is P_3, the firm chooses Q_3^*. You see where I am going? We are just tracking the $MC(Q)$ curve. Therefore, **the firm supply curve is the firm's marginal cost curve**.

Here is another way to think about why the firm supply curve is the same thing as the firm's marginal cost curve. Just like the individual demand curve, a firm's supply curve has two interpretations.

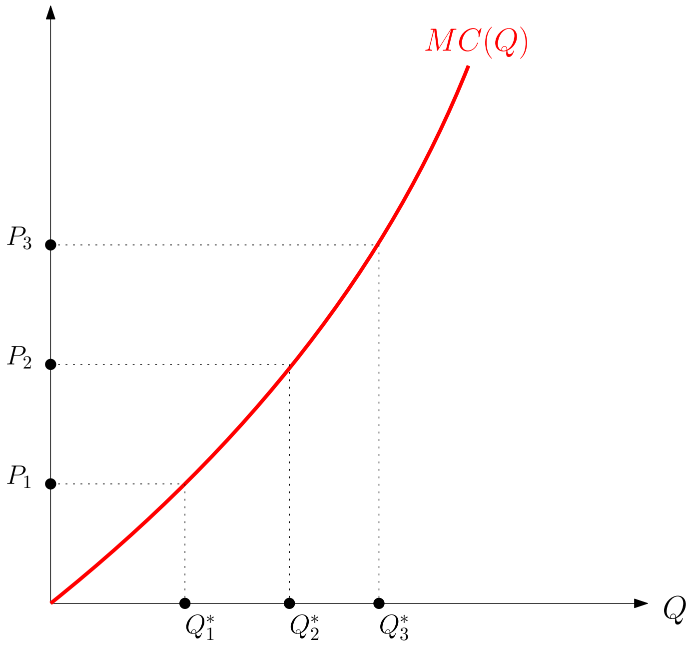


Figure 176.1: Firm Supply Curve when $MC(Q)$ is increasing.

a. (From P to Q) It shows, at each price, the quantity supplied by the producer.

b. (From Q to P) It shows, at each quantity Q, the smallest price there needs to be for the firm to supply at least Q units.

   - But what should the consumers pay for the firm to supply the Q-th unit? Answer: The cost of supplying the Q-th unit, i.e., $MC(Q)$.

8.2 Market Supply Curves

You know what is coming next: just like we constructed a market demand curve from individual demand curves, we will construct a market supply curve from the firm supply curves of each firm in a market. To obtain the market supply curve, we add up the supply of every firm in the market. That is, at every single price, we add up the quantity supplied of each firm at the said price. Figure 177.1 is a representative figure where we add up two firm supply curves. For more than two firms, the process is the same.

So, if we add up every firm's supply curve, we end up with the market supply curve, or a **supply curve** as it is commonly called. Figure 178.1 illustrates a representative supply curve. We will use the letter S to label a supply curve, which stands for "Supply".

A couple of general points about supply curves follow.

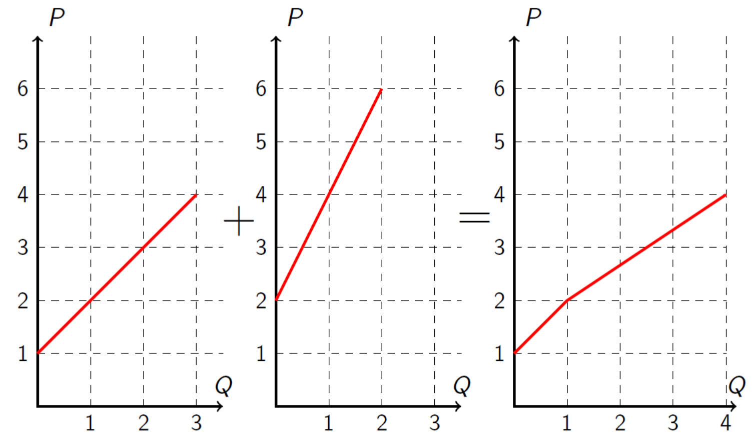


Figure 177.1: Addition of two firm supply curves.

First, note that the supply curve is increasing. This is not surprising: a supply curve is the summation of multiple firm supply curves. Because each firm's supply curve is increasing, their summation has to be an increasing curve!

You shouldn't be surprised to hear that economists call this a "law".

Definition 177.1 [Law of Supply] *Holding everything else constant, when the price of a good rises, the quantity supplied rises.*

I think it is better to understand which assumptions lead to this result, rather than blindly memorizing it as a law. Remember: we started by assuming that the firm's production function exhibits **diminishing returns**. This translates into an **decreasing marginal product of inputs**. This implies **increasing marginal cost**. But then, since the firm supply curve is the firm's marginal cost curve, **the firm supply curve is increasing**. Naturally, then, the market supply curve is increasing, and we end up with the **law of supply**. (This train of thought is also useful in illustrating the value of math in economic analysis: we can keep track of which assumptions lead to which conclusions!)

Second, a simple remark. Most economics textbooks draw supply curves as lines, not curves. That is made for the sake of convenience, but that may be misleading sometimes. **A supply curve can be a line, but it does not have to be.** The only requirement we are imposing on the supply curve is being upward-sloping.

Third, just like a firm supply curve, a market supply curve also has a dual interpreta-

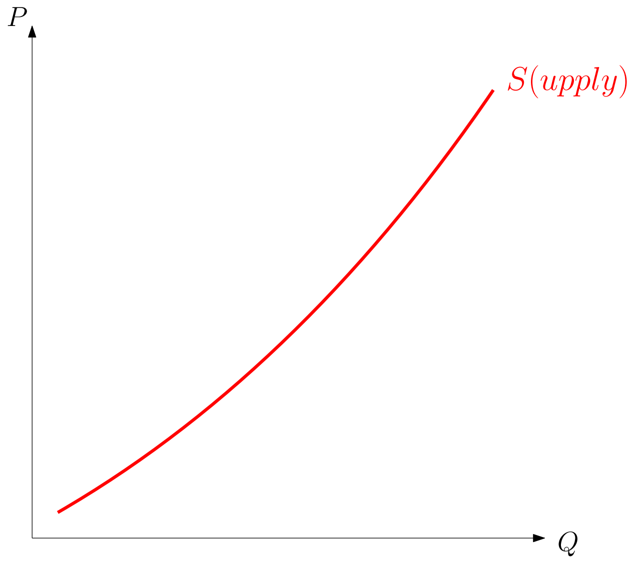


Figure 178.1: A representative supply curve.

tion.

a. (From P to Q) It shows, at each price, the total quantity supplied by the producers in the market.

b. (From Q to P) It shows, at each quantity Q, the marginal cost of the producer that supplies the marginal unit of the good.

Finally, even though we derived the supply curve from each firm's profit maximization decisions, the general analysis also applies for the cases where the supply side of the market does not literally "produce" the good. They may just be the existing owners of some good who are considering selling the goods they own in a market. (If it helps, you can also imagine a bunch of warehouses that own the goods in their stock, and deciding on whether selling their stock in the marketplace.) The only requirement we impose is: at higher prices, more sellers must be willing to sell.

To capture the idea of "potential sellers deciding whether to sell a good they own", we can introduce the notion of a **reservation price**. The reservation price is the minimum price a seller is willing to sell the good she owns in a market. Consider a second hand car market. Literally every owner of a car is a potential seller: if the price in the second hand car market is high enough, they will decide to sell their cars. But to do that, the market price has to be sufficiently high: it must be higher than the valuation they assign to their cars. A car owner's valuation for the car, then, is her reservation price. Naturally, if the market price is higher, it is above the reservation prices of more sellers, which will result in a higher quantity supplied.

If you think about it, the reservation price plays the role of a marginal cost in a market where sellers do not literally engage in production. Well, that should not be surprising. Both the notion of "marginal cost" and "reservation price" are some measures of *opportunity costs* of sellers.

If the good under question is not produced, but merely owned by the sellers, we can reinterpret the supply curve:

a. (From P to Q) It shows, at each price, the total quantity supplied by the sellers in the market.

b. (From Q to P) It shows, at each quantity Q, the reservation price of the seller that supplies the marginal unit of the good.

8.3 Structural Changes in the Economy

So, the *supply curve* of a good (or service) summarizes the relationship between possible prices of the good and the quantities supplied by the producers at these prices. It answers the following question: what happens to quantity supplied if the market price increases?

The issue is: there are other (structural) changes in the economy that would impact the quantity supplied *at each level of price*. We represent these changes with shifts in the supply curve, just like we did with the demand curve.

As an example, suppose the government decides to impose a sales tax of T on producers: for every unit of the good a producer sells, it has to pay an extra T TLto the government. This is effectively equivalent to an increase of T in the marginal cost: from the perspective of the firm, the cost of producing and selling the Q-th unit is $MC(Q)+T$ instead of $MC(Q)$. This change will be reflected as an upward shift in the supply curve. The exact amount of shift is not important, but the qualitative change matters: for every quantity, the smallest price for the firms to supply that quantity is higher. See Figure 180.1: at quantity Q, the new supply curve has P' rather than P. Of course, geometrically, this can also be called a shift to the left. For each value of market price P, the firms are willing to supply a smaller quantity of units because producing is effectively more costly. That is, at price P, the quantity supplied is Q' rather than Q. To be precise with what we mean, we will mostly refer to this as "a shift in the northwestern direction".

Now, consider an alternative policy: the government gives a subsidy of T on producers: for every unit of the good a producer sells, the government pays an extra T TL. This is equivalent to an decrease of T in the marginal cost: from the perspective of the firm, the cost of producing and selling the Q-th unit is $MC(Q)-T$ instead of $MC(Q)$.

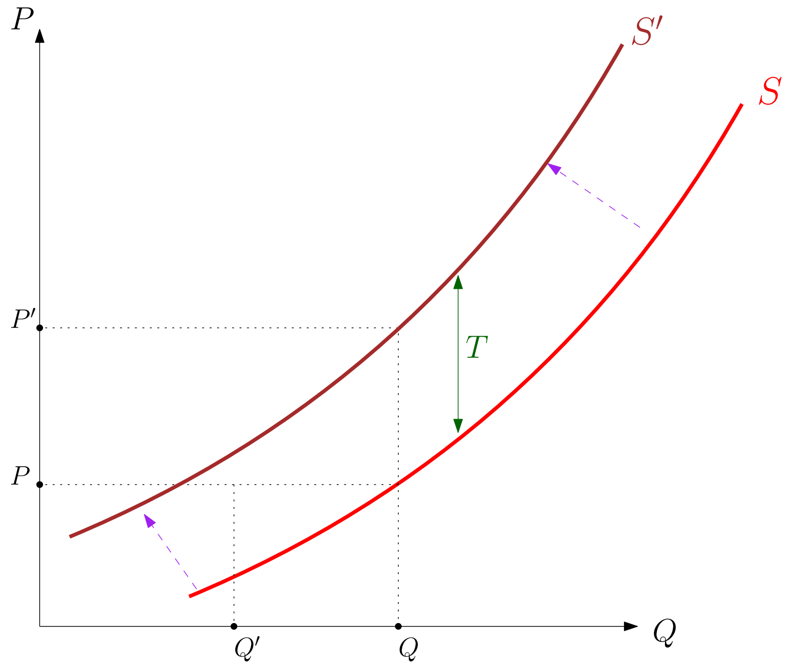


Figure 180.1: A shift of the supply curve shift in the northwestern direction. Some resources also call this a “shift left” or a “shift upwards”.

This change will be reflected as an downward shift in the supply curve: for every quantity, the smallest price for the firms to supply that quantity is lower because the government is already partly paying to the firm. See Figure 181.1: at quantity Q, the new supply curve has P' rather than P. Geometrically, this can also be called a shift to the right. For each value of market price P, the firms are willing to supply a larger quantity of units because the government is subsidizing selling the goods. That is, at price P, the quantity supplied is Q' rather than Q. To be precise with what we mean, we will mostly refer to this as “a shift in the southeastern direction”.

Below is a discussion of several variables that may shift the supply curve.

8.3.1 Costs of Inputs

Just like taxes or subsidies, a change in the costs of inputs will change the marginal cost and therefore shift the supply curve.

- If the cost of an input increases, the supply curve will shift in the northwestern direction. This happens, for instance, if the wages increase due to an increase in minimum wage or due to a change in the labor market conditions.
- If the cost of an input decreases, the supply curve will shift in the southeastern

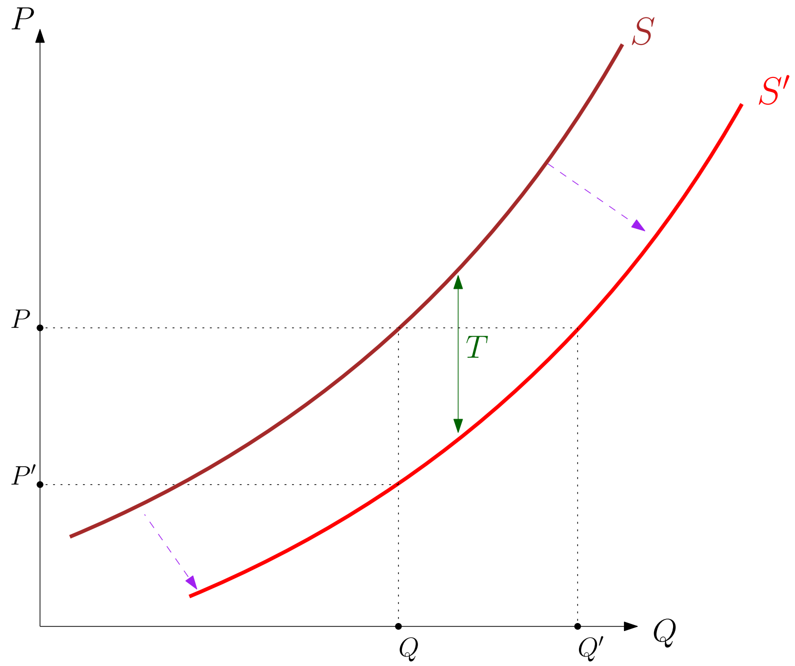


Figure 181.1: A shift of the supply curve in the southeastern direction. Some resources also call this a “shift right” or a “downard shift”.

direction. This happens, for instance, if one of the raw materials used in production becomes cheaper. (The market supply curve for air travel will shift in the southeastern direction if the price of oil, an input in the process of producing air travel, decreases.)

8.3.2 Technological Change

This covers the changes in production technology.

- If the firms find a much more efficient way of producing an output for a given set of inputs, they are willing to supply more at a given price. Thus, the supply curve will shift in the southeastern direction. This is the case, for instance, when a technological breakthrough in the industry occurs. (Steam powered engines, assembly lines, invention of computers etc. are all examples.) For instance, if oil producers figure out a way of extracting oil much more efficiently, the supply curve will shift in the southeastern direction.
- If the firms lose some technology necessary to produce a good or service, this will cause a shift in the northwestern direction. This is the case, for instance, if an earthquake happens and destroys a factory/the machinery necessary to

produce a good.

8.3.3 Prices of Other Goods

Just like consumers, the producers can also switch to/from producing other goods. This will lead to shifts in the supply curve.

We say that two goods are **substitutes in production** if they can alternatively be produced by a firm. Think of smartphones and smartwatches: most firms can use the same set of inputs, machinery, factories etc. they use to produce smartphones to use smartwatches as well. Therefore, a smartphone producing firm can alternatively produce smartwatches. If the price of smartwatches increases, some smartphone producers will switch to producing smartwatches. This will lead to a northwestern shift in the supply curve for smartphones. Alternatively, if the price of smartwatches decreases, some smartwatch producers will switch to producing smartphones. This will lead to a southeastern shift in the supply curve for smartphones.

In general, consider the demand curve for good X, and consider another good Y which is a substitute in production for good X.

- An increase in the price of good Y will cause a northwestern shift in the supply curve for good X.
- A decrease in the price of good Y will cause a southeastern shift in the supply curve for good X.

We say that two goods are **complements in production** if they can be produced together by a firm. For instance, goatherds produce both wool and goat cheese. (Once you start cattling goat, you can sell its wool, but also can produce goat cheese out of their milk.) If the price of goat cheese increases, goatherds will start cattling more goat. But then, they will also end up producing more wool. This will lead to a southeastern shift in the supply of wool.

In general, consider the demand curve for good X, and consider another good Y which is a complement in production for good X.

- An increase in the price of good Y will cause a southeastern shift in the supply curve for good X.
- A decrease in the price of good Y will cause a northwestern shift in the supply curve for good X.

8.3.4 Expected Future Prices

If the producers expect the price of a good to increase in the future, they will want to store some of their produced goods with the hope of selling them in the future for a higher price. This will result in a northwestern shift in the supply curve right now.

These type of changes are especially relevant for the markets where sellers are not literal "producers", and they are merely existing owners that can sell the goods they own. Once again, think of the second hand car market. If the price of second hand cars is expected to increase in the future, fewer people sell their cars because they want to wait until the prices increase. Similarly, if the price of a second hand car is expected to decrease in the future, some people (those who are contemplating selling theır car and buying a new one for themselves) and induced to selling their cars before the prices decrease, leading to the shift in the southeastern direction.

8.3.5 Number of Firms in the Market

Quite easily, if more firms enter the market, the supply curve will shift in the south-eastern direction. Suppose, due to some unexplained mania, thousands of lokma producers start popping up everywhere in the city. The supply curve for lokmas will shift in the southeastern direction. When all those lokma producers start going bankrupt, the supply curve will shift in the northwestern direction.

Here is a picture of lokma for you to finish the lecture. Because why not. Good luck if you are reading this document in the middle of the night.

Figure 183.1: Lokma. https://youtu.be/aPJWq3WPpko

Extra Readings for Chapter 8

Throughout Chapter 8, we assumed that the marginal cost is increasing. What if it was decreasing, i.e., what if the supply curve was downward-sloping? (This is the counterpart to the question of "what if the demand curve was upward-sloping?" for the supply side of the economy. As you recall, we called such a situation *Giffen behavior*. Is there something like a *Giffen behavior* for the supply side?)

For a very interesting exercise, see:

> Einav, Liran, and Amy Finkelstein. "Selection in Insurance Markets: Theory and Empirics in Pictures." *Journal of Economic Perspectives* 25, no. 1 (2011): 115-138.

The authors argue that insurance markets have this intriguing feature of a decreasing marginal cost. The reason is: for a given price, only the consumers with sufficiently high marginal benefit will buy insurance. But those consumers are precisely the consumers who *need* insurance the most, and therefore, they are the *most costly* consumers for the insurance company! As a result, the costs are pretty high. As the price decreases, less costly consumers start buying insurance as well, pulling the costs down.

You should check out the paper to really appreciate how insightful our graphical approach is.

The awkward feature of the insurance markets (those benefit from insurance the most are the most costly consumers) is called an *adverse selection problem*. Adverse selection is a prominent feature of many economic interactions: if a car owner is willing to sell their car at a given price to you, you should be wary – that is a sign that they don't value the car much, which means you wouldn't value it much either. This epiphany led to George Akerlof writing a classic in economic theory:

> Akerlof, George A. "The Market for "Lemons": Quality Uncertainty and the Market Mechanism." *The Quarterly Journal of Economics* 84, no. 3 (1970): 488-500.

Anyway, if you are interested in learning adverse selection more, you should check out Econ 448 (Economics of Information).

Figure 185.1: When Groucho Marx said "I don't want to belong to any club that will accept me as a member", we all felt dat adverse selection problem.

Appendix to Chapter 8

Firm Supply Curve under "Eventually" Increasing Marginal Cost

Before you read this section, you should revisit the Appendix to Chapter 7.

When the marginal cost is "eventually" increasing, the firm supply curve is:

- A vertical line for prices less than the minimal value of $AVC(Q)$.
- The marginal cost curve for prices larger than the minimal value of $AVC(Q)$.

For a further illustration, let's check Figure 186.1.

So, generally speaking, the firm supply curve is first a vertical line, and then it coincides with the marginal cost curve. But from now on, I will not really worry about the vertical part.

- First of all, the vertical part does not exist if $MC(Q)$ is increasing for all Q.
- But even if it exists, it exists for **very small** values of the price. Like, **unrealistically small** prices. Prices so small that the firms shut down rather than getting revenue! If prices remain that low for long, these firms would go bankrupt! But then, because some firms are swept away from the market, eventually the market price should increase. Therefore, such low prices cannot exist over long time

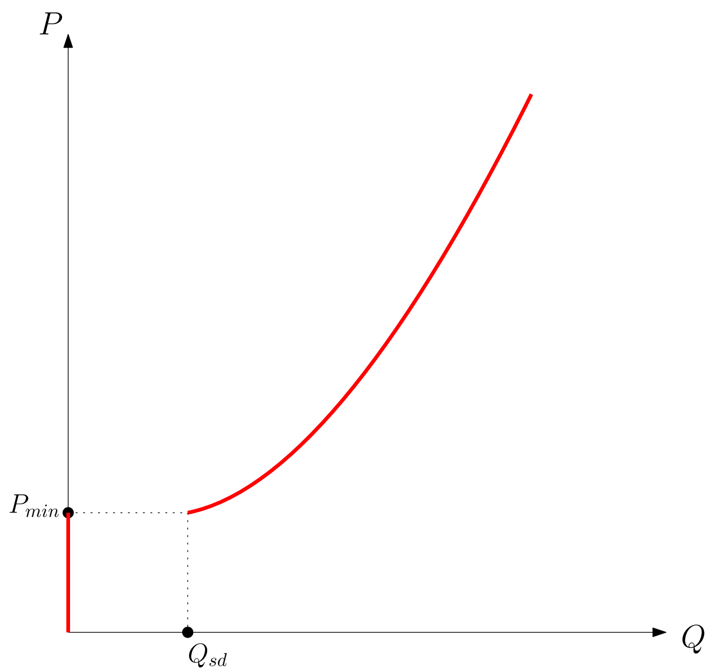


Figure 186.1: Firm Supply Curve when $MC(Q)$ is eventually increasing.

periods.[1]

So from now on, let's agree on saying that the firm supply curve is the firm's marginal cost curve.

[1] You should wait until Econ 203 to formalize this argument.

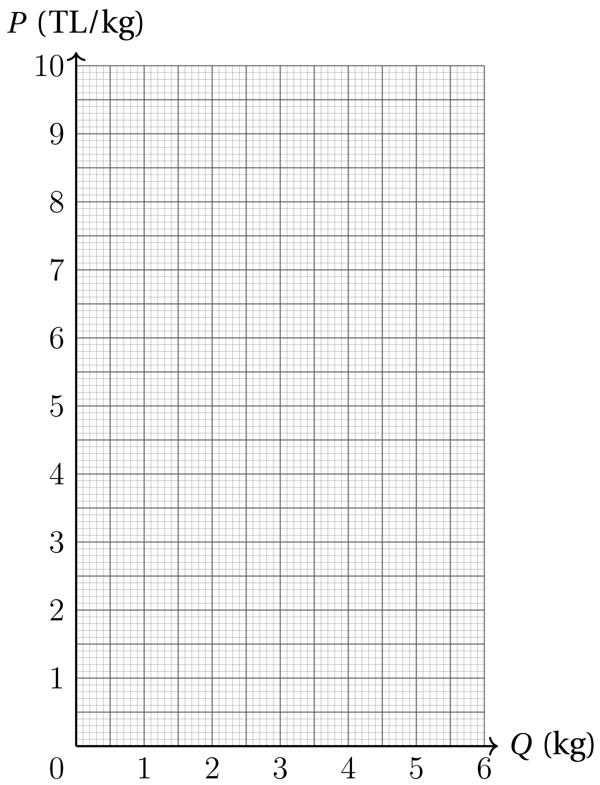
Exercises for Chapter 8

1) Consider a competitive market where the short run total cost function of a typical firm is given by:

$$\text{TC}(Q) = 4 + 2Q + Q^2 \ ,$$

where the quantity Q is measured in kg.

a. Draw the firm supply curve on the graph below:



b. Suppose there are 50 firms in the market with identical cost functions. What is the quantity supplied in the market as a function of P?

Chapter 9

Competitive Equilibrium

This chapter brings everything we built together; namely, the *demand* and *supply*. We consider a market for a particular good, and study the equilibrium of this market. As you know by now, a market is an infrastructure that facilitates the interaction of economic agents. In a simple consumption/production setup, it facilitates the trade of a certain group of buyers and sellers in a certain amount of time for a certain good.

We will consider a *perfectly competitive market* for consumers and producers. As I discussed previously, a **perfectly competitive market** is the one where no participant in the market has the ability to affect the price. Therefore, every consumer and producer is a **price-taker**.

9.1 Competitive Equilibrium

Let's recall what we had in Section 1.1.5: broadly speaking, equilibrium is **the situation in which every agent is optimizing, so nobody would benefit personally by changing her behavior, given the choices of other agents**.

Now we are considering the equilibrium of a perfectly competitive market, and **competitive equilibrium** is just the equilibrium of this market. I will just tailor the broad definition so that it applies to this market.

Definition 189.1 *The* **competitive equilibrium** *of a market is a price P^{eq} and quantity traded Q^{eq} such that:*

a. Given P^{eq}, each consumer in the market chooses the optimal quantity to demand.

b. *Given P^{eq}, each producer in the market chooses the profit-maximizing quantity to supply.*

c. *The total quantity demanded is equal to the total quantity supplied at Q^{eq}, i.e. the market clears.*

Let's go through it step by step:

a. When every consumer chooses the optimal quantity, the aggregate behavior of consumers in the market is represented by the *demand curve.*

b. When every producer chooses the profit-maximizing quantity, the aggregate behavior of producers in the market is represented by the *supply curve.*

c. Therefore, we must have a demand curve and a supply curve. Moreover, the consumers and the producer face the same price P^{eq} and, moreover, the quantity supplied is equal to the quantity demanded. Therefore, the competitive equilibrium is just the **intersection of demand and supply curves.**

All in all, we have the Figure 190.1 illustrating the competitive equilibrium. This is it. This is the infamous figure. This is the one that comes up when you Google "Econ 101".

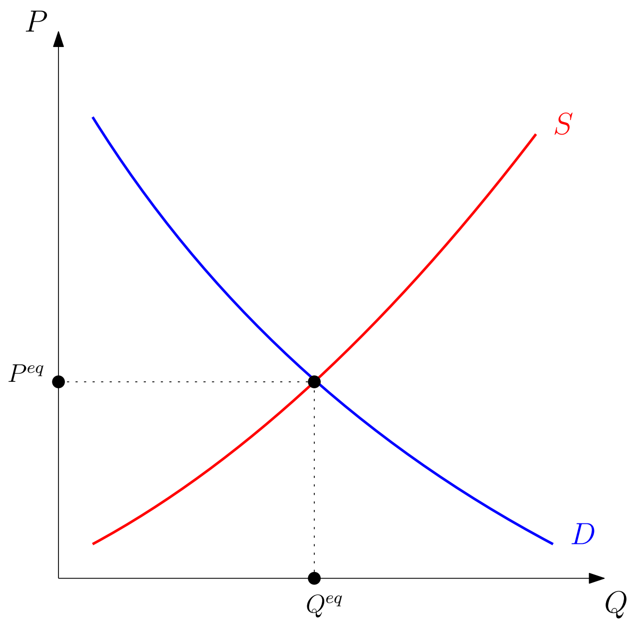


Figure 190.1: Competitive Equilibrium.

A couple of notes:

- This is sometimes also called the "free market" equilibrium.
- Some sources denote is with (P^*, Q^*). Some others denote it with (P^{CE}, Q^{CE}). We denote it with (P^{eq}, Q^{eq}). They all stand for the same thing.

9.1.1 The Single Price

One question you may ask at this point is: how come is there a single price for all the consumers and producers? After all, there are many consumers and producers in the market who are all trading with each other; how come we ensure that they all conduct the trade at the same price?

The very brief answer to that question is: ¯_(ツ)_/¯. We don't know, they somehow converge on a single price. This is the part of the model we do not really specify at the Econ 101 level.

- Maybe there is a huge message board that shows the price of each transaction (think of Wall Street).
- Maybe this is an online marketplace where people can search for other buyers and sellers easily, thereby forcing the sellers to post the same price (think of Hepsiburada).
- Maybe there are a few people who search for multiple sellers extensively and inform other consumers about the prices at different alternatives, thereby making sure the prices remain close to each other (think of my dad, or basically any retired relative of yours who spends all the Wednesday on finding the cheapest white cheese in the whole city).
- Presumably, buyers and sellers just communicate and exchange information with each other. In such cases, introduction of new communication technologies play a huge role in bringing prices closer.

I want to point out an important, possibly confusing, part of a competitive market model. Who sets the price in a competitive market? The answer is: "No one, because this is a competitive market. But also, at the same time, everyone, collectively." As you can see, we imagine market as an aggregate monster who somehow decides on a price as a result in gazillions if tiny communications, and everyone obeys that price. This is the "invisible hand" of Adam Smith at work, choosing how much trade there will be and between whom by deciding the price. As I said, this process of price discovery is beyond the scope of Econ 101.

9.1.2 Shortage and Surplus

Now that we convinces ourselves of a single price, the next question is: how come is the quantity demanded equal to the quantity supplied?

- Suppose quantity demanded is larger than quantity supplied. This happens when the market price is lower than P^{eq}, such as P_2 in Figure 192.1. In this case, a lot of consumers cannot find the good they are willing to consume at price P_2. But then, some consumers would find some producers and start offering them higher prices in the hope of getting the good. This will drive the market price up, increasing the quantity supplied and decreasing the quantity demanded. The process will end when they equalize each other.
- Suppose quantity supplied is larger than quantity demanded. This happens when the market price is higher than P^{eq}, such as P_1 in Figure 192.1. In this case, a lot of producers cannot find the consumers to sell the goods. But then, some producers would start offering lower prices to be able to sell the good. This will drive the market price down, decreasing the quantity supplied and increasing the quantity demanded. The process will end when they equalize each other.

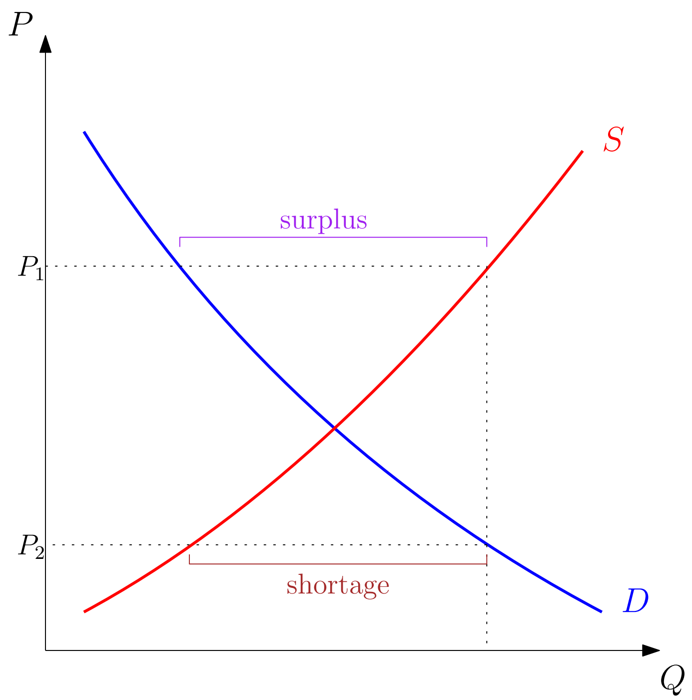


Figure 192.1: Shortage and Surplus

9.1.3 An Example

Consider a perfectly competitive market, where the market supply is given by

$$Q^S = 10P - 100 \tag{193.1}$$

This is the equation for **supply curve**. It says: if the price producers face is $P = 10$, they supply a quantity of $Q^S = 10 \cdot 10 - 100 = 0$. Beyond this price, for every unit increase in price, the quantity supplied increases by 10 units.

The market demand is given by

$$Q^D = 200 - 5P \tag{193.2}$$

This is the equation for **demand curve**. It says: if the price consumers face is $P = 0$, they demand a quantity of $Q^D = 200 - 5 \cdot 0 = 200$. For every unit increase in price, the quantity demanded increases by 5 units.

Let's start by finding the competitive equilibrium price and quantity. There are two methods for doing this, both are equally valid.

a. The straightforward method uses the following idea. We know that, in a competitive equilibrium, consumers and producers face the competitive equilibrium price, i.e., $P = P^{eq}$. Therefore, Equations (193.1) and (193.2) can be written as:

$$Q^S = 10P^{eq} - 100$$
$$Q^D = 200 - 5P^{eq}$$

We also know that markets clear in a competitive equilibrium. That is, there is no shortage or surplus, and quantity demanded equals quantity supplied. Therefore, $Q^S = Q^D = Q^{eq}$. Using the equations above, this implies:

$$10P^{eq} - 100 = 200 - 5P^{eq} \implies P^{eq} = 20$$

To find the equilibrium quantity, you can use the supply equation or demand equation – they will give the same answer by construction. Let's use the supply equation. Then,

$$Q^{eq} = 10P^{eq} - 100 = 10 \cdot 20 - 100 = 100$$

If we used the demand equation instead, we would have

$$Q^{eq} = 200 - 5P^{eq} = 200 - 5 \cdot 20 = 100$$

In any case, we find $P^{eq} = 20$ and $Q^{eq} = 100$.

b. An alternative way to find the competitive equilibrium price and quantity is to find Q^{eq} first and then P^{eq}. Needless to say, this will give the exact same answer, through a slightly longer path. To do this, we write the supply and demand equations in a way that looks like P is a function of Q, rather than the other way around.

   Take the equation for supply curve (193.1) and rearrange it to get:

$$P = 10 + \frac{Q^S}{10} \tag{194.1}$$

   This is the equation for what we call an **inverse supply curve**. ("Inverse" because it now looks like P is a function of Q^S. Of course, we know that nothing is a function of another: both the supply curve and the inverse supply curve are just a list of (P, Q^S) that constitute the optimal decisions of producers.) The inverse supply curve says: to produce a quantity $Q^S = 0$, the producers ask for a price of $P = 10$. For every extra unit of quantity supplied, the producers ask for $\frac{1}{10}$ extra units of price. This is because the marginal cost of the marginal producer increases linearly in Q^S with a slope of $\frac{1}{10}$.

   Similarly, take the equation for demand curve (193.2) and rearrange to get:

$$P = 40 - \frac{Q^D}{5} \tag{194.2}$$

   This is the equation for an *inverse demand curve*. It says that the marginal benefit of the consumers for Q^D'th unit is $40 - Q^D/5$.

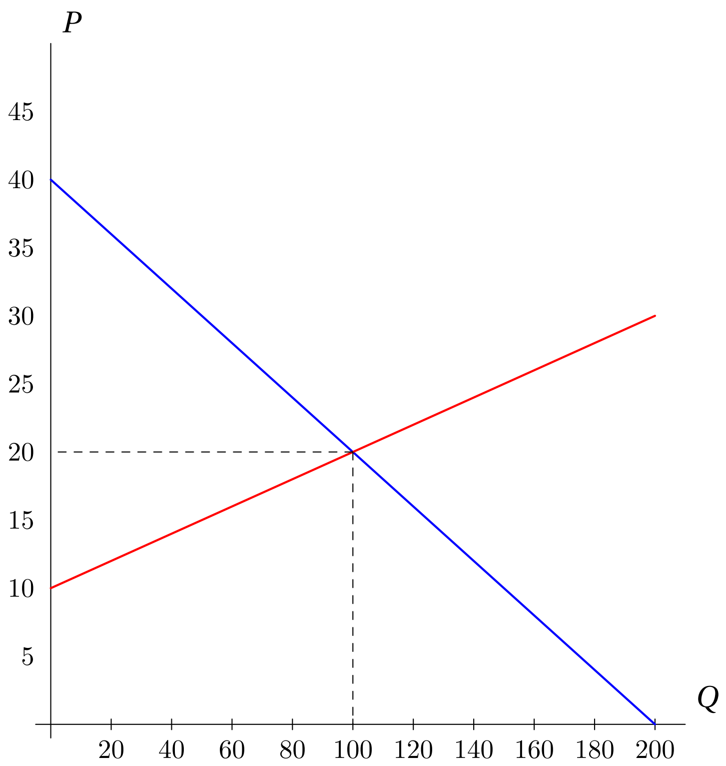
   Now, using $Q^S = Q^D = Q^{eq}$ and $P = P^{eq}$ and substituting into (194.1) and (194.2) yields:

$$10 + \frac{Q^{eq}}{10} = 40 - \frac{Q^{eq}}{5} \implies Q^{eq} = 100$$

   and

$$P^{eq} = 10 + \frac{Q^{eq}}{10} = 20$$

Now, let's draw the graph corresponding to the competitive equilibrium by drawing the curves. (And this is the reason why deriving the equations for *inverse supply* and *inverse demand* curves may be a good idea, because the curves are easier to draw when you have those equations.) See the figure below.



9.2 Structural Changes in the Economy

Here comes the fun part: what happens to the equilibrium price and quantity if there are some structural changes in the economy (so that demand or supply curves shift)? We can conduct a graphical analysis of these cases. I will show a few examples here, but the general insight is that you can "mix-and-match": you have shifted demand and supply curves separately, now the only thing to be added is showing them on the same graph.

9.2.1 Case Study: Fracking

Natural gas extraction is a challenging and difficult process. In mid-to-late 2000s, the industry has perfected a method called hydraulic fracturing ("fracking") to extract natural gas and crude oil from resources. According to BBC,[1] "Fracking is the process of drilling down into the earth before a high-pressure water mixture is directed at the rock to release the gas inside. Water, sand and chemicals are injected into the rock at high pressure which allows the gas to flow out to the head of the well." It is still

[1] https://www.bbc.com/news/uk-14432401

a controversial technology because its environmental impacts are yet to be figured out.[2] Still, the use of fracking in the United States has increased tremendously since 2007.

The effect of fracking on the price and quantity of natural gas can be illustrated on a graph. Fracking is a “technological breakthrough”, which leads to a southeastern shift on the supply curve. Presumably, it does not have any significant effects on the demand. Then, the new intersection of supply and demand curves will be to the southeast of the old intersection. That is, the price of natural gas will decrease and the quantity traded will increase. See Figure 196.1: the supply curve shifts from S to S'. As a consequence, the equilibrium price reduces to $(P^{eq})' < P^{eq}$ and the equilibrium quantity increases to $(Q^{eq})' > Q^{eq}$.

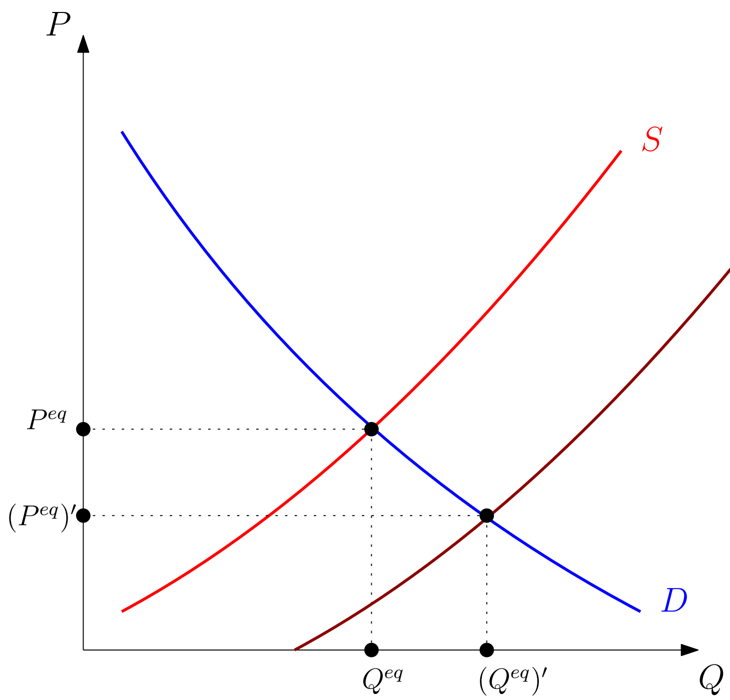


Figure 196.1: Effects of fracking on the market for natural gas.

Does this argument has merit? Looks so. Figure 197.1 plots the natural gas prices (in dollars) since 1997. Note the stable trend of declining prices after 2007 (apart from the surge in 2022 – do you recall what happened?).

Moreover, Figure 197.2 plots the production of natural gas in the United States (which proxies for quantity traded). Note the uptick since 2007!

[2] https://www.nationalgeographic.com/environment/article/how-has-fracking-changed-our-future

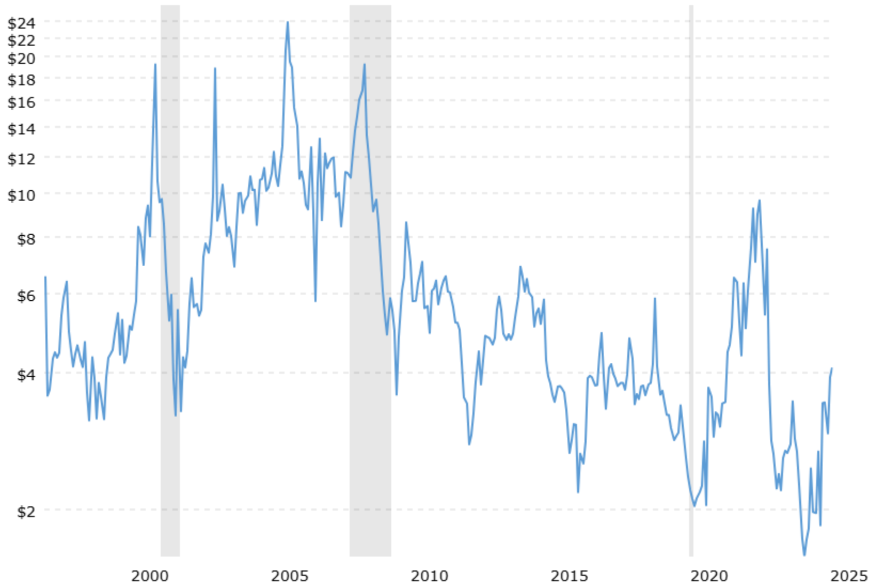


Figure 197.1: Natural Gas Prices. Source: `https://www.macrotrends.net/2478/natural-gas-prices-historical-chart`.

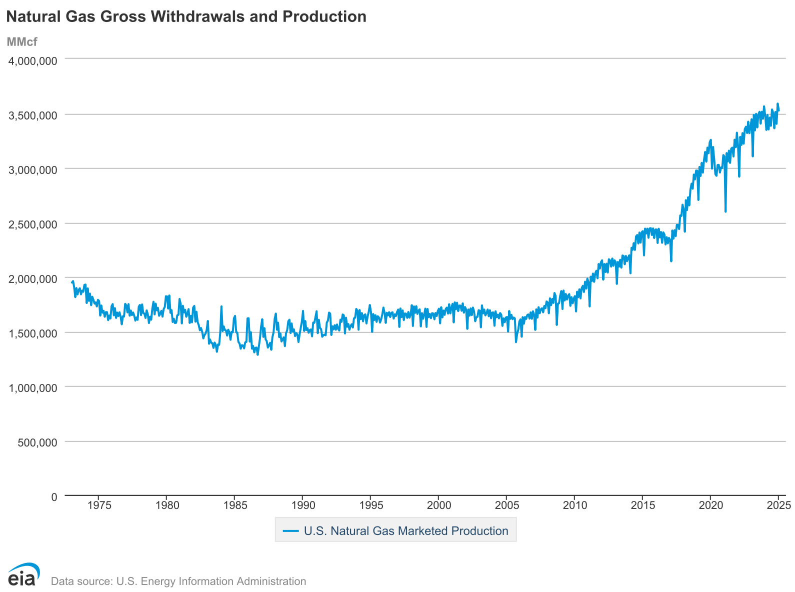


Figure 197.2: Natural Gas Production. Source: `https://www.eia.gov/`.

9.2.2 Case Study: Ridesharing

Uber is a ridesharing platform which brings people looking for a ride (consumers of a ride) and people who are willing to provide a ride (producers of a ride) together. For all purposes of Econ 101, this is a market for ridesharing. It is a brilliant example of a modern marketplace, and one that gets economists very excited (it is very unfortunate that it doesn't exist in Turkey in the manner it exists elsewhere).

One interesting bit about Uber is that, unlike many competitive markets, the price-setting mechanism is pretty transparent. Basically, at any point in a particular location, Uber sets the price of a ride between the consumer and the producer. Uber's objective while setting the price is ensuring that the market clears. This suggests the existence of an interesting laboratory where we can test the principles of Econ 101.

Suppose, for instance, that there is a sudden increase in the number of consumers in a certain location. This may happen, for instance, in national holidays, when a match or concert ends, around 5pm on a workday, etc. We can represent this with a north-eastern shift in the demand curve. See Figure 198.1: the demand curve shifts from D to D'. As a consequence, the equilibrium price increases to $(P^{eq})' > P^{eq}$ and the equilibrium quantity increases to $(Q^{eq})' > Q^{eq}$. In practice, to match the increase in quantity demanded, Uber increases the price of a ride. This attracts more drivers to the market, thereby increasing the quantity supplied and preventing shortage. The procedure of charging high prices during times of high demand is called "surge pricing."

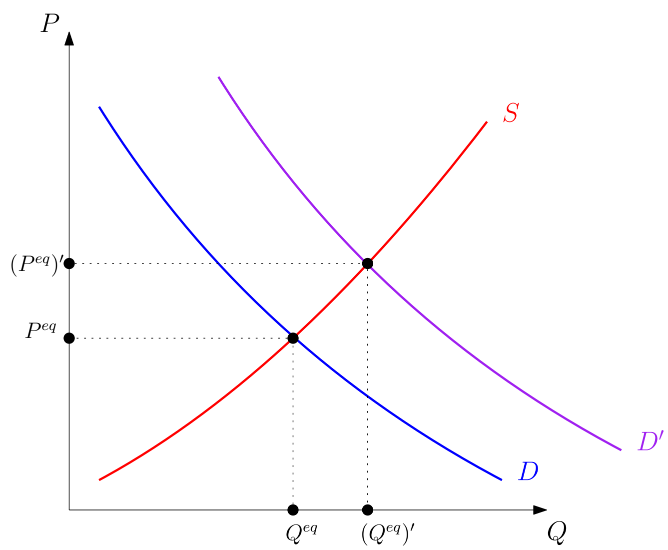


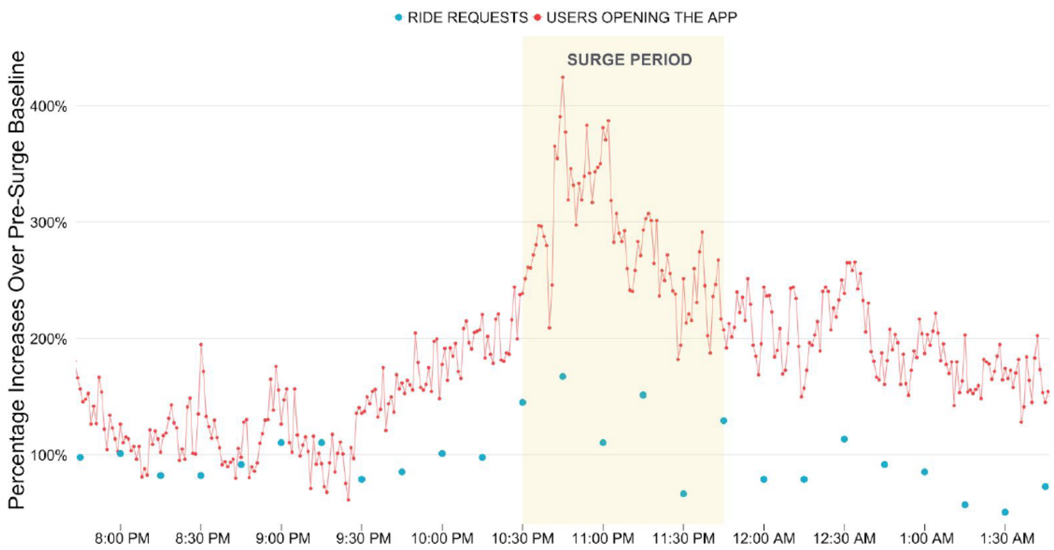
Figure 198.1: Effect of an increase in the number of consumers on the market for ridesharing.

Can we see these ideas in action? Of course. The following examples are taken from a discussion paper:

> Hall, Jonathan, Cory Kendrick and Chris Nosko. "The Effects of Uber's Surge Pricing: A Case Study".

Hall, Kendrick, and Nosko consider a particular example of a sudden shift in demand: the end of a sold-out Ariana Grande[3] concert in Madison Square Garden on March 21, 2015. As you can imagine, by the end of the concert (roughly 10:30pm), there was a huge spike in the ridesharing requests. See Figure 199.1.

Figure 1: Demand for Uber Spikes Following Sold-Out Concert on March 21, 2015



Note: Figure reports the number of users opening the Uber app each minute over the course of March 21, 2015 (in red), as well as the sum of total requests for Uber rides in 15-minute intervals over the same time period (blue circles). Data is for a restricted geospatial bounding box containing Madison Square Garden in New York City, roughly 5 avenues long and 15 streets wide, for uberX vehicles only. Pure volume counts have been normalized to a pre-surge baseline, defined as the average of values between 9:00 and 9:30 PM that evening, before surge turned on. "Surge period" (yellow box) is the time over which the surge multiplier increased beyond 1.0x.

Figure 199.1: The increase in the number of consumers.

To match the increase in demand, Uber started surge pricing: the time where surge pricing applies is illustrated with the yellow region in Figure 199.1 (between 10:30pm and 11:45pm). You can imagine that high prices will attract more drivers, and thus the quantity supplied will increase. This is illustrated in Figure 200.1.

The increase in supply matches the increase in demand in equilibrium, so that the completion rate remains at 100% even though ridesharing requests almost double.

[3]QWEEN.

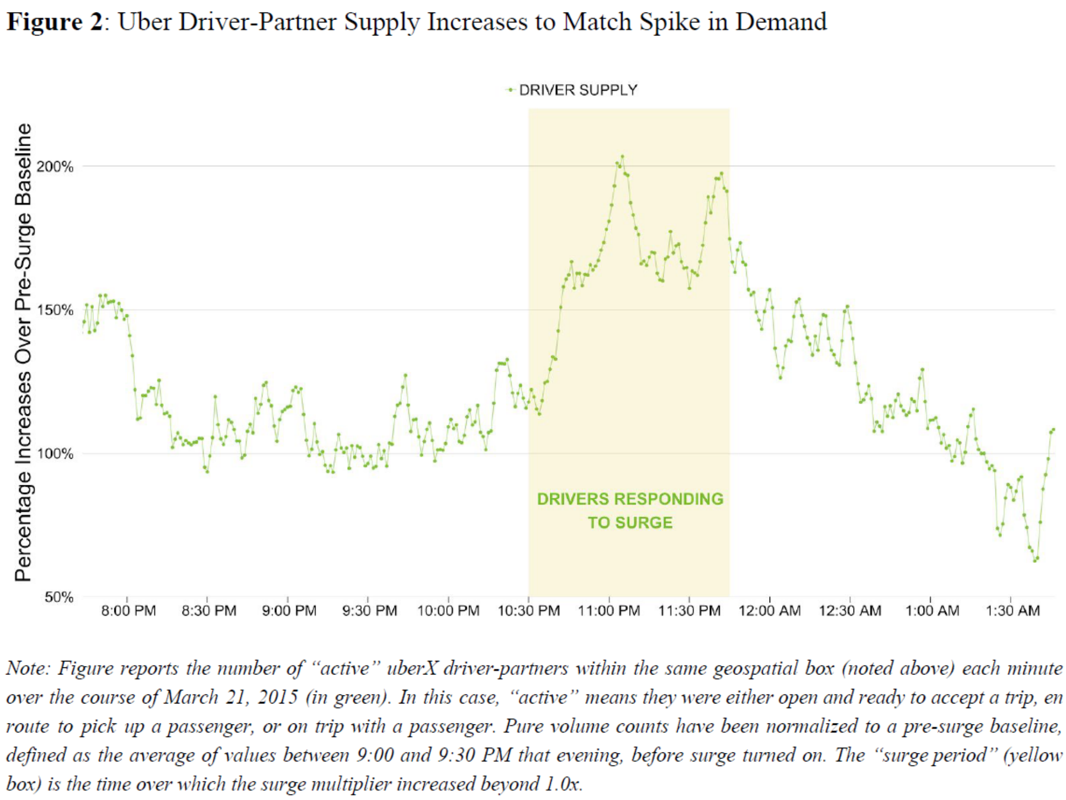


Figure 200.1: The effect of higher price on the number of producers.

In other words, market clears as a result of surge pricing. See Figure 201.1.

Shortage in a Ridesharing Market

What happens if Uber fails to set the market-clearing price? This happens, for instance, when the Uber algorithm "forgets" about the increase in the number of consumers and keeps charging the usual price for a ride. The end result will be a **shortage** as we discussed before. See Figure 201.2. In this figure, the demand shifts from D to D'. When Uber keeps charging the old price P, the quantity demanded Q^D is larger than quantity supplied Q^S, which results in a shortage. As a consequence, many riders will not be able to find a ride (which, take my word for it, is a very frustrating experience).

We actually have an example for such shortage! During New Year's Eve in 2015' Uber's pricing algorithm had a glitch, which resulted in a "surge outage" for 26 minutes. That is, Uber kept charging the "usual price" to the consumers and producers instead of the higher price. See Figure 202.1.

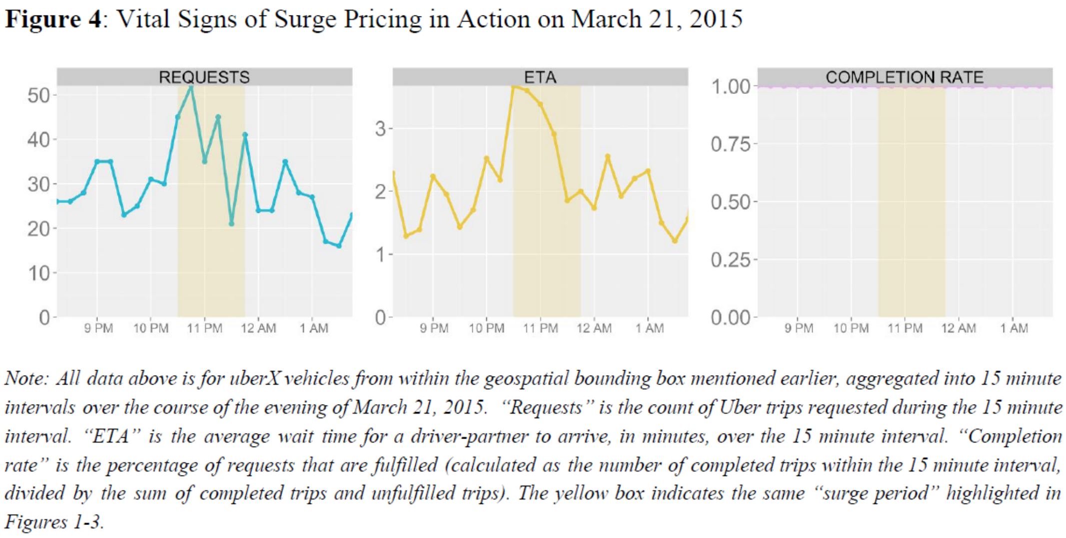


Figure 201.1: Market clearing.

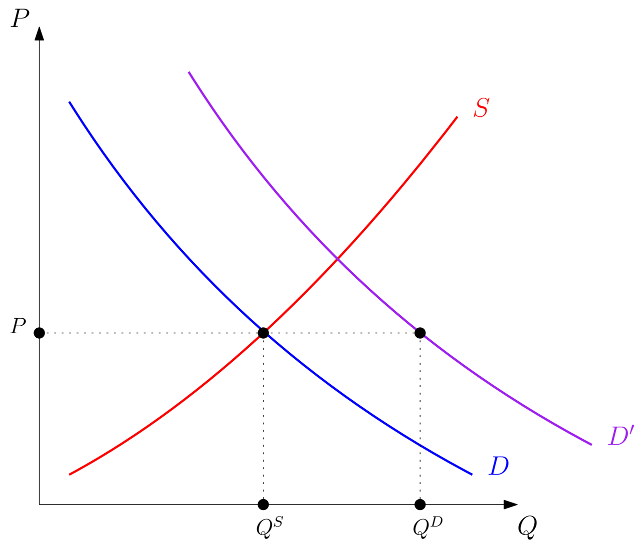


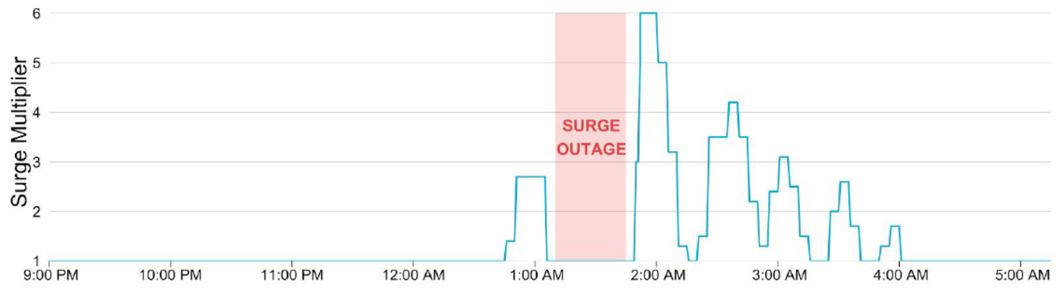
Figure 201.2: Shortage in a ridesharing market.

The resulting shortage due to this glitch led to a dramatic decrease in the completion rate, which went below 20% during the surge outage. This is illustrated in Figure 202.2.

9.2.3 Application: Market for Corn

You may actually go one step further and think about structural changes that affect both demand and supply curves. Consider, for instance, a city that primarily pro-

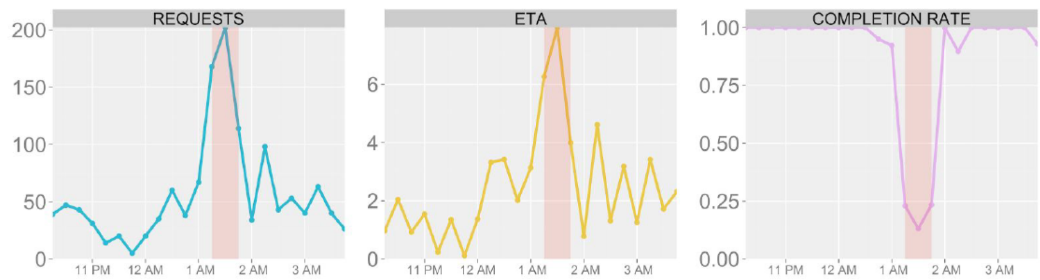
Figure 5: Twenty Minutes Without Surge on New Year's Eve (January 1, 2015)



Note: Figure reports the surge multiplier for a given minute over the course of New Year's Eve, December 31, 2014 to January 1, 2015, for uberX vehicles within the geospatial bounding box noted earlier (blue line). "Surge outage" (red box) is the time period during which Uber's surge pricing algorithm broke down due to a technical glitch, from 1:24am to 1:50am EST.

Figure 202.1: The surge outage.

Figure 7: Vital Signs of a Surge Pricing Disruption on New Year's Eve (January 1, 2015)



Note: All data above is for uberX vehicles from within the geospatial bounding box mentioned earlier, aggregated into 15 minute intervals over the course of New Year's Eve, December 31, 2014 to January 1, 2015. "Requests" is the count of Uber trips requested during the 15 minute interval. "ETA" is the average wait time for a driver-partner to arrive, in minutes, over the 15 minute interval. "Completion rate" is the percentage of requests that are fulfilled (calculated as the number of completed trips within the 15 minute interval, divided by the sum of completed trips and unfulfilled trips). The red box indicates the same "surge outage" highlighted in Figure 6.

Figure 202.2: Shortage as a result of the surge outage.

duces and consumes corn. Suppose, due to some policy that mandates the increase in minimum wage, the wages in the city increases. This will affect both curves.

- On the demand side, the demand curve will shift because consumers' income increases. If corn is a normal good, this will lead to a northeastern shift in the demand curve.
- On the supply side, the cost of a vital input in the corn production (labor) in-

creases. This will lead to a northwestern shift in the supply curve.

What will be the end result? We will definitely have an increase in the equilibrium price. The equilibrium quantity may increase (as in Figure 203.1) or decrease (as in Figure 203.2), depending on which effect is stronger.

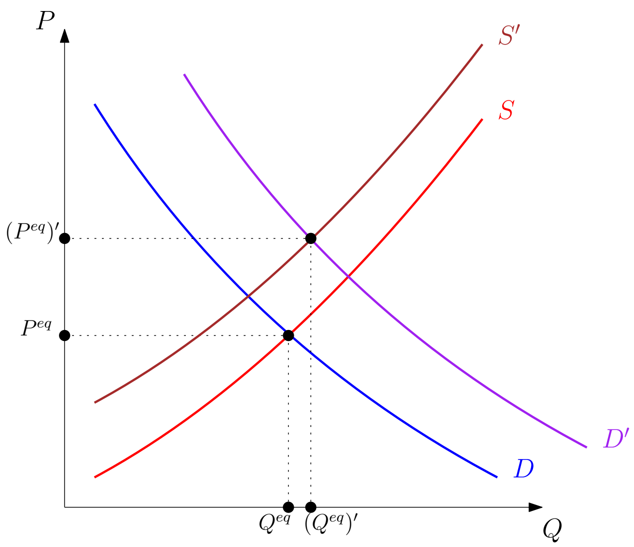


Figure 203.1: An increase in quantity traded in the market for corn.

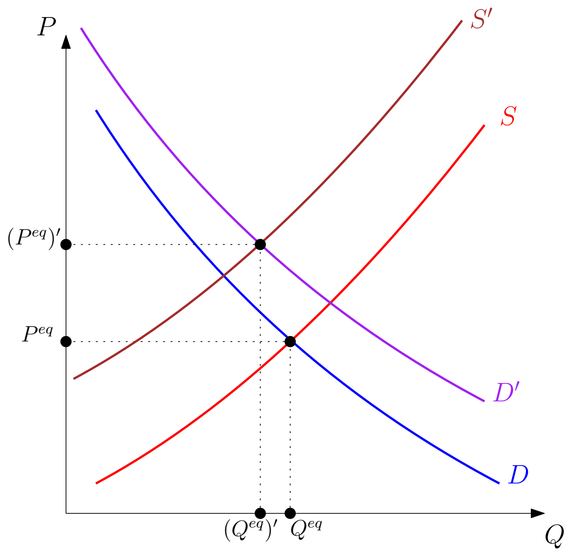


Figure 203.2: A decrease in quantity traded in the market for corn.

9.2.4 Taking Stock

Overall, you may have realized the value of using the geographical notation to keep track of shifts. To make sure we are on the same page:

- If only one of the curves shift, the equilibrium will shift in the same direction.
  - If demand curve shifts northeast, the equilibrium also shifts northeast. That is, the equilibrium price will increase and quantity traded will increase.
  - If demand curve shifts southwest, the equilibrium also shifts southwest. That is, the equilibrium price will decrease and quantity traded will decrease.
  - If supply curve shifts southeast, the equilibrium also shifts southeast. That is, the equilibrium price will decrease and quantity traded will increase.
  - If supply curve shifts northwest, the equilibrium also shifts northwest. That is, the equilibrium price will increase and quantity traded will decrease.
- If both curves shift, we “add up” the effects.
  - If demand curve shifts northeast and supply curve shifts southeast, the equilibrium also shifts east, and it may shift north or south. That is, the quantity traded will increase and equilibrium price may increase or decrease.
  - If demand curve shifts northeast and supply curve shifts northwest, the equilibrium also shifts north, and it may shift east or west. That is, the equilibrium price will increase and quantity traded may increase or decrease.
  - ...

Extra Readings for Chapter 9

When discussing the assumption of a single price, I mentioned that "introduction of new communication technologies play a huge role in bringing prices closer". To see this idea in action, you can check:

> Jensen, Robert. "The Digital Provide: Information (Technology), Market Performance, and Welfare in the South Indian Fisheries Sector" *Quarterly Journal of Economics* 122. 3 (2007), pp. 879-924.

This paper investigates the introduction of cell phones to some fisheries in Southern India. Figure IV in that paper is one of the most remarkable figures in economics.

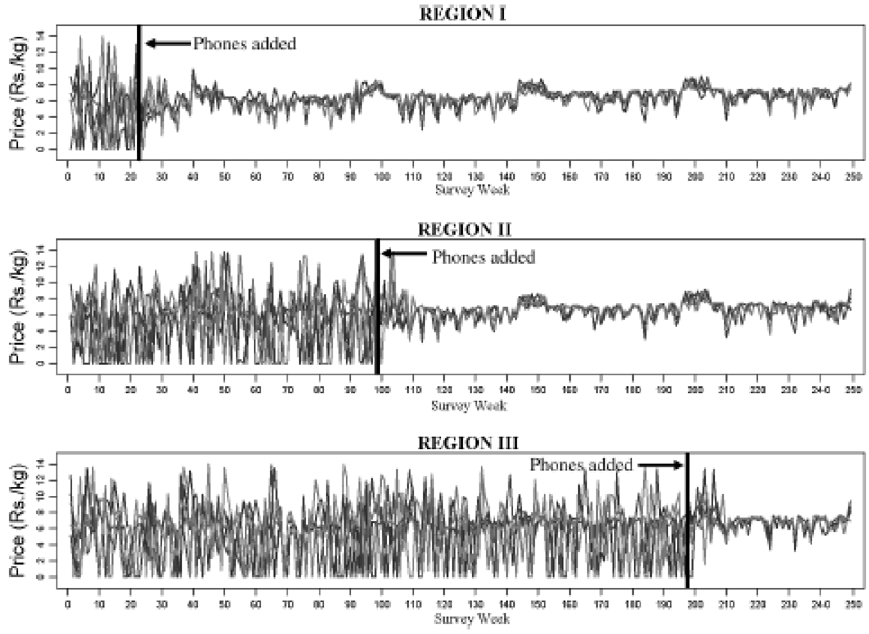


FIGURE IV
Prices and Mobile Phone Service in Kerala
Data from the Kerala Fisherman Survey conducted by the author. The price series represent the average 7:30–8:00 A.M. beach price for average sardines. All prices in 2001 Rs.

Figure 205.1: Figure IV of Jensen (2007).

There are variety of approaches on modeling the price formation process in a market. A line of research considers a market as a place where a seller meets with multiple buyers over time and runs small *auctions* every time. For an example of this approach, see:

Satterthwaite, Mark, and Artyom Shneyerov. "Dynamic Matching, Two-sided Incomplete Information, and Participation Costs: Existence and Convergence to Perfect Competition." *Econometrica* 75, no. 1 (2007): 155-200.

Another line of research considers a market as a place where buyers and sellers randomly encounter each other and *bargain* over the price. For and example of this approach, see:

Osborne, Martin J., and Ariel Rubinstein. *Bargaining and Markets.* Academic Press Limited, 1990.

and

Gale, Douglas. *Strategic Foundations of General Equilibrium: Dynamic Matching and Bargaining Games.* Cambridge University Press, 2000.

Exercises for Chapter 9

1) Assume you are analyzing the orange market, where the competitive equilibrium has already been achieved. Discuss the effects of the following on the equilibrium price and quantity:

   a. There is a frost in the Mediterranean region where the oranges grow. The frost hits the orange trees really hard.

   b. There is an influenza epidemic and the newspapers report the tremendous curing effect of orange juice.

   c. The price of tangerines, a substitute to oranges in consumption, decreases.

   d. The price of fertilizers used on orange trees falls.

   e. The technology of picking oranges from trees improves. At the same time tourists from all over the world travel to Turkey, significantly increasing the population during this period.

Chapter 10

Why Competitive Equilibrium is (Sometimes) Good

In this chapter, we will revisit some territories we usually hesitate to visit: we will get *normative*. That is, we will go a little beyond the question on *what happens* in an economy, and we will start asking *what should happen*. To this end, we will occasionally make some *value judgments*. In this sense, some of the ideas we explore will remind you of Chapter 3 (in particular, the First Welfare Theorem). However, we will go a few steps beyond discussing why competitive equilibrium allocation is "desirable". We will also discuss how various government interventions distort the outcome, and take us away from the desirable outcomes.

The overarching conclusion of this chapter is that: sometimes, competitive markets are a decent way to organize economic activities. I want to warn you in advance: this is the chapter that a lot of people mentally get stuck in. Once you learn the concepts we are about to cover, some of you will have an urge to say: "Markets are amazing! Why intervene in them at all?" I would kindly ask you to resist that urge as much as possible. In order to do that, please remember how we ended up here: we basically made a bunch of assumptions (we assumed consumers are rational, they know the prices, their preferences satisfy diminishing marginal rate of substitution, the goods in question are ordinary goods, producers are not cash constrained, their productions function satisfy the law of diminishing returns, everyone is a price taker...) What you should remember here is: *under these assumptions*, competitive markets are a decent way to organize economic activity. This is why the title for this chapter has the word "(Sometimes)" in it.

Beginning with next chapter, we will start checking out what happens when we change some of these assumptions. For instance, we will study what happens when not everyone is a price taker, or when one economic agent's economic activity affects others. But for now, let's stick with these assumptions.

10.1 Gains from Trade

As it happens, in order to make value judgments, we need to have some measure of *value*. Let's move there with a quick mental exercise.

Suppose that you are a **social planner** who controls a market for a certain good or service. That is, there is a certain good or service, and there is a particular set of producers and a set of consumers you control. Moreover, suppose that you are extremely powerful: you have the power to go any producer, make them produce the good by any amount, take these goods and give them to any consumer by any amount. (At this point, you are not even bound by the price mechanism: you can extort the producers into producing the goods and you can force consumers into consuming them.) The question you face is: **what is the quantity you will require to be produced and consumed? Who will produce and who will consume?**

To answer this question, we need to specify what you, the social planner, care about. And this is precisely the part we get normative. In particular, you may want to create the best outcome for the consumers, the best outcome for the producers, or a particular group of consumers etc... This is the question of what you *value* the most, and it is an ethical question: it does not have any right answer. Still, we will extend our boundaries a little and assume the following: **the social planner wants to maximize the gains from trade**.

"But what is the gains from trade?", you might say. Basically, whenever the social planner makes the producer produce a good and a consumer consumes it, she is creating some *gains* in the economy. A measure of these gains is the difference between how happy it makes the consumers *and* how costly it is for the producers. The good news is: we already have a name for these objects! In particular, when Q'-th unit of the good is produced by the marginal producer and is consumed by the marginal consumer:

- $\mathrm{MB}(Q')$, the marginal benefit of the marginal consumer, is how much the marginal consumer values the consumption of the Q'-th unit (in monetary terms).
- $\mathrm{MC}(Q')$, the marginal cost of the marginal producer, is how costly it is for the marginal producer to produce the Q'-th unit (in monetary terms).

All in all, the gains from producing and consuming the Q'-th unit is:

$$\mathrm{MB}(Q') - \mathrm{MC}(Q')$$

Note: this can even be negative. That would mean that producing and consuming the Q'-th unit is not socially desirable (according to what the social planner cares about, i.e., when the objective is maximizing gains from trade).

Note: by now this should give an idea on why we defined the "dual interpretations" of the market demand and market supply curves. For a quantity Q', the market demand curve illustrates $\mathrm{MB}(Q')$ and the market supply curve illustrates $\mathrm{MC}(Q')$. Therefore, the gains from producing and consuming the Q'-th unit is the vertical difference between the demand and supply curves. See Figures 211.1 and 211.2 below.

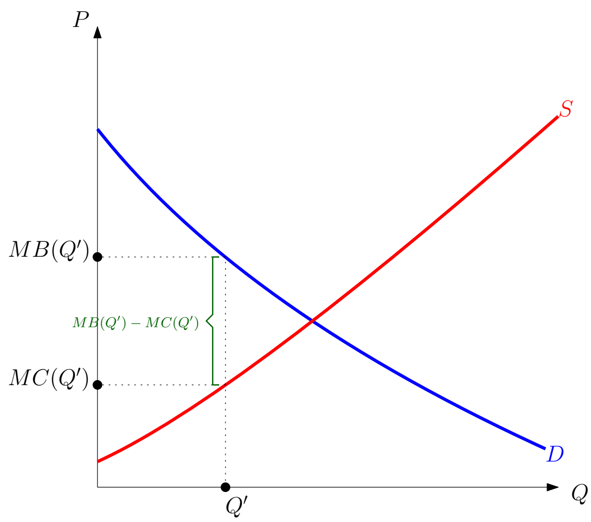


Figure 211.1: Gains from trading Q'-th unit, where $MB(Q') - MC(Q') > 0$.

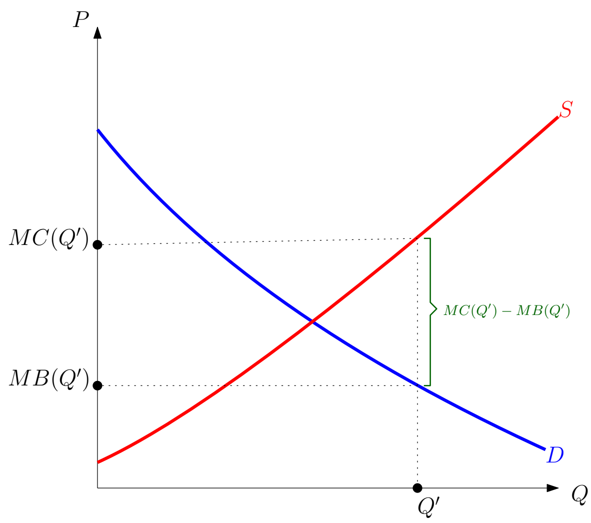
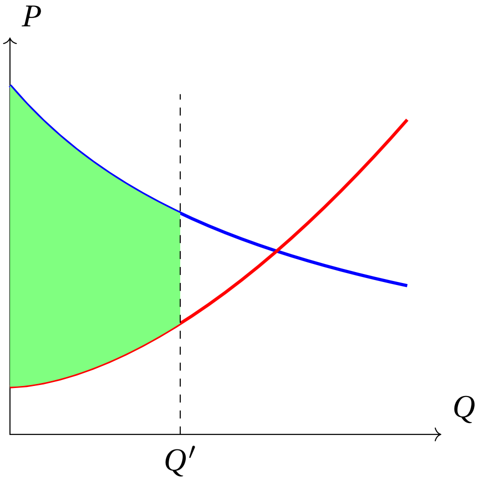


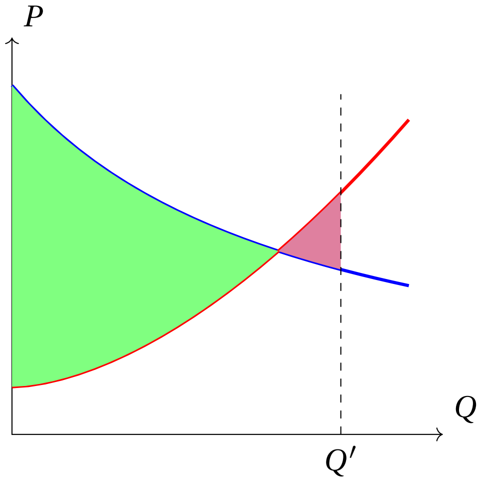
Figure 211.2: Gains from trading Q'-th unit, where $MB(Q') - MC(Q') < 0$.

The total gains from trading Q' units is called the **economic surplus** from producing and consuming Q' units. This is simply the sum of all the gains from trading all Q-th units, where Q varies between 0 and Q'.

From now on, we will assume that the social planner always finds the producer who produces the marginal unit at the lowest marginal cost, and the consumer who values the marginal unit most (i.e., who has the highest marginal benefit).[1] Then, the **economic surplus** from trading Q' units is the area below the market demand and above the market supply from $Q = 0$ to $Q = Q'$ (area of regions in which supply is above demand, i.e., marginal cost is higher than marginal benefit, is taken as negative).



Economic surplus from producing Q' units and consuming it is given by the green area.



Economic surplus from producing Q' units and consuming it is given by the green area minus the purple area.

10.1.1 The Efficient Quantity

Okay, the social planner just wants to maximize the economic surplus by choosing the quantity Q. So, what will be the quantity chosen by the social planner? Put simply, the social planner will require the producers to produce and require the consumer to consume the quantities as long as $\text{MB}(Q) \geqslant \text{MC}(Q)$. When $\text{MB}(Q)$ is decreasing (i.e., when the law of demand is satisfied) and $\text{MC}(Q)$ is increasing (i.e., when the law of supply is satisfied), this process stops at the point where there are no more gains from trade to exhaust. This quantity is called the **efficient quantity**, and is denoted Q^*. This is the value of Q^* that satisfies:

$$\text{MB}(Q^*) = \text{MC}(Q^*)$$

[1] It is kind of obvious that the social planner always targets these people, but we will not get into the details here.

See Figure 213.1 below.

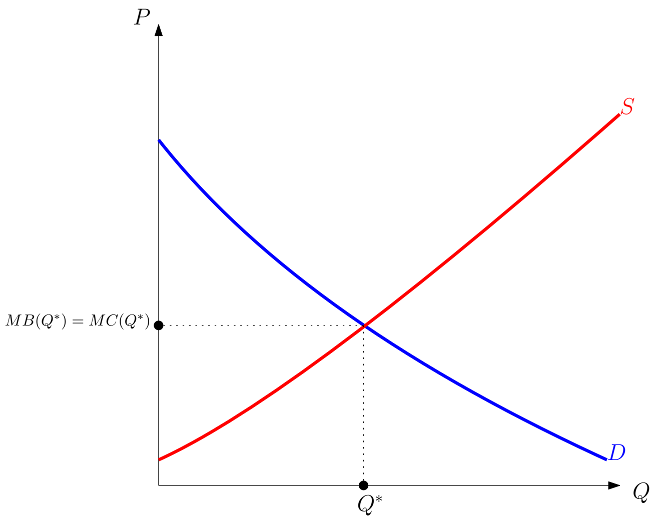


Figure 213.1: The total gains from trade is maximized at the quantity Q^*.

The gains from trade under quantity Q^* is the maximum value of **economic surplus**. See Figure 214.1 below.

All in all, the social planner maximizes the economic surplus by:

a. Calculating Q^*,

b. Finding the producers with the lowest marginal cost, making them produce Q^* units of good,

c. Finding the consumers with the highest marginal benefit, making them consume Q^* units.

10.1.2 What Can Go Wrong?

This looks simple enough in theory, but in practice, can a social planner do that easily? I would argue not. I can think of three potential, very serious, problems.

a. The social planner may not be able to find the producers with the lowest marginal cost.

b. The social planner may not be able to find the consumers with the highest marginal benefit.

c. Perhaps most importantly, the social planner may not be able to calculate Q^* correctly. After all, market demand curves and market supply curves are some

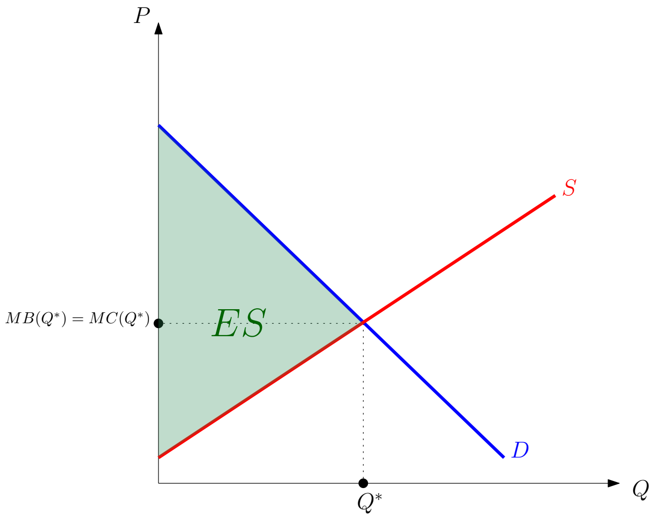


Figure 214.1: The total gains from trade at quantity Q^* is the economic surplus (ES), illustrated with the green area.

analytical devices we designed to better understand economic interactions. There is no such thing as a market demand curve or a market supply curve out there in the wild. If I asked someone to draw the market demand curve for milk in Turkey for 2024, would they be able to draw it? I suspect not.

So, what will happen is the social planner is unable to set Q^* correctly, i.e. what if she sets $Q' \neq Q^*$? We will have a lower economic surplus than what is achievable.

- If the social planner sets $Q' < Q^*$, then the society has *too little trade*. This results in some gains from trade not being realized, and the society losing a small triangle. The economic surplus in this case will the the maximum economic surplus possible minus a triangle.
- If the social planner sets $Q' > Q^*$, then the society has *too much trade*. This results in some units being produced even though the cost of producing them is higher than the gains. We end up with a small triangle of negative surplus. The economic surplus in this case will be the maximum economic surplus possible minus a triangle (because the triangle enters to the equation as negative).

In any case, we have a triangle that the society is "losing". The area of the triangle is the **difference between the maximum economic surplus and the economic surplus from trading Q' units**. We call this difference **deadweight loss**, or DWL for short. See Figures 215.1 and 215.2.

So, to recap: we have a definition of economic surplus (or gains from trade) we want to maximize. We know the solution that maximizes the economic surplus, but guar-

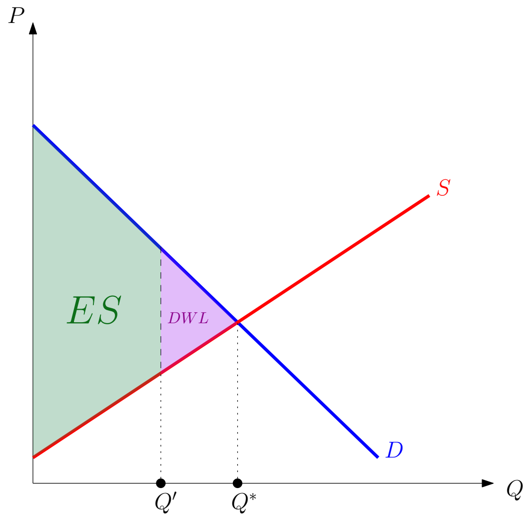


Figure 215.1: Consequences of setting $Q' < Q^*$.

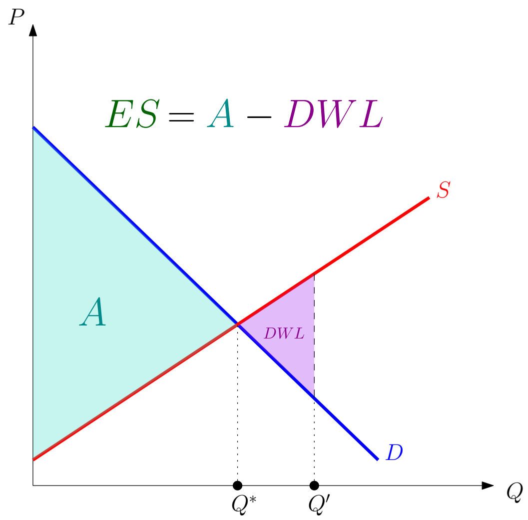


Figure 215.2: Consequences of setting $Q' > Q^*$.

anteeing that we find this solution is indeed very difficult. What can we do? If only there was a mechanism that guarantees maximum economic surplus... Wait, we do!

10.2 Competitive Equilibrium Maximizes Economic Surplus

This may have occurred to you by now, but let's spill the beans. **Competitive equilibrium of a market maximizes the economic surplus.** It does so by making sure that the market demand and market supply curves intersect. Then,

$$P^{eq} = \mathrm{MB}(Q^{eq}) = \mathrm{MC}(Q^{eq})$$

Moreover, any consumer who consumes the Q'-th unit at the competitive equailibrium price has $\mathrm{MB}(Q') \geqslant P^{eq}$ and any producer who produces that unit has $\mathrm{MC}(Q') \leqslant P^{eq}$. That is, the competitive equilibrium automatically finds the consumers with highest value and producers with lowest cost through the price mechanism.

Isn't this fantastic? Basically, you can just lean on your back and let the market "work its magic" by finding the efficient quantity, the highest-value consumers and the lowest-cost producers. The *invisible hand* of the market acts as a social planner who maximizes the economic surplus! **Competitive equilibrium is "good".**

This is a cute finding, but as I told you before: please do not fall in love with markets just because of this. There are many, many assumptions that go behind this result. If some of these assumptions are violated, we may not have the competitive market achieving the maximum economic surplus. This is why **competitive equilibrium is "sometimes" good.**

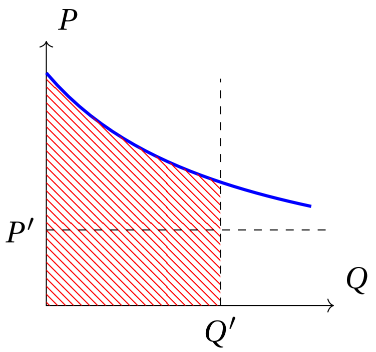
10.2.1 Consumer Surplus and Producer Surplus

So, competitive equilibrium results in maximum economic surplus. There are some total gains from trading in a competitive equilibrium. But what part of those gains goes to consumers, and what part goes to the producers? This is important to know because soon, we will analyze some government policies and investigate who are the winners and who are the losers from those policies (i.e., whose surplus becomes larger and whose becomes smaller). To calculate the fraction of total surplus that goes to either side, we decompose it into the **consumer surplus** and **producer surplus**.

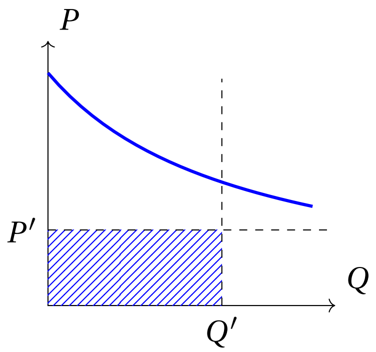
Consumer Surplus

Consumer surplus, denoted CS, is a measure of a consumers' total gains from trade in a market. It is the amount by which the total amount the consumer is willing to pay exceeds what she actually pays.

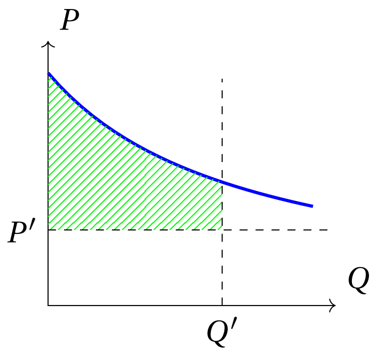
The **consumer surplus** from consuming Q' units of a good at the price of P' is the difference between the highest payment that the consumers are willing to pay to consume Q' units (the benefit of the consumers from consuming Q' units) and the amount they have to pay ($P' \cdot Q'$).



The shaded area gives the maximum amount the consumers are willing to pay in order to consume Q' units of the good.



The shaded area gives the amount that the consumers have to pay in order to consume Q' units of the good, when the price of the good is P'.



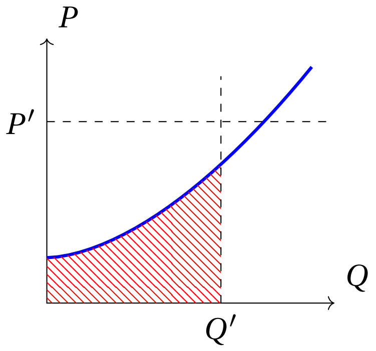
The shaded area gives the difference between the maximum amount that the consumers are willing to pay and what they have to pay in order to consume Q' units of the good at the price of P'. This is the consumer surplus from consuming Q' units which the consumer purchased at the price P'.

Given a market demand, **consumer surplus** from consuming Q' units at the price of P' is a measure of the gain of the consumers from consuming Q' units at the price P'. The area between the market demand (the marginal benefit) and the price line $P = P'$ from $Q = 0$ to $Q = Q'$ gives the consumer surplus.

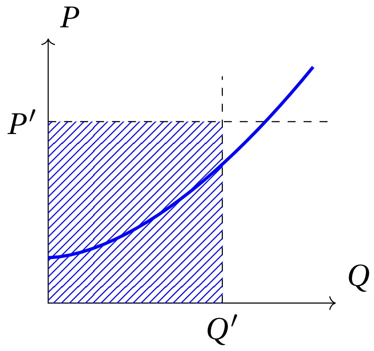
Producer Surplus

Producer surplus, denoted PS, is a measure of a producers' total gains from trade. It is the amount by which the total payment the producers receive exceeds the minimum amount that the producers would require to produce the good.

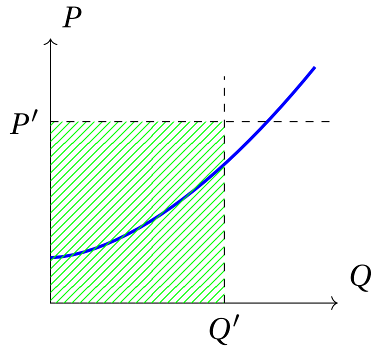
The **producer surplus** from producing (and selling) Q' units of a good at the price of P' is the difference between the payment the producers receive from selling Q' units of the good at the price of P' ($P' \cdot Q'$) and the minimum payment the producers would require to produce Q' units of the good.



The shaded area gives the minimum payment the producer would require in order to produce Q' units of the good.



The shaded area gives the amount that the producer receives when it sell Q' units of the good at the price of P'.



The shaded area gives the difference between the minimum payment that the producer requires and what it is payed when it sells Q' units of the good at the price of P'. This is the economic surplus from producing Q' units and selling it at the price of P'.

Given a market supply, **producer surplus** from producing Q' units and selling at the price P' is a measure of the gain of the producers from this action. The area between supply (market marginal cost) and the price line $P = P'$ from $Q = 0$ to $Q = Q'$ gives producer surplus.

Total Surplus

In a market, the sum of CS and PS gives the economic surplus ES. This is also sometimes referred to as the **total surplus**, denoted TS. In a competitive equilibrium without any external interventions (e.g., in a **free market**), the total surplus is equal to the maximum economic surplus. That is, there are no deadweight losses in a free-market competitive equilibrium. In a moment, we will see that government interventions create deviations from the free-market competitive equilibrium and generate deadweight losses.

10.2.2 The "Meaning" of Prices

While we are on it, let me take a slight detour and get a little philosophical. What is the "meaning" of a price in competitive equilibrium? As in, if we know the price of a good being traded in a market, what does the price say about that good?

Our answer to this question, based on our analysis so far, is: "Prices reflect very little information about the good, if any." All in all, in a competitive market, the price is used for making sure that there is no shortage or surplus. **In a competitive market, the price is nothing but a coordination mechanism.**[2] The consumers see the prices and decide on quantity demanded. The producer see the prices and decide on quantity supplied. The price adjusts such that the quantity demanded is equal to quantity supplied.

An implication of this idea is that: when I, as a consumer, buy a good or service at a price P, it says very little about my valuation of that good. The only information it conveys is that: my marginal benefit of consuming that particular good has to be above P. No other information is conveyed. For instance, when I buy a shirt at 100 TL, it does not mean that I value this particular shirt at 100 TL, and it also does not mean that the cost of producing this particular shirt is 100 TL. It only says that somewhere, some consumer has a marginal benefit of 100 TL and some other producer has a marginal cost of 100 TL.

This may seem like an obvious idea to you, but it wasn't that obvious to 18th-19th century thinkers! If you went to David Ricardo or Karl Marx and asked what the price of this shirt meant, they would steer towards saying that it says something inherent about this particular shirt (it reflects my valuation for this shirt, or the cost of producing this shirt). What we call the **marginal revolution** of 19th century, i.e., the discipline of thinking at the margins, allowed us to take a break from this reasoning. On a broad level, this is the main reason why you are learning **neoclassical economics** in this class: we are breaking down the connection between prices/valuations/costs, and only relating them through marginal benefit and marginal cost.

Just one more thing. What I said does not mean that other theories of economics (sometimes called *heterodox economics*) are wrong. We happen to teach neoclassical economics, and it is a disciplined way of drawing some conclusions from some assumptions. It is up to you to evaluate the reasonable-ness of these assumptions and conclusions.

The Diamond-Water Paradox

Here is an example of a "paradox" that bothered a lot of 18th century thinkers because they did not witness the marginal revolution yet: "Water, by all means, is a much more essential commodity than diamond. We could not survive without water, but we can keep living in the same manner without diamonds. Therefore, water is much more valuable. Then why it is so cheaper compared to diamonds?"

[2] Friedrich Hayek, one of the most inluential thinkers of 20th century, won a Nobel Prize in 1974 for formulating this idea – in a much, much more sophisticated manner, of course.

Adam Smith is believed to be the one who posited this question, but variants of it go back to Ancient Greece. Based on our discussion so far, we can say that this is hardly a paradox any more.

- First of all, we said over and over that the price of a good is not a measure of its value. It is merely a measure of its marginal benefit, evaluated at the equilibrium quantity. The equilibrium quantity is the key here: it does not only depend on the demand curve, but also the supply curve. Indeed, if we only looked at the demand curve, we would see that the marginal benefit of water is higher than the marginal benefit of diamond **at a given quantity**. (This is especially true for small quantities: if you are in the middle of the desert, you would be willing to pay much more for a gram of water than a gram of diamond.) Yet, it turns out that the supply of diamond is very limited, and thus the equilibrium quantity is limited, which results in a higher marginal benefit of diamonds at the equilibrium quantity.
- If you insist of having a measure of a good's total value, it is the **consumer surplus** in equilibrium. And as you can see, we may very easily have a low price for water but a very high consumer surplus. This actually follows easily from our discussion of elasticities. It is not hard to imagine that the demand for water is very inelastic (after all, it the *the* necessity). Presumably, the supply of water is very elastic (i.e., the supply curve is very flat – probably the marginal cost of extracting water does not increase very fast). Altogether, then, we conclude that the consumer surplus is very high in the water market, and the producer surplus is very low. For the diamond market, the reverse is true. See Figures 220.1 and 221.1.

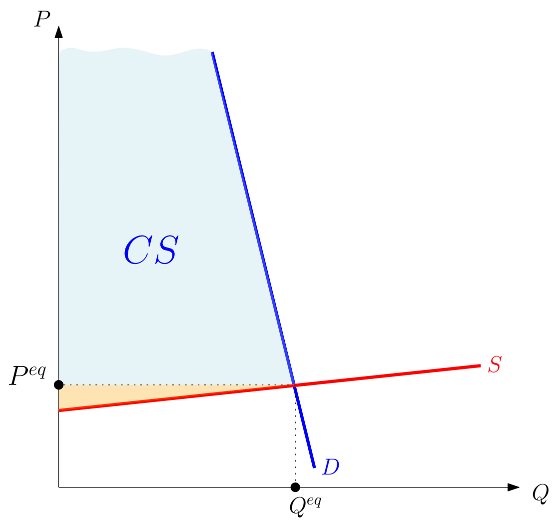


Figure 220.1: Equilibrium of the water market.

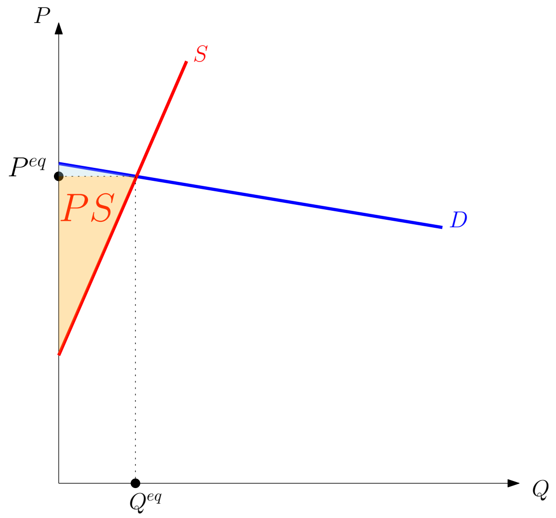


Figure 221.1: Equilibrium of the diamond market.

10.3 Government Interventions

Having defined the crucial concepts, we are now in a good position do discuss some "popular" government interventions. The overarching conclusion of these examples will be: government interventions in a competitive equilibrium create deadweight losses. We will, indeed, go beyond this observation and see who the winners and losers of such interventions will be. We can do this because we already defined the total gains of consumers from trade (CS) and the total gains of producers from trade (PS). Thus, for instance, if PS decreases, we will say: "Overall, producers are the losers from this government intervention.

10.3.1 Price Ceilings

Suppose that we have a (free market) competitive equilibrium (Q^{eq}, P^{eq}). Then, the government comes in and says the following: "I have decided to impose a price ceiling in this market. From now on, the price of the good cannot be above P^c liras."

Examples: price ceilings are frequently used in agricultural markets (onion/potato price are sometimes set by the government), in sectors where public health concerns are prevalent (masks etc.) Perhaps the most notable example is **rent controls**.

What will happen with a price ceiling?

- If $P^c \geqslant P^{eq}$, we have an **ineffective price ceiling**: the free market competitive equilibrium price is already below the price ceiling. The equilibrium price will remain at P^{eq} and the equilibrium quantity will remain at Q^{eq}. There will be no changes in CS, PS, ES, and there will be no DWL.

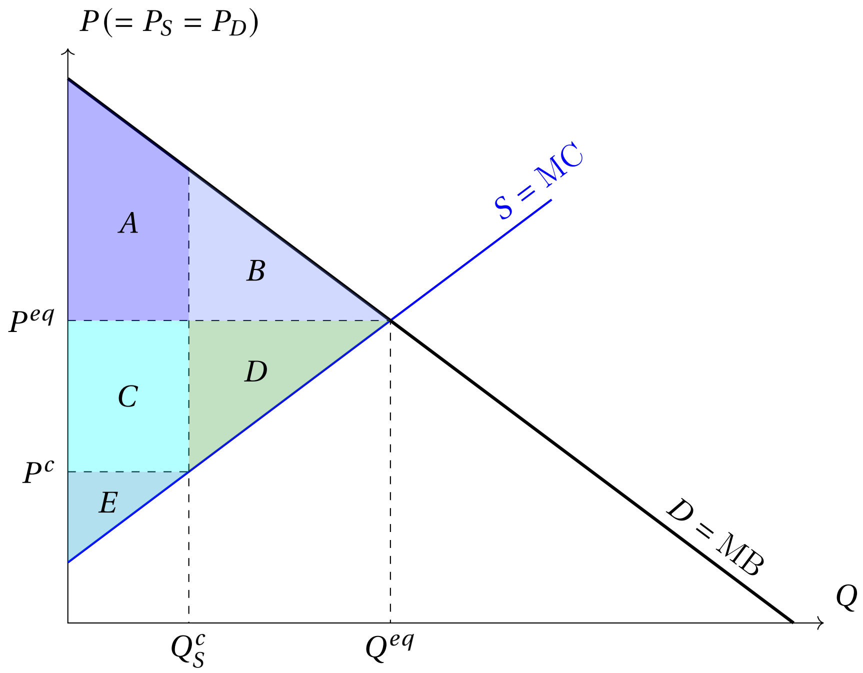
- If $P^c < P^{eq}$, we have an **effective price ceiling**. In this case, the quantity demanded under price P^c, Q_D^c, is larger than the quantity supplied under price P^c, Q_S^c. Indeed, by the law of demand and the law of supply and $P^c < P^{eq}$, we have:

$$Q_S^c < Q^{eq} < Q_D^c$$

  Therefore, there is **shortage** in the market with price P^c. Without any government interventions, the price would rise back to P^{eq} to prevent the shortage. Yet, with an effective price ceiling, the government does not allow the prices to rise. As a result, the equilibrium quantity traded will be dictated by the smaller of two quantities: Q_S^c.

We conclude: with an effective price ceiling, the price will be P^c and the quantity traded will be $Q^c = Q_S^c$. What happens to the consumer and producer surplus? Let's investigate the figure:

Economic Surplus and Deadweight Loss with an Effective Price Ceiling P^c



	Free Market	With Price Ceiling
CS	$A+B$	$A+C$
PS	$C+D+E$	E
Economic Surplus = CS + PS	$A+B+C+D+E$	$A+C+E$
Maximum Economic Surplus	$A+B+C+D+E$	$A+B+C+D+E$
DWL	0	$B+D$

A couple of notes:

- An effective price ceiling unambiguously reduces the producer surplus (it reduces from $C + D + E$ to E). This is because:
  - Some producers reduce their supply of the good at the lower price – recall that this is the reason why quantity traded decreases from Q^{eq} to Q_S^c. This leads to the loss of D in producer surplus.
  - Moreover, for every unit sold, the producers are receiving a lower payment. This leads to a loss of C in producer surplus.

  Both of these effects reflect negatively on PS.

- An effective price ceiling may increase or decrease the consumer surplus ($A + B$ may be larger or smaller from $A + C$, depending on the shape of the demand curve.)
  - Some consumers cannot buy the good they could consume before the price ceiling. This is because without a price ceiling quantity traded is Q^{eq}, whereas with a price ceiling it reduces to to Q_S^c. This leads to the loss of B in consumer surplus.
  - However, for every unit bought, the consumers are paying a lower price. This leads to a gain of C in consumer surplus.

  The net effect on CS may be positive or negative. But it is useful to keep in mind that **there are winners and losers from an effective price ceiling among the consumers.** Some of them enjoy the price ceiling (they are paying lower prices!) and some of them hate it (they cannot find the goods to buy, even though they are willing to pay a higher price than P^c!)

- In net, however, the sum of consumer and producer surplus unambiguously decrease. This is because some profitable trades are lost with an effective price ceiling. The result is a DWL of $B + D$.

So... with an effective price ceiling, producers lose, some consumers gain, and some consumers lose. There may still be justifications to use price ceilings (maybe the government really cares about the winners?) but it is important to keep the consequences in mind.

Consider rent controls: with an effective upper limit on rent, it seems like the government is protecting the tenants in the city. This is partly true, but is missing some parts of the picture. **First,** the landlords are the unambiguous losers of this policy. **Second,** because the rents are lower, some landlords decide not to rent their houses at all (they may use the houses themselves, or use it as a warehouse etc.) As a result, the total supply of houses available for rental decreases. Due to this decrease, some tenants (who were able to find houses before the rent controls) cannot find houses any more. **Third,** on net, fewer gains from trade are created because some houses are not rented. This reflects a typical *cautionary tale* of policy-making: you always have to think about the winners and losers, and be aware of the consequences.

10.3.2 Price Floors

Suppose that we have a (free market) competitive equilibrium (Q^{eq}, P^{eq}). Then, the government comes in and says the following: "I have decided to impose a price floor in this market. From now on, the price of the good cannot be below P^f liras."

Examples: Some agricultural products. Also, minimum wages (as we will discuss in a second.)

What will happen with a price floor?

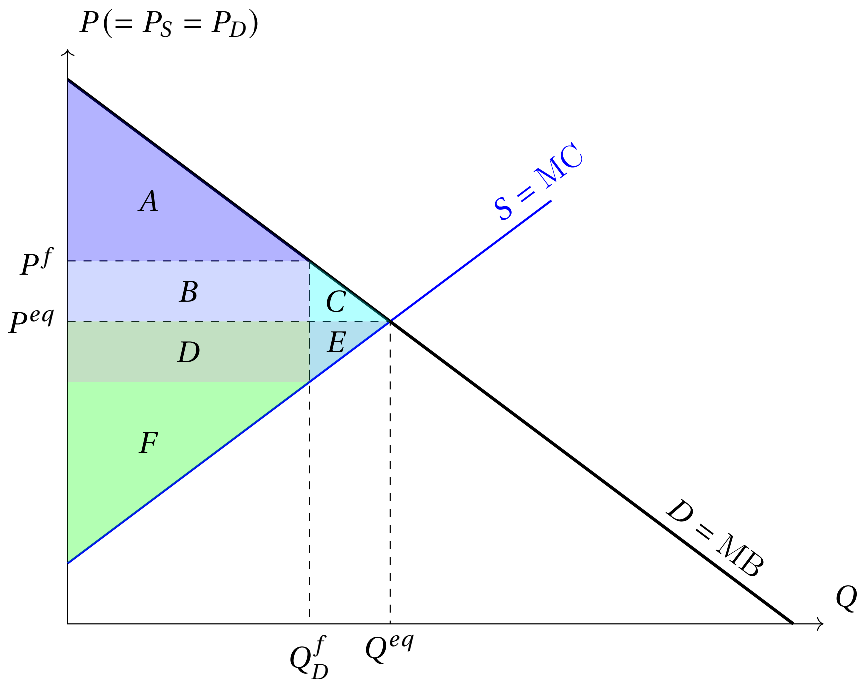
- If $P^f \leqslant P^{eq}$, we have an **ineffective price floor**: the free market competitive equilibrium price is already above the price ceiling. The equilibrium price will remain at P^{eq} and the equilibrium quantity will remain at Q^{eq}. There will be no changes in CS, PS, ES, and there will be no DWL.
- If $P^f > P^{eq}$, we have an **effective price floor**. In this case, the quantity supplied under price P^f, Q_S^f, is larger than the quantity demanded under price P^f, Q_D^f. Indeed, by the law of demand and the law of supply and $P^f > P^{eq}$, we have:

  $$Q_D^f < Q^{eq} < Q_S^f$$

  Therefore, there is **surplus** in the market with price P^f. Without any government interventions, the price would fall back to P^{eq} to prevent the surplus. Yet, with an effective price floor, the government does not allow the prices to fall. As a result, the equilibrium quantity traded will be dictated by the smaller of two quantities: Q_D^f.

We conclude: with an effective price floor, the price will be P^f and the quantity traded will be $Q^f = Q_D^c$. What happens to the consumer and producer surplus? Let's investigate the figure:

Surplus and Deadweight Loss with an Effective Price Floor P^f



	Free Market	With Price Floor
CS	$A+B+C$	A
PS	$D+E+F$	$B+D+F$
Economic Surplus = CS + PS	$A+B+C+D+E+F$	$A+B+D+f$
Maximum Economic Surplus	$A+B+C+D+E+F$	$A+B+C+D+E+F$
DWL	0	$C+E$

Once again, couple of notes:

- An effective price floor unambiguously reduces the consumer surplus (it reduces from $A+B+C$ to A). This is because:
  - Some consumers reduce their demand of the good at the higher price – recall that this is the reason why quantity traded decreases from Q^{eq} to Q_D^f. This leads to the loss of C in consumer surplus.
  - Moreover, for every unit sold, the consumer are paying a higher price. This leads to a loss of B in consumer surplus.

  Both of these effects reflect negatively on CS.

- An effective price floor may increase or decrease the producer surplus ($B+F+D$ may be larger or smaller from $E+F+D$, depending on the shape of the supply curve.)
  - Some producers cannot find the consumer they could find to sell before the price ceiling. This is because without a price floor quantity traded is Q^{eq}, whereas with a price floor it reduces to to Q_D^f. This leads to the loss of E in producer surplus.
  - However, for every unit sold, the producers are receiving a higher price. This leads to a gain of B in producer surplus.

  The net effect on PS may be positive or negative. But it is useful to keep in mind that **there are winners and losers from an effective price floor among the producers.** Some of them enjoy the price floor (they are getting higher prices!) and some of them hate it (they cannot find the consumers to sell, even though they are willing to sell at a lower price than P^f!)

- In net, however, the sum of consumer and producer surplus unambiguously decrease. This is because some profitable trades are lost with an effective price ceiling. The result is a DWL of $C+E$.

So... with an effective price floor, consumers lose, some producers gain, and some producers lose. There may still be justifications to use price floors (maybe the government really cares about the winners?) but it is important to keep the consequences in mind.

10.3.3 Minimum Wage in a Competitive Labor Market

A very famous policy that corresponds to a price floor is a **minimum wage** in a labor market.

To conceptualize a labor market, we should think about labor as a commodity. Here, the workers are the "producers" and the frms are the "consumers" of labor.

- The quantity is the employment, L.
- The price is wage, w.
- The firms are at the demand side of the labor market. Consider the short run where the capital is fixed at $\bar{K}$. In a competitive labor market, firms see wage w and decide how much labor L to hire. Recall from Equation 153.1 that the firms choose their labor demand L^D such that:

  $$w = MPL(L^D, \bar{K})\ P$$

  Thus, this is the equation for the *inverse demand curve* at the labor market (i.e., this equality expresses w as a function of L^D). (With many firms, this is the choice of the *marginal firm* that hires the *marginal worker*.)

  Due to the diminishing marginal product, $MPL(L^D, \bar{K})\ P$ is decreasing in L^D. This is the famous law of demand: if the wage is higher, firms hire fewer workers.

- The workers are at the supply side of the labor market. In a competitive labor market, each worker observes the wage w and decide how much to work, i.e., how much labor to supply. Behind this optimization problem lies a decision that considers the benefits of working (wage) and the opportunity costs of working (spending time on home production, spending time with family, spending time on recreational activities...) As a result of this optimization problem, we observe an *inverse supply curve* that expresses w as a function of L^S. Let's call this equation $w(L^S)$.

  We will assume that $w(L^S)$ is an increasing function of L^S. This is the famous law of supply: if the wage is higher, workers are willing to work for more hours.

All in all, the labor market can be represented with a diagram of labor demand and labor supply curves. See Figure 229.1 below.

Now, suppose that we have a (free market) competitive equilibrium (L^{eq}, w^{eq}) in the labor market. Then, the government comes in and says the following: "I have decided to impose a minimum wage in this market. From now on, the wage cannot be below w^{min} liras." Adapting what we have learned about price floors:

- With an effective minimum wage $w^{min} > w^{eq}$, there is **surplus** in the labor market: $L^D < L^{eq} < L^S$. That is, the workers are willing to work a lot for the minimum wage but firms are not willing to hire people. As a result, the market will be cleared at $L^{min} = L^D < L^{eq}$. Compared to the competitive equilibrium, minimum wages create unemployment...

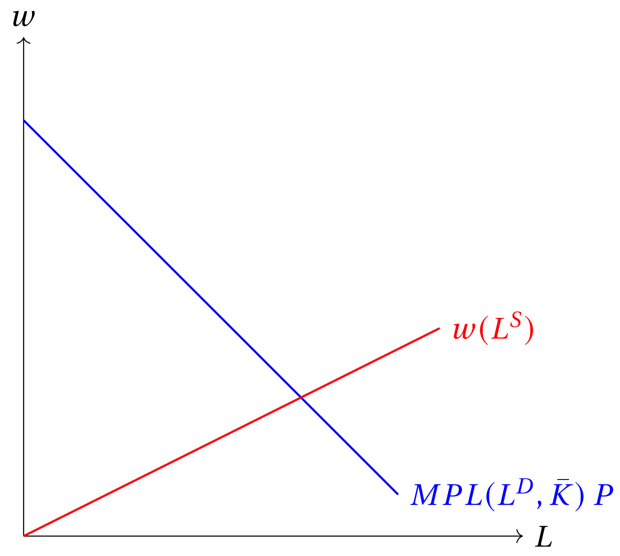


Figure 229.1: A labor market.

- Firms (consumers in the labor market) are unambiguously worse off. They hire fewer workers a higher prices.
- Some workers are happy (these are the workers who can still find jobs under the minimum wage, even though there is less employment), and some workers are upset (these are the workers who lost their jobs due to the minimum wage). Total surplus of workers may increase or decrease.
- In net, economic surplus decreases. Intuitively, the minimum wage prevents some employment arrangements, even though there are some firms and workers who would be willing to sign contracts for wages below w^{min}.

VERY IMPORTANT NOTE: these consequences really depend on the competitive market assumption (most importantly, the assumption that everyone is a price taker, and no firm is powerful enough to change wages unilaterally.) We will see in the next chapter that if the labor market is not competitive, these conclusions do not hold. It is up to you to determine whether this is a reasonable assumption for many labor markets.

Also note: the government may still choose to impose a minimum wage, on the grounds that she cares about the gains to workers who keep their jobs. Or even, the government may say that it does not approve a lower wage on ethical grounds: it is just inhumane to employ people at wages below poverty levels. All of these are valid justifications. Still, it is important to be aware of the consequences.

10.3.4 Taxes on Producers

Suppose that we have a (free market) competitive equilibrium (Q^{eq}, P^{eq}). Then, the government comes in and says the following: "I have decided to impose a per unit sales tax on the producers in this market. From now on, for every unit that is sold, the producer who sells the good has to pay me T liras."

Examples: abundant. Basically, any sales tax.[3]

What will happen with a tax on producers?

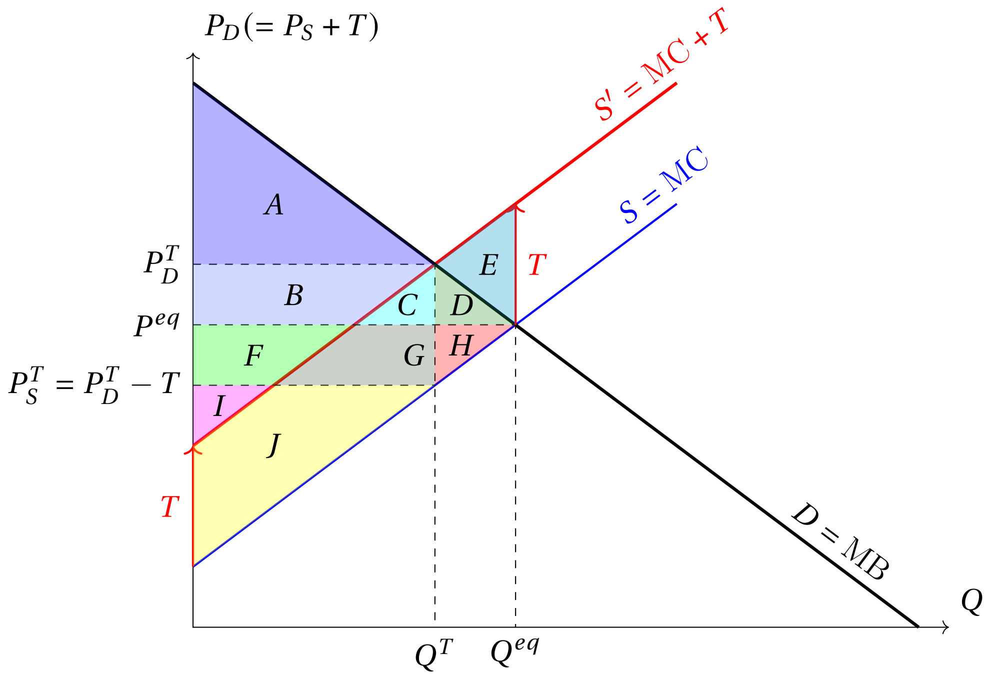
- Recall what we said in Chapter 8 when we were discussing the shifts in the supply curve. A tax on producers will cause a northwestern shift on the supply curve. This is because for each quantity Q, the marginal cost of producing and selling the Q-th unit is now $\mathrm{MC}(Q) + T$ instead of $\mathrm{MC}(Q)$. Therefore, the supply curve will shift upwards by T liras.
- As a result, the equilibrium will shift in the northwestern direction: the quantity traded under taxation will be $Q^T < Q^{eq}$ and the price that the consumers pay will be $P_D^T > P^{eq}$.
- Note, however, that the price producers receive will be lower than the price consumers pay. This is because the producers have to pay T liras to the government per each unit sold. Therefore, the price producers receive in equilibrium is $P_S^T = P_D^T - T$. The (Q^T, P_S^T) point lies on the original supply curve. We calculate the producer surplus based on the area between the original supply curve and P_S^T.[4]
- The tax revenue of the government (TR) is the quantity traded (Q^T) times the per unit tax (T). Therefoe, $\mathrm{TR} = Q^T \cdot T$.

Perhaps it is best to investigate the figure.

[3] A sales tax is typically is a percentage of the sales price, but the overall conclusions would not differ much.

[4] Equivalently, we can calculate it as the area between the shifted supply curve and P_D^T. It would give the same answer.

Surplus and Deadweight Loss with Unit Tax T on Producers



	Free Market	With Taxation
CS	$A + B + C + D$	A
PS	$F + G + H + I + J$	$I + J$
Tax Collected (TR)	0	$B + C + F + G$
Economic Surplus = CS + PS + TR	$A + B + C + D + F + G + H + I + J$	$A + B + C + F + G + I + J$
Maximum Economic Surplus	$A + B + C + D + F + G + H + I + J$	$A + B + C + D + F + G + H + I + J$
DWL	0	$D + H$

A couple of notes:

- Both CS and PS unambiguously decrease as a result of taxation. The price that consumers pay is larger than the free market price, and the price producers receive is smaller than the free market price.
- We are including the tax revenue in the economic surplus. The implicit assumption is that: the tax collected by the government does not "disappear", and the government feeds it back to the economy in a way that creates welfare. In a sense, this is the "best case" scenario.
- Still, even when the tax revenue is fully included in the economic surplus, there is some deadweight loss. Intuitively, this is because of the following: the government says "Every time a consumer and a producer make a trade, I demand T." When the gains from producing and consuming a unit is less than T (i.e., when $\mathrm{MB}(Q) - \mathrm{MC}(Q) < T$), the parties of the transaction stop trading, because it does not generate enough gains to cover for what they pay to the government. The result is a DWL of $D + H$.
- The tax revenue is $B+C+F+G$. Even though the producers are legally required to pay the tax, in equilibrium, the producers and consumers share the tax burden. Note that: $B+C$ is taken away from the **consumer surplus**: effectively, this is the tax burden that the consumers pay. $F+G$ is taken away from the **producer surplus**: this is the tax burden that the producers pay. Therefore, **even though one side of the market is required to pay the tax, both sides of the market share the tax burden**. We will come back to this observation.

10.3.5 Taxes on Consumers

Suppose that we have a (free market) competitive equilibrium (Q^{eq}, P^{eq}). Then, the government comes in and says the following: "I have decided to impose a per unit sales tax on the consumers in this market. From now on, for every unit that is bought, the consumer who buys the good has to pay me T liras."

Examples: abundant. Basically, any type of consumption tax.[5]

What will happen with a tax on producers?

- A tax on consumers will cause a southwesterm shift on the supply curve. This is because for each quantity Q, the marginal benefit of consuming the Q-th unit is now $\mathrm{MB}(Q) - T$ instead of $\mathrm{MB}(Q)$. Therefore, the supply curve will shift upwards by T liras.

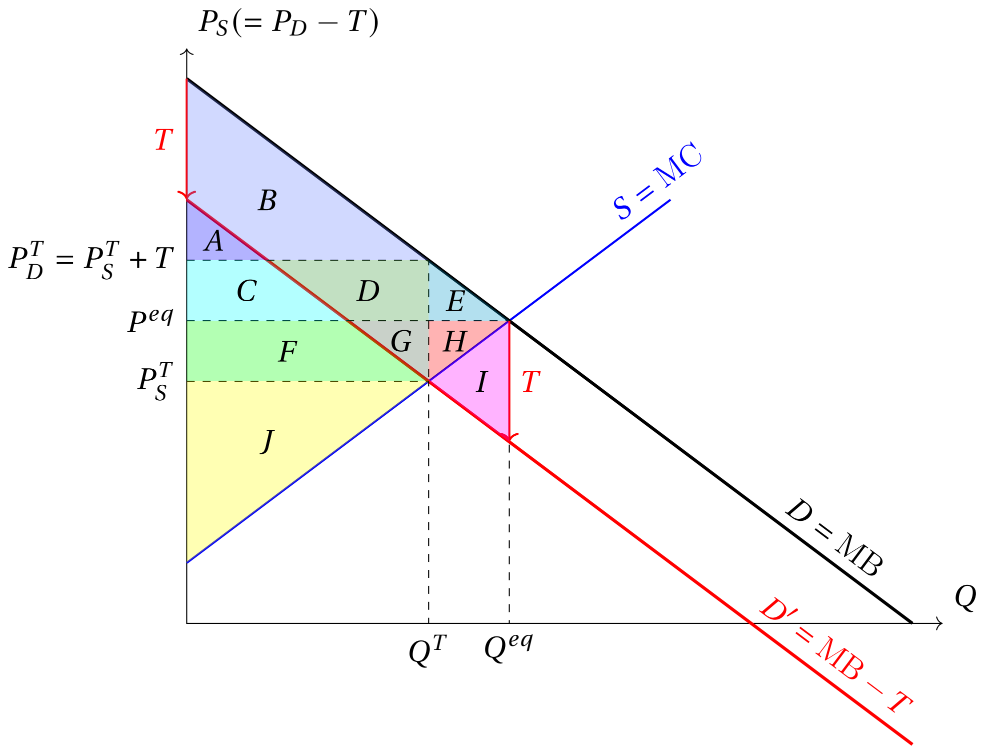
[5] Is a sales tax (for instance, the value added tax) formally paid by the consumer or the producers? Unclear, and also, as we will see in a moment, it does not matter.

- As a result, the equilibrium will shift in the southwestern direction: the quantity traded under taxation will be $Q^T < Q^{eq}$ and the price that the producers receive will be $P_S^T < P^{eq}$.
- Note, however, that the price consumers pay will be higher than the price producers receive. This is because the consumers have to pay T liras to the government per each unit bought. Therefore, the price consumers pay in equilibrium is $P_D^T = P_S^T + T$. The (Q^T, P_D^T) point lies on the original demand curve. We calculate the consumer surplus based on the area between the original supply curve and P_D^T.[6]
- The tax revenue of the government (TR) is the quantity traded (Q^T) times the per unit tax (T). Therefoe, $\mathrm{TR} = Q^T \cdot T$.

Let's investigate the figure.

[6]Equivalently, we can calculate it as the area between the shifted demand curve and P_S^T. It would give the same answer.

Surplus and Deadweight Loss with Unit Tax T on Consumers



	Free Market	With Taxation
CS	$A+B+C+D+E$	$A+B$
PS	$F+G+H+J$	J
Tax Collected (TR)	0	$C+D+F+G$
Economics Surplus = CS + PS + TR	$A+B+C+D+E+F+G+H+J$	$A+B+J+C+D+F+G$
Maximum Economics Surplus	$A+B+C+D+E+F+G+H+J$	$A+B+C+D+E+F+G+H+J$
DWL	0	$E+H$

A couple of notes:

- Both CS and PS unambiguously decrease as a result of taxation. The price that consumers pay is larger than the free market price, and the price producers receive is smaller than the free market price.
- Once again, we are including the tax revenue in the economic surplus.
- There is some deadweight loss. Intuitively, the government still says "Every time a consumer and a producer make a trade, I demand T." When $\mathrm{MB}(Q)-\mathrm{MC}(Q) < T$, the parties of the transaction stop trading. The result is a DWL of $E+H$.
- The tax revenue is $C+D+F+G$. Even though the consumers are legally required to pay the tax, in equilibrium, the producers and consumers share the tax burden. $C+D$ is taken away from the **consumer surplus**, and $F+G$ is taken away from the **producer surplus**. Therefore, **even though one side of the market is required to pay the tax, both sides of the market share the tax burden**.

10.3.6 Tax Incidence

If you had a careful look at the figures representing taxes on consumers and taxes on producers, you may have realized by now: these figures are exactly the same. That is, CS, PS, TR and DWL in both cases are identical. In both cases, DWL is a triangle whose left side has a height of T. This is not a coincidence! **It does not matter who is legally required to pay the tax, the implications of a per unit tax is always the same.**

This bears the following question: "In any case, both parties share the tax burden. But **who pays the larger share of the tax burden?**" The answer to this question is: **"Whichever side of the market has lower elasticity."** You can convince yourself of this by drawing a very inelastic demand curve: you will see that the prices consumers pay increase almost as much as T. Alternatively, if the supply curve is very inelastic, the price producers receive decrease almost as much as T.

This actually makes a lot of sense. Consider a market with very inelastic demand, for instance, the market for cigarettes. (As you know, cigarettes are addictive, which makes it almost impossible for people to adjust their consumption habits in response to price.) Suppose the government increases the tax on cigarettes: it says "From now on, for every pack of cigarette sold, I will get 5 TLmore." Do you think Phillip Morris gets upset about this tax? Not at all. This is because they can increase the price of cigarettes by 5 TLand get almost the same quantity demanded, and the same profits. Th real losers of this policy is the consumers. (As a side note, this is also one of the reasons why governments like taxing cigarettes and alcohol: they can extract a lot of tax revenue out of such goods. Of course, this is not the only, or even primary, reason for taxing cigarettes. You need to wait for the next chapter to hear about the other reasons.)

10.3.7 Subsidies to Producers

Suppose that we have a (free market) competitive equilibrium (Q^{eq}, P^{eq}). Then, the government comes in and says the following: "I have decided to impose a per unit sales subsidy on the producers in this market. From now on, for every unit that is sold, the producer who sells the good will receive an extra B liras from me."

Examples: agriculture markets, medical products.

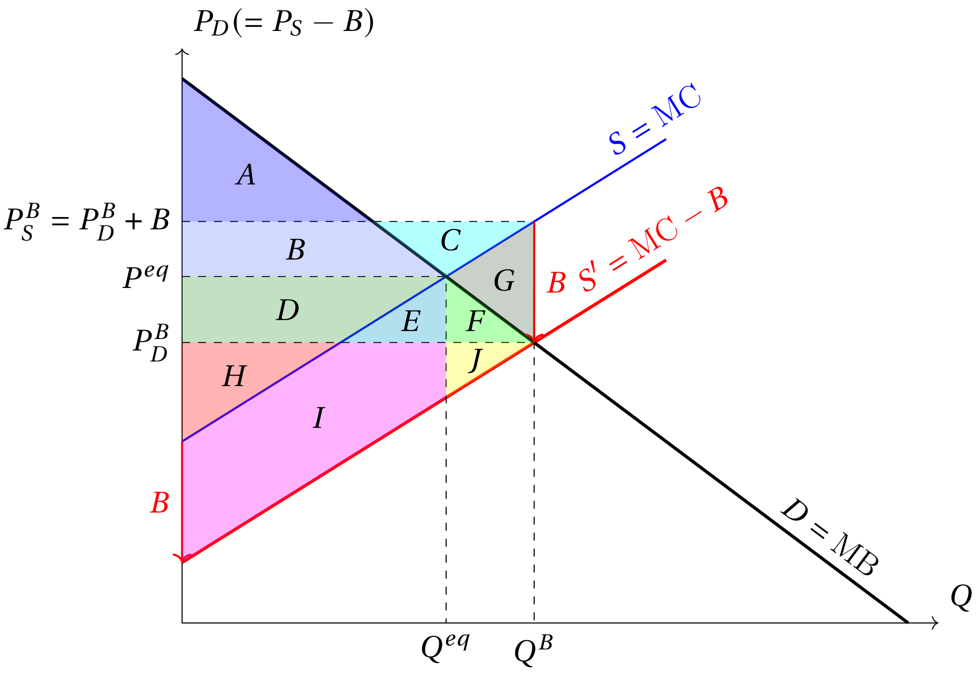
What will happen with a subsidy on producers?

- Based on what we said in Chapter 5, a subsidy on producers will cause a southeastern shift on the supply curve. This is because for each quantity Q, the marginal cost of producing and selling the Q-th unit is now $\mathrm{MC}(Q) - B$ instead of $\mathrm{MC}(Q)$. Therefore, the supply curve will shift downwards by B liras.
- As a result, the equilibrium will shift in the southeastern direction: the quantity traded under taxation will be $Q^B > Q^{eq}$ and the price that the consumers pay will be $P_D^B < P^{eq}$.
- Note, however, that the price producers receive will be higher than the price consumers pay. This is because the producers receive an extra B liras deom the government per each unit sold. Therefore, the price producers receive in equilibrium is $P_S^B = P_D^B + B$. The (Q^B, P_S^B) point lies on the original supply curve. We calculate the producer surplus based on the area between the original supply curve and P_S^B.[7]
- The subsidy paid the government (TS) is the quantity traded (Q^B) times the per unit subsidy (B). Therefoe, $\mathrm{TS} = Q^B \cdot B$.

Let's investigate the figure.

[7]Equivalently, we can calculate it as the area between the shifted supply curve and P_D^B. It would give the same answer.

Surplus and Deadweight Loss with Subsidy B to Producers



	Free Market	With Subsidy
CS	$A+B$	$A+B+D+E+F$
PS	$D+H$	$B+C+D+H$
Subsidy distributed by government (TS)	0	$B+C+D+E+F+G$
Economics Surplus = CS + PS − TS	$A+B+D+H$	$A+B+D+H-G$
Maximum Economics Surplus	$A+B+D+H$	$A+B+D+H$
DWL	0	G

Notes:

- Both CS and PS unambiguously increase as a result of subsidies. The price that consumers pay is smaller than the free market price, and the price producers receive is larger than the free market price.
- We are subtracting the subsidy distributed in the economic surplus. This is because that subsidy does not fall from the sky: the government has to finance it somehow! (I was being very generous in the tax analysis by adding the tax revenue, not I am being equally harsh by subtracting the subsidy.)
- When the subsidy distributed is subtracted from the economic surplus, there is some deadweight loss. Intuitively, this is because of the following: the government says “Every time a consumer and a producer make a trade, I will give an extra B.” When the gains from producing and consuming a unit is less than $-B$ (i.e., when $\mathrm{MC}(Q) - \mathrm{MB}(Q) < B$), the parties of the transaction have incentives to trade and share this extra money offered by the government. This results in **too much trade**. The result is a DWL of G.
- The subsidy distributed is $B+C+D+E+F+G$. Even though the producers are legal recipients of the subsidy, in equilibrium, the producers and consumers share the gains. Note that: $D + E + F$ is added to the **consumer surplus**: effectively, this is the subsidy that the consumers receive. $B + C$ is added to the **producer surplus**: this is the subsidy that producers receive. Therefore, **even though one side of the market is the legal recipient of the subsidy, both sides of the market benefit**.

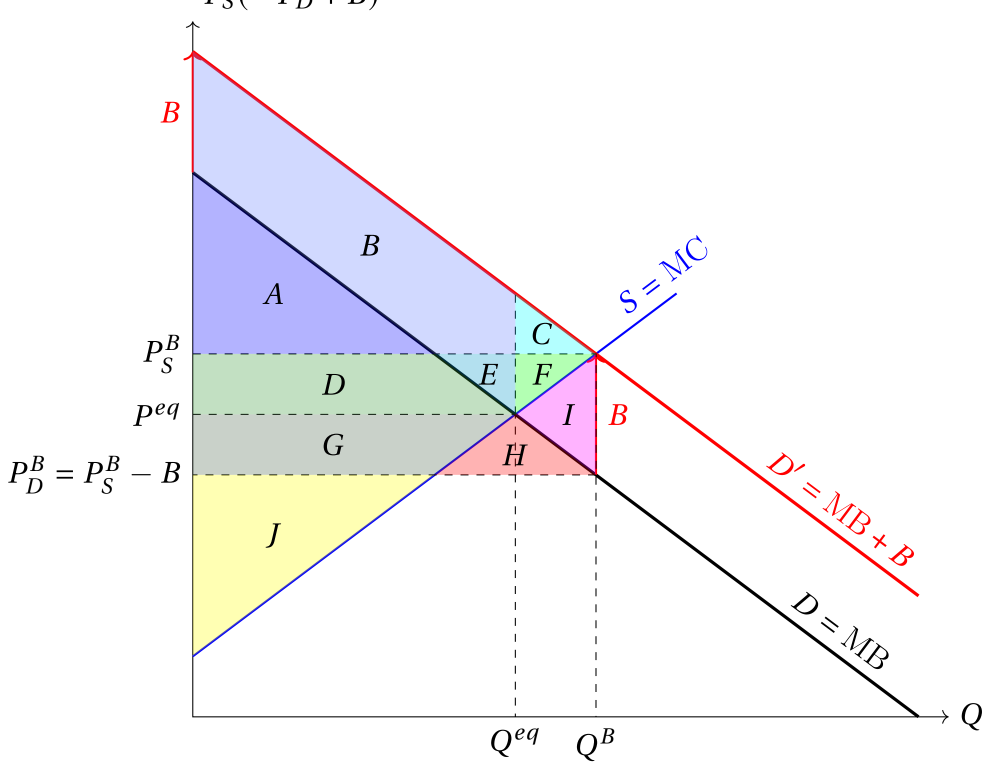
10.3.8 Subsidies to Consumers

Suppose that we have a (free market) competitive equilibrium (Q^{eq}, P^{eq}). Then, the government comes in and says the following: “I have decided to impose a per unit sales subsidy on the consumers in this market. From now on, for every unit that is consumed, the consumer who buys the good will receive an extra B liras from me.”

By now, this should be obvious to you. I will save you from the burden by just presenting the figure. See the next page.

So... What have we learned? Overall, it is not a great idea for the government to intervene in a perfectly competitive market. Beginning with the next chapter, we start studying the deviations from the perfectly competitive market. Then, you will see that some of these conclusions will be overturned as well.

Surplus and Deadweight Loss with Subsidy B to Consumers



	Free Market	With Subsidy
CS	$A + D$	$A + D + G + H$
PS	$G + J$	$D + E + F + G + J$
Subsidy distributed by government (TS)	0	$D + E + F + G + H + I$
Economics Surplus $= \text{CS} + \text{PS} - \text{TS}$	$A + D + G + J$	$A + D + G + J - I$
Maximum Economics Surplus	$A + D + G + J$	$A + D + G + J$
DWL	0	I

Extra Readings for Chapter 10

Have you ever wondered how people measure the consumer surplus? I mean, conceptually, we know that consumer surplus is the area between the demand curve and the price, up until quantity traded. The issue is: demand curves do not exist in nature, and hence the exercise of “finding the area below a curve” becomes challenging when we do not know how the curve looks like.

For a recent take on this question, check out:

> Cohen, Peter, Robert Hahn, Jonathan Hall, Steven Levitt, and Robert Metcalfe. *Using Big Data to Estimate Consumer Surplus: The Case of Uber.* National Bureau of Economic Research Working Paper No. w22627, 2016.

Using the huge dataset generated by the price changes in Uber, the authors draw Uber’s demand curve (see Figure 6 in the paper). Based on this, they estimate the consumer surplus to be $ 6.8 billion in the United States in a year.

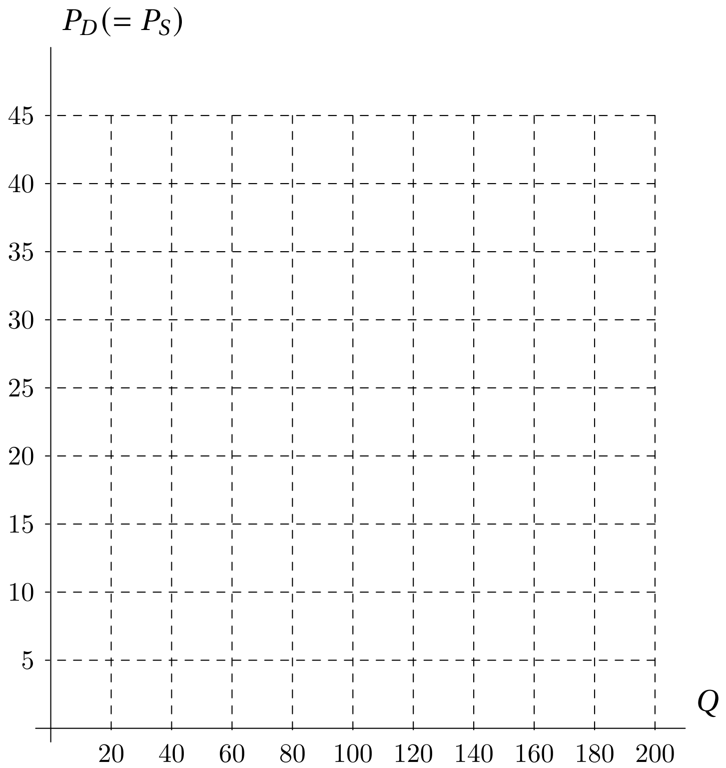
Note that this paper would not have been possible without the “big data” generated and collected by a firm like Uber. The type of analysis that could be conducted with big data is a source of excitement for many economists.

Contrary to the case of entire consumer surplus, if you want to measure the loss in consumer surplus and producer surplus due to a tax, there are only two things you need to know: (i) the amount of tax, (ii) the elasticity of demand at P^{eq}, and (iii) the elasticity of supply at P^{eq}. In other words, (i)-(ii)-(iii) are the **sufficient statistics** to calculate the welfare consequences of a tax. This observation is at the heart of Raj Chetty’s sufficient statistics approach to estimate welfare. If you are interested, see:

> Chetty, Raj. “Sufficient Statistics for Welfare Analysis: A Bridge Between Structural and Reduced-Form Methods." *Annu. Rev. Econ.* 1, no. 1 (2009): 451-488.

Exercises for Chapter 10

1) Consider a competitive market where the market supply is given by the function $Q_S = 10P_S - 100$ and the market demand is given by $Q_D = 200 - 5P_D$.



a. Find the free market equilibrium price and quantity.

b. Find the consumer surplus, producer surplus and the economic surplus (under free market conditions) at the equilibrium.

c. Assume that the government sets a price ceiling of 15 TL/unit on this good.

   (a) Find the equilibrium price and quantity traded.

   (b) Find the consumer surplus, producer surplus, economic surplus and deadweight loss at the equilibrium under price ceiling.

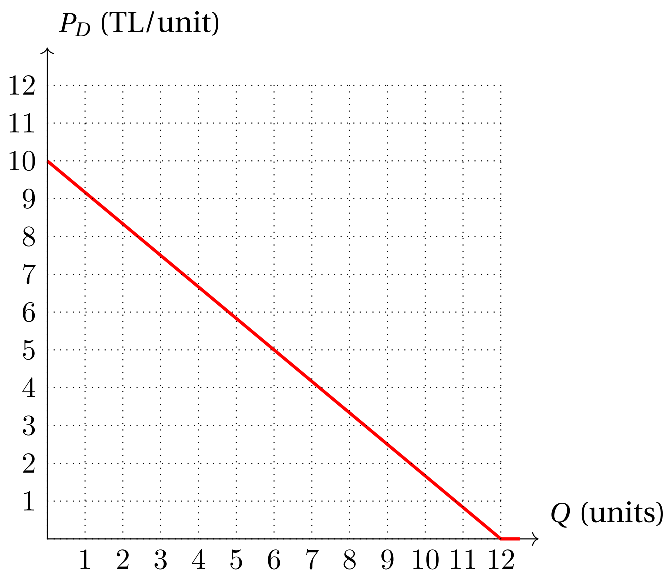
d. Assume that a unit tax of 3 TL/unit is imposed on the producers.

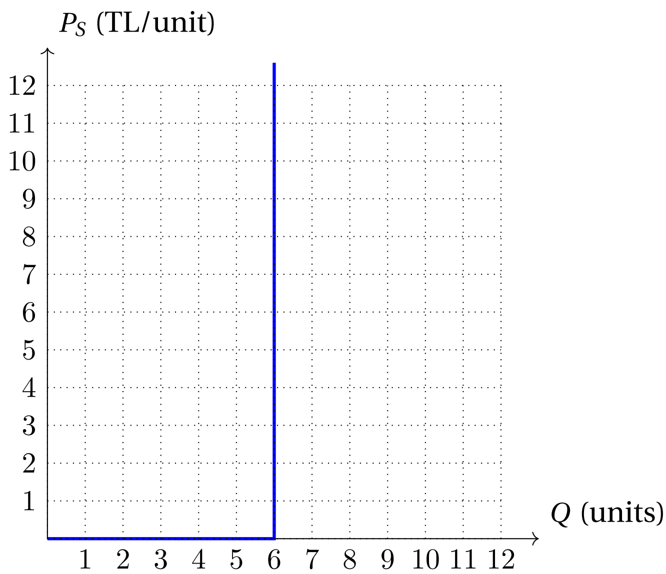
   (a) Find the equilibrium price that the consumers face, the equilibrium price that the producers face, the equilibrium quantity traded under taxation.

   (b) Find the consumer surplus, producer surplus, the tax revenue (tax collected by government), economic surplus and the deadweight loss, at the equilibrium under taxation.

(c) What percentage of the tax burden is passed on to the consumers?

2) Consider a market with the following market demand and market supply (note that the market supply is perfectly inelastic):





a. Find the free market equilibrium price and quantity.

b. Find the consumer surplus, producer surplus and the economic surplus (under free market conditions) at the equilibrium.

c. Assume that a unit tax of 2 TL/unit is imposed on the consumers.

(a) Find the equilibrium price that the consumers face, the equilibrium price that the producers face, the equilibrium quantity traded under taxation.

(b) Find the consumer surplus, producer surplus, the tax revenue, economic surplus and the dead-weight loss, at the equilibrium under taxation.

(c) What percentage of the tax burden is passed on to the producers?

d. Now assume that a unit tax of 2 TL/unit is imposed on the producers.

(a) Find the equilibrium price that the consumers face, the equilibrium price that the producers face, the equilibrium quantity traded under taxation.

(b) Find the consumers' surplus, producers' surplus, the tax revenue, economic surplus and the dead-weight loss, at the equilibrium under taxation.

(c) What percentage of the tax burden is passed on to the consumers?

Chapter 11

Market Power

Until now, we have studied the "benchmark model" of perfectly competitive markets. I have repeated it several times, by let me say that again: this model includes a bunch of (very demanding) assumptions.

- Consumers have well-defined preferences and they make their decisions under complete information about prices and their preferences,
- Producers maximize profits and they are not credit-constrained,
- Everyone is a price-taker and market clears,
- The gains from trade is given by the difference between the consumer's marginal benefit and the producer's marginal cost (i.e., third parties are not affected by the economic activity between the consumer and producers),
- ...

Under these assumptions, we concluded that markets lead to "desirable" outcomes. Admittedly, these assumptions rarely hold in real life; nevertheless, it is useful to study this idealized benchmark. This allows us to understand how conclusions change when we change one of the basic assumptions of the benchmark model.

The rest of this course (and frankly, the rest of your undergraduate education) will be about studying deviations from this benchmark. What happens when consumers' preferences are not well-defined? If the consumers have uncertainty about the value of a good? If the producers have long-term objectives? If producers are credit-constrained? If some economic agents can influence the price of a product (i.e, when they are not price-takers)? If economic activity between two agents affect third parties? We have years to answer all of these questions (believe me, we will!) – but for the rest of this course, we will tackle some of them.

The analogy is: so far what we have done was Newtonian physics in a vacuum and frictionless environment. Slowly we will introduce frictions, and then electromagnetic forces, and then nuclear forces... etc.

Let's start by investigating what happens if some economic agent (firm) is not a price-taker; rather, the firm can influence the price of a product. When that is the case, we say that the firm has **market power**. But first things first.

11.1 Why Do Firms Have Market Power?

You may ask: *how on earth would a firm be able to influence the price of a product?* If a firm is so powerful, that would be a very attarctive industry for other entrepreneurs. As a result, whenever some firm has market power, other firms would rush into the market. This would result in many sellers selling identical products, and hence a competitive market. But for this reasoning to work, we need other firms to be able to enter the market. Sometimes, new firms cannot enter the market due to some obstacles. We refer to these obstacles as **barriers to entry**. Whenever there are severe barriers to entry, the market moves away from the competitive benchmark.

There are barriers to entry in a market whenever (1) new firms cannot produce the identical product as the existing firms, (2) even when they can produce, they cannot steal the existing firms' consumers. Let's walk through some common examples.

11.1.1 New Firms Cannot Produce...

This is the case when the existing firms **control** some **key inputs**. Examples:

- Saudi Arabia maintains control over a vast oil supply – new firms cannot simply extract oil and enter the oil market.
- Apple has signed long-term deals with its assemblers – new firms cannot just enter and use the assembly facilities that Apple gets to use.

Sometimes the government just makes it illegal to enter a market through **regulation**. Examples:

- Patents – new firms cannot just enter and produce a vaccine.
- Licensing requirements – we cannot just buy a car and offer a taxi service.
- Prohibitions on competition – we cannot just offer the same postal service as the state-run postal service, that's illegal.

11.1.2 Even When They Can Produce...

There is always some **product differentiation**. For instance, now matter how hard we try, we will never be able to produce the identical burger to the one in *Burger King* – it has a particular taste and texture that allows Burger King to live without constant fear of losing all their customers at once.

Sometimes, there are **switching costs** that prevent consumers from going to the cheaper alternatives. Examples:

- Airline loyalty programs make it costly to switch carriers – "but I have accumulated so many miles!".
- Software ecosystems can be incompatible – try moving from Apple to Windows without a few headaches.
- Search costs – have you ever attempted to find a better health insurance plan? Those plans have all the obscure and confusing terms, making it hard to compare alternatives.
- Some goods have the characteristics that make them **network goods**. This is the type of good that becomes more valuable as more people use it. Think of TikTok or LinkedIn – they are valuable only because othe people are using it. This means: even if we can produce the identical product as TikTok, we may not be able to attract people.

11.2 Monopoly

There are various forms of non-competitive markets, depending on how many firms with market power there are. When there are many firms with market power this is called an **oligopoly**, with two firms we have a **duopoly**, and, with a single firm, we have **monopoly**. As you can see, a monopoly is the most extreme case of market power. In this course we will analyze the decision of a monopolist. (Analyzing decisions of duopolists/oligopolists requires an understanding of how multiple firms with market power strategically respond to each other. To be able to conduct this analysis, you need to learn a little bit of *game theory*. Hence, we will leave it for future courses. In contrast, a monopolist faces a simple optimization problem, which is a type of problem we have mastered in Econ 101.)

Definition 247.1 *A **monopolist** is a firm that is the sole supplier of a good in the market.*

This has a stark contrast with the firm in a competitive market, which competes with many other firms selling the identical product.

The key assumption we will make in this chapter is: we will assume that the monopolist sets a **single price** P that applies to every unit sold. In other words, we assume that the monopolist does not set different prices for different people/units, i.e., the monopolist does not engage in *price discrimination*. Price discrimination is a fascinating topic, but we will also leave it for future semesters.

All in all, we will assume that there is a single seller who sets a single price. We are mostly after understanding the nature of distortions that occur under this setting – I promise you that similar distortions occur in richer settings.

11.2.1 How is a Monopoly Different than a Competitive Market?

Our monopolist is, by definition, the only seller of the good in a market. This means the monopolist **owns** the demand curve of the market. Therefore, the monopolist can choose any price-quantity pair (P, Q) as long as they are on the demand curve. This is the sense in which the monopolist has market power: it is free to choose any price (but not any quantity demanded; after all, consumers will make the consumption decisions). See Figure 248.1 below.

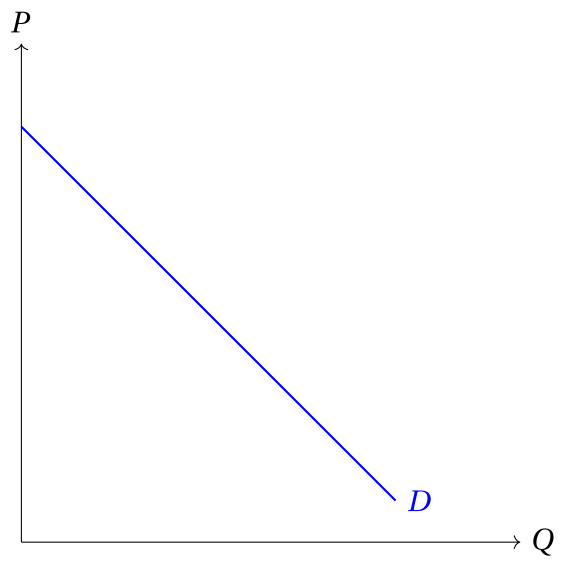


Figure 248.1: The monopolist can choose any (Q, P) pair on the blue curve.

Let's compare this with the problem faced by a firm in a competitive market. This firm is a price-taker: it realizes that its competitors are selling the same good at a price of P^{eq}. The competitive firm simply chooses Q, realizing that P^{eq} is beyond its control.

Thus, the competitive firm can choose any (P^{eq}, Q) pair. This is as if the competitive firm is choosing (P, Q) pairs from a horizontal curve. See Figure 248.1 below.

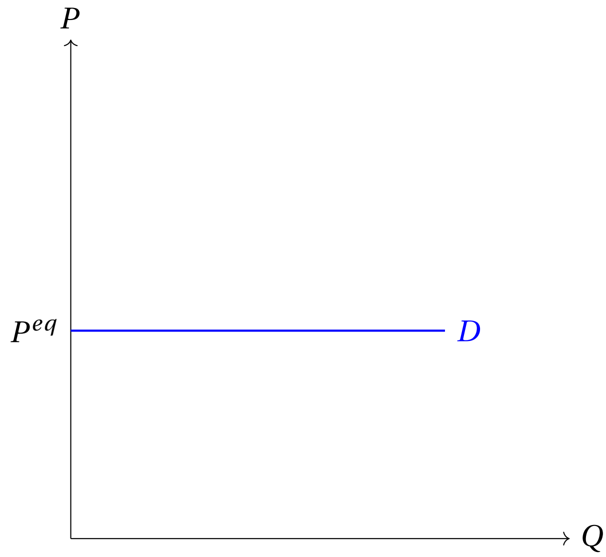


Figure 249.1: The competitive firm can choose any (Q, P) pair on the blue curve.

Comparing Figures 248.1 and 249.1 reveals an important observation: a competitive firm is just like a monopolist, with a demand curve perfectly elastic at P^{eq}. To put it another way, what makes a firm a monopolist (rather than a competitive firm) is having a *downward-sloping* demand curve (rather than a horizontal demand curve).

This means that competitive firm is a special case of a monopolist. This is crucial observation, because the monopoly is defined with respect to the market. *If we define a market narrowly enough, any firm can be thought as a monopolist.* **Example:** Serdar Ortaç, by definition, has a monopoly over Serdar Ortaç songs. Nobody else can compose a Serdar Ortaç song.

However, remember Section 5.5.2: if we define a market very narrowly, inevitably, we will have a very elastic demand curve. That means, every firm can be made a monopolist if the market is define a narrowly enough, but if we have to define it too narrowly so that the demand curve becomes very elastic, the firm will not have a meaningful monopoly. It will only *technically* be a monopoly. For a monopoly to be meaningful, the market demand still needs to be sufficiently inelastic by the time we narrowed down. **Example:** Does Serdar Ortaç have a meaningful monopoly? Presumably, other people can compose very similar songs. Thus, the demand curve for Serdar Ortaç songs is probably very elastic, hence his monopoly is not very meaningful.

Once again, we go back to the idea: as a producer, you should always strive to have

Figure 250.1: Did you know that Mert Ekren composed "İki Deli"? You would be almost certain that it was a Serdar Ortaç song.

inelastic demand – this gives you *real* market power beyond technicalities.

11.2.2 How Is a Monopolist's Problem Different than a Competitive Firm's Problem?

So far, we have established:

- Competitive firm = Horizontal demand curve
- Monopolist = Downward-sloping demand curve

This means the monopolist will face a different problem than a competitive firm. If a competitive firm wants to sell one more unit, it just produces one more unit and sells it at P^{eq}. However, if a monopolist sells one more unit, it needs to decrease the price P. Because the monopolist sets a single price, this means the monopolist needs to reduce the price of *all* units it sells (not just the extra unit it produces). This newly added trade-off is reflected in the monopolist's marginal revenue.

Recall from Section 7.2.1 that the total revenue is the price times quantity sold:

$$\mathrm{TR}(Q) = P \cdot Q$$

and the marginal revenue is the rate of change of total revenue:

$$\mathrm{MR}(Q) = \frac{d\mathrm{TR}(Q)}{dQ}.$$

- For a competitive firm, P is fixed at P^{eq}. Hence, $\mathrm{TR}(Q) = P^{eq} \cdot Q$, and $\mathrm{MR}(Q) = P^{eq}$.

- For a monopolist, P is **not** fixed. It is a function of Q (this captures the fact that if a monopolist wants to sell an extra unit, it needs to reduce the price). Therefore, for a monopolist, the price is $P(Q)$. This is what we call the **inverse demand curve** from Section 9.1.3. Then,

  $$\text{TR}(Q) = P(Q) \cdot Q$$

  and

  $$\begin{aligned}\text{MR}(Q) = \frac{d\text{TR}(Q)}{dQ} &= \frac{dP(Q)\,Q}{dQ} \\ &= P(Q) + P'(Q)\,Q\,.\end{aligned}$$

  Therefore, a monopolist's marginal revenue is the sum of two terms. The first term is the usual price term – this is the term we have in the competitive firm's problem. The second term is the new addition. It is there because, for a monopolist, $P'(Q) \neq 0$, that is, the price is not fixed. Moreover, because the demand curve is downward-sloping, $P'(Q) < 0$. Then, the second term is negative.

Intuitively, the monopolist's marginal revenue consists of two parts that capture the monopolist's trade-off. When the monopolist sells an extra unit, there are two opposing forces:

a. The total revenue tends to increase because the firm increases its sales. This is captured by the $P(Q)$ term.

b. The total revenue tends to decrease because the firm needs to reduce its price for all units it sells. This is captured by the $P'(Q)\,Q$ term. (The price needs to decrease by $P'(Q)$ and Q units are sold, hence the multiplication.)

11.2.3 A Monopolist's Marginal Revenue Curve

So, $\text{MR}(Q) = P(Q) + P'(Q)\,Q$. Moreover, $P(Q)$ is the equation of the demand curve, and since the demand is downward-sloping, $P'(Q) < 0$. Combining all these facts, we have an insightful observation: $\text{MR}(Q) < P(Q)$ for every positive Q. In other words, **marginal revenue curve is below the demand curve at every positive quantity**. See Figure 252.1 below.

Before we go on, you can confirm that if the demand curve is horizontal (the special case of competitive firm), $P'(Q) = 0$ and therefore the marginal revenue curve is the demand curve.

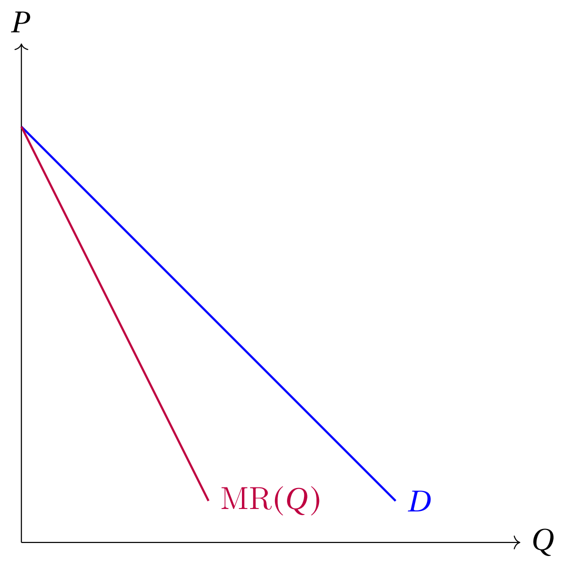


Figure 252.1: The monopolist's demand and marginal revenue curves.

11.2.4 A Monopolist's Choice

Time to characterize the monopolist's choice... A monopolist chooses a price-quantity pair on the demand curve to maximize its profits. Let's call the monopolist's choice (P^m, Q^m). Since this pair is on the demand curve,

$$P^m = P(Q^m)\ .$$

Moreover, by the "golden rule of profit maximization" (Theorem 166.1), Q^m satisfies:

$$\mathrm{MR}(Q^m) = \mathrm{MC}(Q^m)\ .$$

See Figure 253.1 below.

Example 252.1 Suppose the inverse demand curve is given by:

$$P(Q) = 24 - Q\ .$$

Then, we have:

$$\begin{aligned} \mathrm{TR}(Q) &= P(Q)\, Q = (24 - Q)\, Q = 24Q - Q^2 \\ \mathrm{MR}(Q) &= \frac{d}{dQ}(24Q - Q^2) = 24 - 2\,Q. \end{aligned}$$

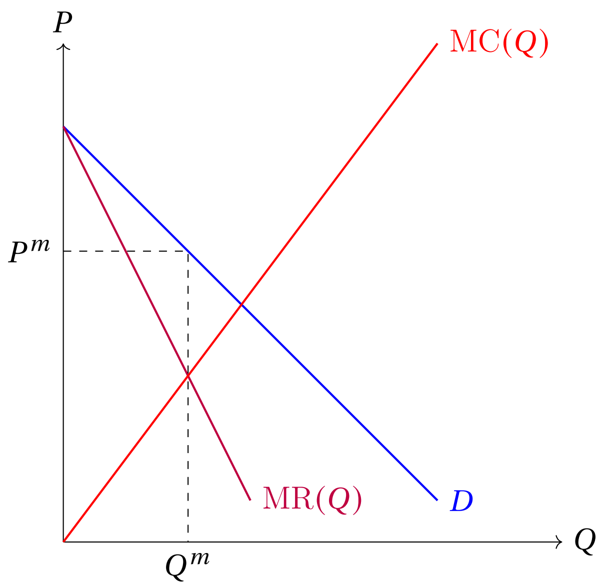


Figure 253.1: The monopolist's choice (P^m, Q^m).

Suppose the total cost is:

$$\text{TC}(Q) = Q^2 + 12\,.$$

Then, we have:

$$\text{MC}(Q) = \frac{d}{dQ}\text{TC}(Q) = 2\,Q\,.$$

Since $\text{MR}(Q^m) = \text{MC}(Q^m)$,

$$24 - 2\,Q^m = 2\,Q^m \implies Q^m = 6\,.$$

and

$$P^m = P(Q^m) = 24 - Q^m = 24 - 6 = 18\,.$$

The monopolist produces 6 units and charges a price of 18.

11.2.5 Welfare Cost of Monopoly

Let's compare the monopolist's decision to the efficient quantity. Recall from Section 10.1.1 that the efficient quantity Q^* satisfies: $\text{MB}(Q^*) = \text{MC}(Q^*)$. Since the marginal benefit is the demand curve, $\mathrm{MB}(Q^*) = P(Q^*)$ and therefore:

$$P(Q^*) = \mathrm{MC}(Q^*)\,.$$

On the other hand, monopolist's quantity Q^m satisfies $\mathrm{MR}(Q^m) = \mathrm{MC}(Q^m)$. Because the marginal revenue curve is below the demand curve, the intersection with marginal revenue occurs at a smaller quantity than the intersection with the demand curve. Therefore, $Q^m < Q^*$: the monopolist produces too little compared to the efficient outcome. As a result, there is a deadweight loss. See Figure 254.1.

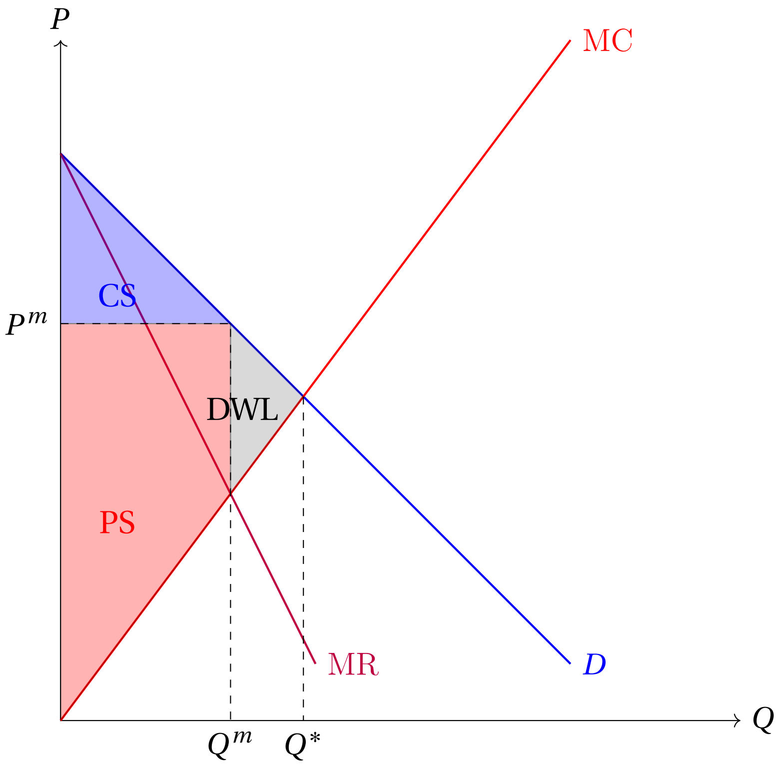


Figure 254.1: Consumer surplus, producer surplus and deadweight loss under a monopoly.

Let's compare what happens under the monopoly with the competitive market benchmark. If the monopolist was replaced by a competitive supply side with the same marginal cost as the monopolist, we would have a competitive equilibrium at the intersection of demand and marginal cost curves. Then, the quantity would be efficient, $Q^{eq} = Q^* > Q^m$. Moreover, since demand is downward-sloping, $Q^m < Q^*$ implies $P^m > P^*$. In other words, **compared to the competitive benchmark, a monopoly results in too little output and too high prices.**

Moreover, take a look at Figure 254.1 to convince yourself that the consumer surplus under monopoly is smaller than the consumer surplus under the competitive benchmark. This is not surprising: the consumer are buying fewer items at higher prices.

I will also argue that the producer surplus under monopoly is higher than the producer surplus under the competitive benchmark. You do not need to investigate any figures for this. This follows from the basic principles: the monopoly could set (P^{eq}, Q^{eq}) and imitate the competitive benchmark if it wanted to. The fact that the monopolist chooses to do something else immediately implies that the producer surplus must be higher.

11.2.6 Key Takeaways

To wrap up, the key takeaways are: compared to the competitive benchmark,

a. A firm with market power reduces output.

b. By doing so, it raises the prices and the producer surplus at the expense of consumer surplus.

c. This results in a deadweight loss: some mutually beneficial trades do not happen.

These insights are fairly generalizable to the other cases of market power (duopoly/oligopoly). In sum, policy makers are worried that the market power will result in too little trades and too high prices, which will result in a welfare loss.

The last bullet point is a curious one. Why does a monopolist refuse to sell more, even though it would be mutually beneficial? To see why, it helps me to think about the following mental exercise. Suppose you are the only dentist in a small town (thus you are the monopolist). You have set your price of your services at 1000 TL per hour, and your marginal cost of giving these services (including the opportunity cost, e.g., the value of time spent with family instead of working) is 500 TL per hour. It is 5pm on a Friday, and you are about to lock your office and leave. All of a sudden, a patient comes in and say that their teeth hurts and they would like you to take a look at it. You say, "Fine, my appointment costs 1000 TL per hour." The patient says, "I am only willing to pay you 800 TL for the next hour." At that point, you realize that this is a mutually beneficial exchange: it only costs you 500 TL to open your office and take a look at the patient's tooth, and you can receive 800 TL for it. **Nevertheless,** you also realize the following: if you receive 800 TLfrom this patient, you will have to reduce your price to 800 TL per hour for *every* patient. That would be a tremendous loss in revenue! As a result, you refuse to treat the patient, lock your office and leave the patient in agony.

Cruel, huh? But this is precisely the reason why we should be concerned about market power.

11.3 Regulating a Monopoly

Let me begin by emphasizing that this is the first time we have seen a result that says: left on its own, an economy results in an undesirable allocation. Consequently, government intervention now can fix things rather than distorting them.

Let's talk about several forms of government interventions that can fix the deadweight loss of a monopoly.

11.3.1 Marginal Cost Subsidies

The fundamental problem of the monopoly is that the firm produces too little output. Hence, government intervention should convince the firm to produce more. The most obvious candidate is a **quantity regulation** (requiring the firm to produce at least Q^* units), a slightly less obvious one is **subsidizing the firm**.

In particular, a per unit sales subsidy of B reduces the firm's marginal cost to $\mathrm{MC}(Q) - B$. If the government chooses B such that

$$\mathrm{MR}(Q^*) = \mathrm{MC}(Q^*) - B\,,$$

it will successfully restore the efficient output level. See Figure 256.1.

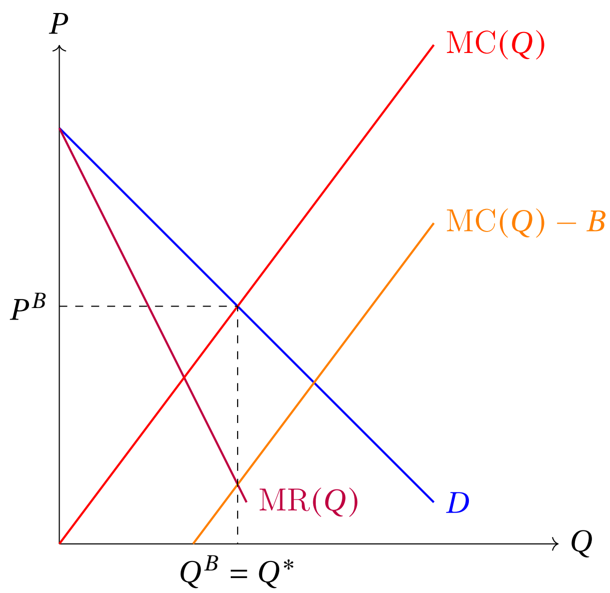


Figure 256.1: The monopolist's choice (P^B, Q^B) under subsidy B.

The subsidy policy sounds easier than it is to implement, though.

a. First of all, to calculate the correct amount of subsidy to the firm, the government needs to know that MC curve. This is very difficult to know without asking the firm. If the government asks the firm what the marginal cost curve looks like, there are no guarantees that the firm will be truthful.

b. Even when the government has the correct estimate of the marginal cost curve, subsidizing the firm kills the firm's incentives to invest in reducing the marginal cost curve. Thus, subsidies may be an obstacle to innovation.

c. Even if the government does every calculation right, the subsidies are very costly to implement. Can you imagine the government saying: "We believe Amazon is a very powerful monopoly; it essentially rips people off by providing less-than-optimal service at high prices. To resolve this issue, we decided to... pay money to Amazon." Taxpayers would be furious!

   Sometimes, to prevent this backlash, the governments make an active effort to declare that the subsidy is financed through other channels than tax revenue. In many cases, governments sell the monopoly rights (via selling licenses to operate), and use the revenue from the sale of monopoly rights to finance the subsidies.

   Sometimes the government pays the subsidies in a more covert way. For instance, in many countries, the government has a monopoly in public transportation. It is also a widely accepted fact that public transportation regularly loses money and the government covers those losses. This is a form of subsidy, even when it is not explicitly defined so.

11.3.2 Price Ceilings

Another government intervention that would fix the deadweight loss goes through realizing that the monopolist charges too high prices. Solution? Imposing a price ceiling at the price of P^{eq}, i.e., the price that would realize under the competitive benchmark. See Figure 258.1.

Mechanically, price ceiling works because it *flattens* the demand curve: the firm cannot go above P^c anyway; thus, the firm cannot decrease the price in order to sell more units in the region where price ceiling binds. As a result, the marginal revenue curve also flattens. The result is a horizontal marginal revenue curve in this region, which is the marginal revenue of a competitive firm. Efficiency restored!

While a price ceiling is politically more feasible than the marginal cost subsidy, some practical issues still remain. Knowing the "correct" level of price ceiling requires that

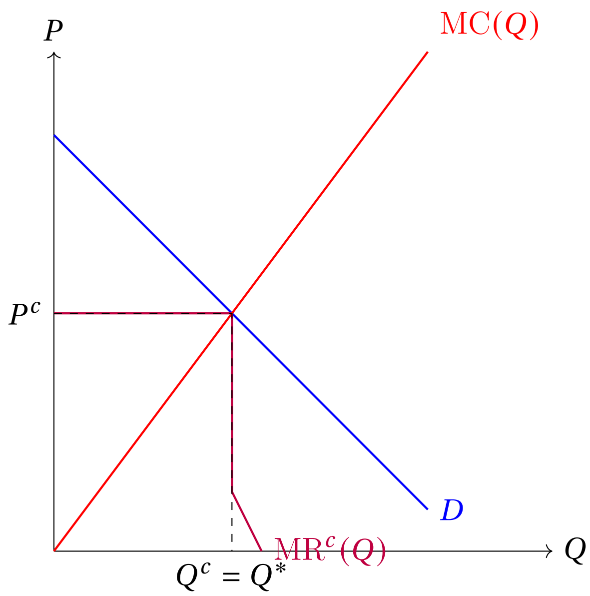


Figure 258.1: The monopolist's choice (P^c, Q^c) under price ceiling $P^c = P^{eq}$.

the government knows about the demand and marginal cost curves. Good luck extracting that information from the firm truthfully.

11.3.3 Encouraging Competition

The government can impose subsidies or price ceilings, but they tend to be impractical solutions. Is there a more fundamental solution to the welfare cost of monopoly? To answer this question, we have to go back to the basics and realize that the deep issue at hand is the *lack of competition*. So, perhaps, the best solution to a monopoly is encouraging competition, i.e., a monopoly does not arise to begin with?

Encouraging competition is the task of every regulatory government agency. They do so by:

- Splitting up monopolies/preventing mergers that would lead to monopolies,
- Lowering entry barriers, subsidizing competitors,
- Privatizing government enterprises and deregulating markets
- ...

If you are interested, you should take a course on the economics of competition (Econ 438 in our department).

11.4 Market Power in Labor Markets

Recall the labor markets we discussed in Section 10.3.3. What if the labor market is not competitive?

- This may be because there is a single seller in the labor market – think of labor unions that has the power to set wages. This would be the **monopoly** in the labor market, and the reasoning we had so far applies.
- But this may also be because there is a single buyer in the labor market – think of a factory that is the sole employer in a small town. The "factory town" was the employment model of mid-century United States. This is called a **monopsony**. This is like a mirror image of a monopoly.

11.4.1 Modeling Monopsony

A monopsonist is, by definition, the only buyer in a labor market. This means the monopsonist **owns** the supply curve. Therefore, the monopolist can choose any wage-employment pair (w, L) as long as they are on the labor supply curve. See Figure 259.1 below.

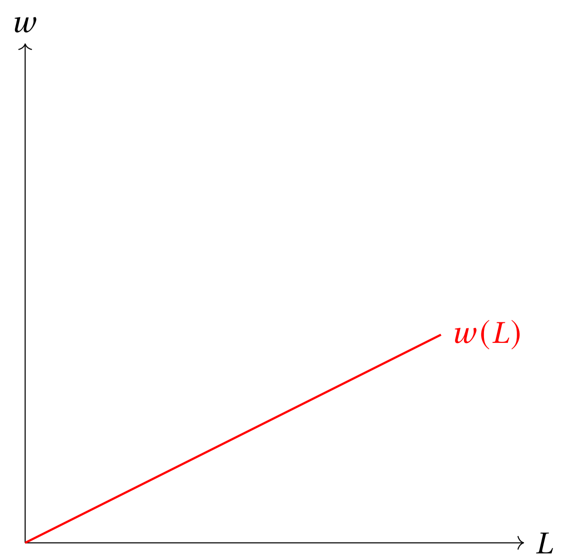


Figure 259.1: The monopsonist can choose any (w, L) pair on the red curve.

11.4.2 A Monopsonist's Marginal Expenditure Curve

For reasons that will be clear in a second, let me define a monopsonist's **total expenditure** as the total wage bill it pays when it hires L units of labor:

$$TE(L) = w(L) \cdot L$$

and the marginal expenditure as the rate of change of total expenditure:

$$\begin{aligned} ME(L) &= \frac{d\,TE(L)}{d\,L} \\ &= w(L) + w'(L)\,L\,. \end{aligned}$$

A monopsonist's marginal expenditure is the sum of two terms. The first term is the usual wage term – if you hire an extra unit of labor, you have to pay them the wage. The second term is there because a monopsonist needs to increase the wage for **every worker** hired. (Wage needs to increase by $w'(L)$ and L units are hired, hence the multiplication.) Since $w(L)$ is increasing, the second term is positive.

Since the second term is positive, we conclude: $ME(L) > w(L)$ for every positive L. In other words, **marginal expenditure curve is above the supply curve at every positive quantity**. See Figure 260.1 below.

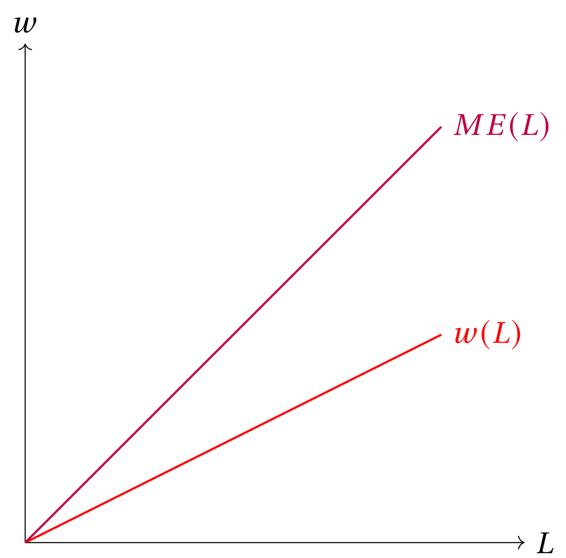


Figure 260.1: The monopsonist's supply and marginal expenditure curves.

11.4.3 A Monopsonist's Choice

A monopsonist chooses a wage-employment pair on the labor spply curve to maximize its profits. Let's call the monopsonist's choice (w^m, L^m). Since this pair is on the demand curve,

$$w^m = w(L^m)\,.$$

But what is L^m? To find it, consider the short-run where capital is fixed at $\bar{K}$. Then, the firm's profit from hiring L units of labor is:

$$\begin{aligned}\pi(L) &= f(L, \bar{K}) \cdot P - w(L) \cdot L \\ &= f(L, \bar{K}) \cdot P - TE(L)\end{aligned}$$

Taking the derivative of $\pi(L)$ and equating to 0 at the optimum L^m yields the equation for L^m:

$$MPL(L^m, \bar{K}) \cdot P = ME(L^m)\,.$$

See Figure 261.1 below.

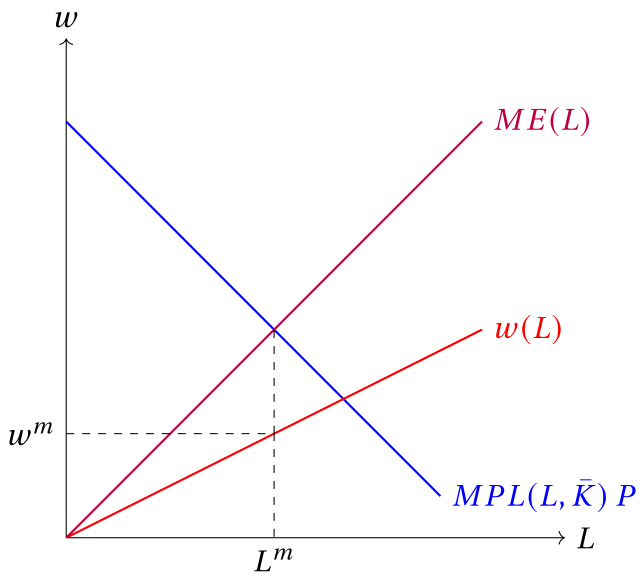


Figure 261.1: The monopsonist's choice (w^m, L^m).

11.4.4 Welfare Cost of Monopsony

The efficient employment level L^* equates the benefits and costs of employment at the margin: $MPL(L^*, \bar{K})\ P = w(L^*)$. On the other hand, monoposnist's employment

L^m satisfies $MPL(L^m, \bar{K})\dot{P} = ME(L^m)$. Because the marginal expenditure curve is above the labor supply curve, the intersection with marginal expenditure occurs at a smaller employment level than the intersection with the supply curve. Therefore, $L^m < L^*$: **the monopsonist hires too few people compared to the efficient outcome**. As a result, there is a deadweight loss. See Figure 262.1.

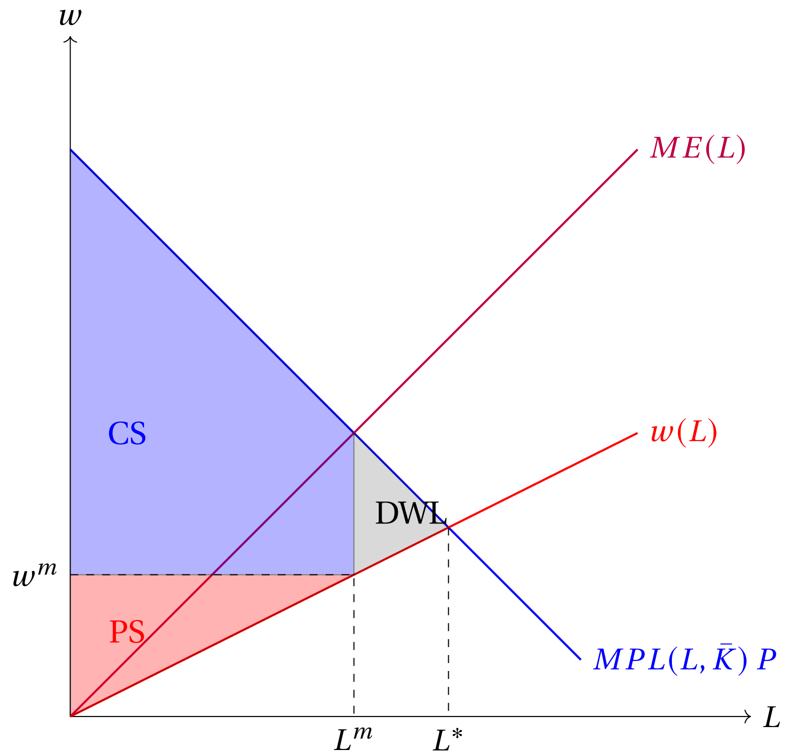


Figure 262.1: Consumer surplus, producer surplus, and deadweight loss under a monopsony.

Let's compare what happens under the monopoly with the competitive market benchmark. If the monopolist was replaced by a competitive supply side with the same marginal cost as the monopolist, we would have a competitive equilibrium at the intersection of demand and marginal cost curves. Then, the quantity would be efficient, $Q^{eq} = Q^* > Q^m$. Moreover, since demand is downward-sloping, $Q^m < Q^*$ implies $P^m > P^*$. In other words, **compared to the competitive benchmark, a monopoly results in too little output and too high prices.**

To wrap up, the key takeaways are: compared to the competitive benchmark,

a. A firm with market power in the labor market hires too few workers.

b. By doing so, it keeps the wages low and profits high, at the expense of workers' welfare.

c. This results in a deadweight loss.

11.4.5 Regulating a Monopsony

Given that the monopsony hires too few workers, policies to encourage the firm to hire more workers (direct quantity regulations that require the firm to produce at least a certain number of units/hire at least L^* workers, subsidies to encourage hiring workers) would work. But once again, they require information and they are politically challenging to impose.

An alternative, widely adapted, policy to alleviate the welfare losses of a monopsony is imposing a **minimum wage**. This would work in the same manner as a price ceiling works for a monopoly: it flattens the firm's marginal expenditure curve. See Figure 263.1.

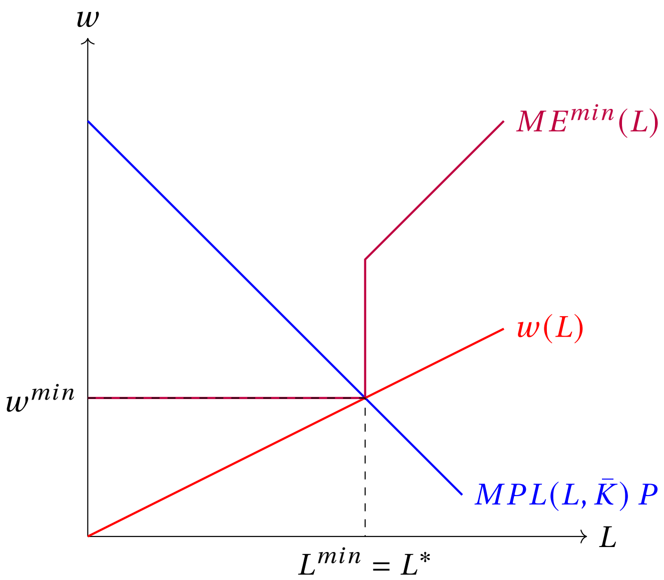


Figure 263.1: The monopsonist's choice (w^{min}, L^{min}) under minimum wage w^{min}.

11.4.6 "It Depends."

In Section 10.3.3, we found out that:

> "In a competitive labor market, minimum wages cause unemployment and welfare losses."

We have now found out that:

> “In a monopsonistic labor market, minimum wages **create employment and welfare gains**.”

I really, really want to you appreciate the drastic change in conclusions based on the assumptions. The simple assumption of price-taking behavior has very important welfare and policy implications.

We want you to take this observation to heart. Economics **does not** say whether minimum wages are good or bad. (To take it a step further; anyone who claims that economics says whether minimum wages are good or bad is not a serious economist.) It only says “under such and such conditions, minimum wages lead to such and such outcomes.” In other words, when someone asks you whether minimum wages are good or bad, you should say: “It depends.”

A common source of disdain for economists is that most of our answers to policy questions are: “It depends.” People have a right to hate this answer, but in my opinion, it is a better answer than giving unequivocal support to some policies without understanding how things work.

Figure 264.1: Harry S. Truman once said: *“Give me a one-handed Economist. All my economists say ‘on one hand...’, then ‘but on the other...’ ”* So relatable, and now you know why.

Speaking of support to policies... Should the government impose a minimum wage? To answer this question, we need to know whether the labor markets are better represented by the competitive model or the monopsony model. So, which model is a better approximation to reality? To be able to fully answer this question, you first need to learn econometrics and then take a course on labor economics (Econ 458).

Extra Readings for Chapter 11

To continue with where we are left off, consider the question: "Are labor markets better approximated by the competitive model, or the monopsony model?"

The proxy question that we can ask is the following: "Does an increase in minimum wage create unemployment or not?" If the answer is yes, this is suggestive that the labor market is competitive. If the answer is no, we may conclude that labor market is non-competitive.

In one of the most famous articles written in economics profession, David Card and Alan Krueger compare the employment in the fast-food industry in two neighboring states, New Jersey and Pennsylvania, after New Jersey increased its minimum wage in 1992. They find that, relative to Pennsylvania, employment in New Jersey did not change significantly, and they conclude that the labor market is not competitive.

> Card, David, and Alan B. Krueger. "Minimum Wages and Employment: A Case Study of the Fast-Food Industry in New Jersey and Pennsylvania." *The American Economic Review* 84, no. 4 (1994): 772-793.

Card and Krueger's study has been incredibly influential on labor economics – there has been a number of studies dedicated to understanding the impact of minimum wages on employment. There seem to be results that lean either way. For a more recent example that follows the Card-Krueger line of showing that minimum wages do not create unemployment, see:

> Cengiz, Doruk, Arindrajit Dube, Attila Lindner, and Ben Zipperer. "The Effect of Minimum Wages on Low-Wage Jobs." *The Quarterly Journal of Economics* 134, no. 3 (2019): 1405-1454.

Exercises for Chapter 11

1) Consider a firm with a total cost function of:

$$\mathrm{TC}(Q) = 40 + 0.1 \cdot Q^2$$

Assume that the market demand for the product of this firm is given by

$$Q(P) = 100 - 10 \cdot P \quad .$$

When the firm is a monopoly, in equilibrium,

a. What is the quantity supplied by the firm, and what would the market price of the good be?

b. What is the profit of the firm in equilibrium?

c. What is the consumer surplus, producer surplus, and deadweight loss in equilibrium?

d. Suppose the government wants to eliminate the deadweight loss of a monopoly by offering a subsidy of B TLper each unit sold. What value of B fully eliminates the deadweight loss?

e. What other government policies can eliminate the deadweight loss of a monopoly?

2) Consider a firm which only uses labor as its input (there is no capital in the production function). The firm operates in a perfectly competitive output market where the price of a unit of output is $P = 5$ TL/unit.

The firm chooses haw many hours of labor to hire. When the firm hires L hours of labor, the output is:

$$f(L) = 10 \cdot L \text{ units.}$$

That is, each hour of labor hired produces 10 units of the good. Note that this implies:

$$MPL(L) = \frac{d}{dL} f(L) = 10 \text{ units.}$$

That is, each hour of labor produces an extra 10 units of output.

Suppose that the labor supply function is given by

$$L(w) = 0.2 \cdot w \text{ hours.}$$

That is, when the hourly wage is w TL/hour, the workers are willing to supply $0.2 \cdot w$ hours of labor. Note that this implies the “inverse labor supply curve” has the equation:

$$w(L) = 5 \cdot L \text{ TL/hours}$$

You can think of this equation as conveying the following information: the opportunity cost of L-th hour of labor supplied is $5 \cdot L$ TL/hours.

a. What is the efficient quantity of labor, i.e. the quantity of labor that maximizes the economic surplus? [Hint: let the efficient quantity of labor be denoted L^{eff}. L^{eff} must be chosen such that the marginal revenue brought by the L^{eff}-th unit of labor is equal to the opportunity cost of L^{eff}-th hour of labor.]

b. Suppose the firm is a monopsonist in the labor market. That is, the firm does not take the wage as given; instead, if the firm hires L units of labor, it needs to set the hourly wage at $w(L)$ TL/hour. What is the firm's total revenue, total expenditure, and profits from hiring L hours of labor?

c. In equilibrium, how many hours of labor does the firm hire? What is the hourly wage? What is the firm's profits?

d. What is the deadweight loss in equilibrium?

e. What government policies can eliminate the deadweight loss of a monopsony?

Chapter 12

Externalities and Coase Theorem

In Chapter 10, we discussed why free, competitive markets are "good". This was because they maximize the *gains from trade*. To recap, the gains from trading Q-th unit was:

$$MB(Q) - MC(Q) .$$

- Here, $MB(Q)$ was the benefit accrued to the consumer who consumes the Q-th unit. These are related to the consumer's preferences – which may be about anything, but a preference is something the consumer cares about. To emphasize that the marginal benefits accrue to the particular consumer and no one else, we will call these *private marginal benefits* ($PMB(Q)$). In other words, throughout this chapter, we will posit that the demand curve is the $PMB(Q)$ curve.
- Similarly, $MC(Q)$ was the cost accrued to the producer who sells the Q-th unit. These costs are things that the particular producer cares about. To emphasize that the marginal costs accrue to the particular producer and no one else, we will call these *private marginal costs* ($PMC(Q)$). In other words, throughout this chapter, we will posit that the supply curve is the $PMC(Q)$ curve.

Notably, $PMB(Q) - PMC(Q)$ is calculated according to the private benefits and private costs of agents involves in the transaction. If no one else is affected by the transaction, this is a good measure of gains from trade. But what if other agents are affected by the transaction? This would correspond to an existence of externality.

Definition 269.1 *There is* **externality** *due to an economic activity if:*

a. an economic agent is affected by the activity she is not directly involved in, and, *b. the agents involved in the activity does not take into account how others are affected.*

Less formally, there is an externality if "bystanders" are affected by an activity, and those engaging in activity does not care about the bystanders.

In this chapter, we will show that: when there are externalities, competitive markets do not lead to desirable outcomes. In other words, there is **market failure** in the presence of externalities. Consequently, there is room for government intervention. We will explore what externalities are, how they affect welfare, and how they can be corrected.

12.1 Types of Externalities

Let's begin by discussing various forms of externalities.

12.1.1 Positive vs. Negative Externalities

An externality can be positive or negative!

There are **negative externalities** when an economic activity harms others. Examples:

- Pollution (when a factory releases toxins in the air and does not care about how others are affected),
- Smoking (when a smoker does not care about its impact on people around them or on the public health system),
- Cheating in an exam (when a student does not care that the grade distribution will be distorted and others are negatively affected),
- Systemic risk (when a bank takes too much risk and creates a risk of financial crisis),
- ...

There are **positive externalities** when an economic activity benefits others. Examples:

- Immunization (when an individual takes a vaccine it also helps others by reducing the risk of contagion),
- Wearing a face mask when sick,

- Having a good education (when the diploma earner also benefits others by engaging in entrepreneurial activities/creating jobs),
- Playing piano with windows open (if one plays the piano well)
- ...

The key point to notice in all these examples are: the economic activity also has a cost/benefit on the economic agent who engages şn the activity. For instance, smoking imposes a cost on the smoker (personal health risks, the smell). Taking risks already increases the risk of bankruptcy for the risk-taker. Good education directly provides a benefit to the receiver of that education. Those direct effects are already taken into account when the agent does that activity (i.e., they are included in the PMB and PMC). What matters is that there is an *external effect* that the decision makes *does NOT internalize*. As long as there is such a situation, there is an externality.

12.1.2 Externalities due to Consumption vs. Production

In the market setting we consider here, an economic activity (trade) involves two parties (consumer and producer).

We say that there is an *externality due to consumption* if the activity that creates externality is consumption. Examples:

- When a smoker buys a pack of cigarettes from Phillip Morris, it is the consumption activity that creates a negative externality.
- When a student receives a diploma from Bilkent, it is the act of the student that creates a positive externality.

We say that there is an *externality due to production* if the activity that creates externality is production. Examples:

- When I buy a product from a company that pollutes the environment, it is the production activity that created a negative externality.
- When a construction company buys an abandoned building, flips it and sells it, the neighbors enjoy a positive externality due to production due to a rise in the value of their own homes.

12.1.3 Conceptualizing Externalities

Recall that $PMB(Q) - PMC(Q)$ is a measure of the benefits and costs of the buyer and seller directly involved in the transaction. When there are externalities, others are also affected by this transaction. As a society, we care about the collective costs and benefits (i.e., the costs and benefits of the *society*, which involves the buyer and seller but also the others affected). Let the society's benefit from the consumption of Q-th unit be: $SMB(Q)$ ("social marginal benefit"). Let the society's cost due to the production of Q-th unit be $SMC(Q)$ ("social marginal cost"). Then, the gains from trading the Q-th unit is:

$$SMB(Q) - SMC(Q)$$

There are **externalities** whenever:

$$\underbrace{SMB(Q) - SMC(Q)}_{\text{what the society cares about}} \neq \underbrace{PMB(Q) - PMC(Q)}_{\text{what the buyer and seller care about}}$$

There are **negative externalities** when:

$$SMB(Q) - SMC(Q) < PMB(Q) - PMC(Q)$$

and there are **positive externalities** when:

$$SMB(Q) - SMC(Q) > PMB(Q) - PMC(Q) .$$

We can now return to the taxonomy of externalities:

a. There is a **negative externality due to consumption** when:

$$SMB(Q) < PMB(Q)$$

The difference between the social marginal benefit and private marginal benefit is the negative impact of consumption on others, not taken into account by the consumer. We will call this the **external marginal benefit**, $EMB(Q) = SMB(Q) - PMB(Q)$. Whenever there are negative externalities due to consumption, $EMB(Q)$ is negative:

$$EMB(Q) = SMB(Q) - PMB(Q) < 0 .$$

b. There is a **negative externality due to production** when:

$$SMC(Q) > PMC(Q)$$

The difference between the social marginal cost and private marginal cost is the negative impact of production on others, not taken into account by the producer. We will call this the **external marginal cost**, $EMC(Q) = SMC(Q) - PMC(Q)$. Whenever there are negative externalities due to production, $EMC(Q)$ is positive:

$$EMC(Q) = SMC(Q) - PMC(Q) > 0 .$$

c. There is a **positive externality due to consumption** when:

$$SMB(Q) > PMB(Q)$$

Whenever there are positive externalities due to consumption, $EMB(Q)$ is positive:

$$EMB(Q) = SMB(Q) - PMB(Q) > 0\,.$$

d. There is a **positive externality due to production** when:

$$SMC(Q) < PMC(Q)$$

Whenever there are positive externalities due to production, $EMC(Q)$ is negative:

$$EMC(Q) = SMC(Q) - PMC(Q) < 0\,.$$

12.2 Negative Externalities: Overproduction

Because the society cares about $SMB(Q) - SMC(Q)$, the **efficient quantity** Q^* satisfies:

$$SMB(Q^*) = SMC(Q^*)$$

The traded quantity in a competitive market, Q^{eq}, satisfies:

$$PMB(Q^{eq}) = PMC(Q^{eq})$$

When there are no externalities, $SMB(Q) - SMC(Q) = PMB(Q) - PMC(Q)$ and thus $Q^{eq} = Q^*$. Market produces the efficient quantity, as we have seen. But what if there are externalities? Clearly, $Q^{eq} \neq Q^*$.

If there are negative externalities, we will show that $Q^{eq} > Q^*$. In words, the market *overproduces.* Intuitively, this is because the market participants do not internalize the negative impact on others and end up overdoing an activity, compared to the socially desirable level.

12.2.1 Negative Externalities due to Consumption

When there are negative externalities due to consumption, $SMB(Q) < PMB(Q)$. That is, the social marginal benefit curve is below the private marginal benefit (demand) curve. The result is $Q^{eq} > Q^*$ and a deadweight loss, as seen in Figure 274.1.

Basically, left to market forces, people end up smoking too much.

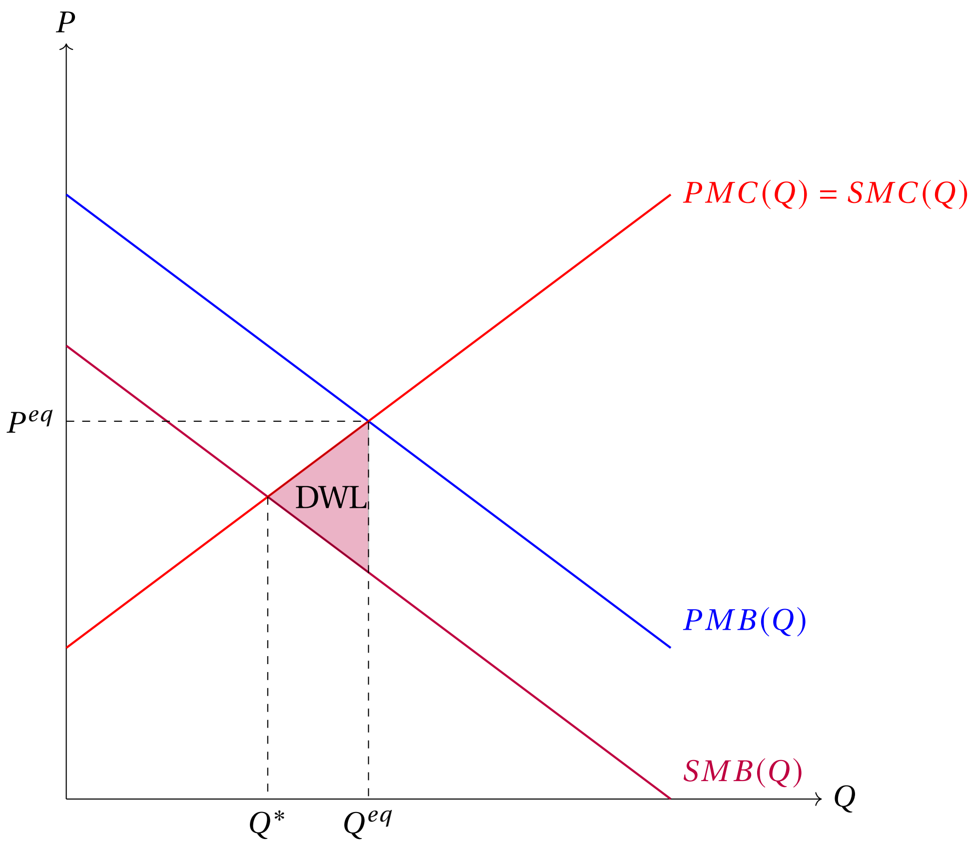


Figure 274.1: Overproduction and deadweight loss when there are negative externalities due to consumption.

12.2.2 Negative Externalities due to Production

When there are negative externalities due to production, $SMC(Q) < PMC(Q)$. That is, the social marginal cost curve is above the private marginal cost (supply) curve. Once again, this results in $Q^{eq} > Q^*$ and a deadweight loss, as seen in Figure 275.1.

Basically, left to market forces, factories end up polluting the environment too much.

12.2.3 Government Responses to Negative Externalities

We have seen that a negative externality results in too much of a bad thing. This means the government can intervene and fix things. Here are some government interventions that can alleviate the issues due to negative externalities.

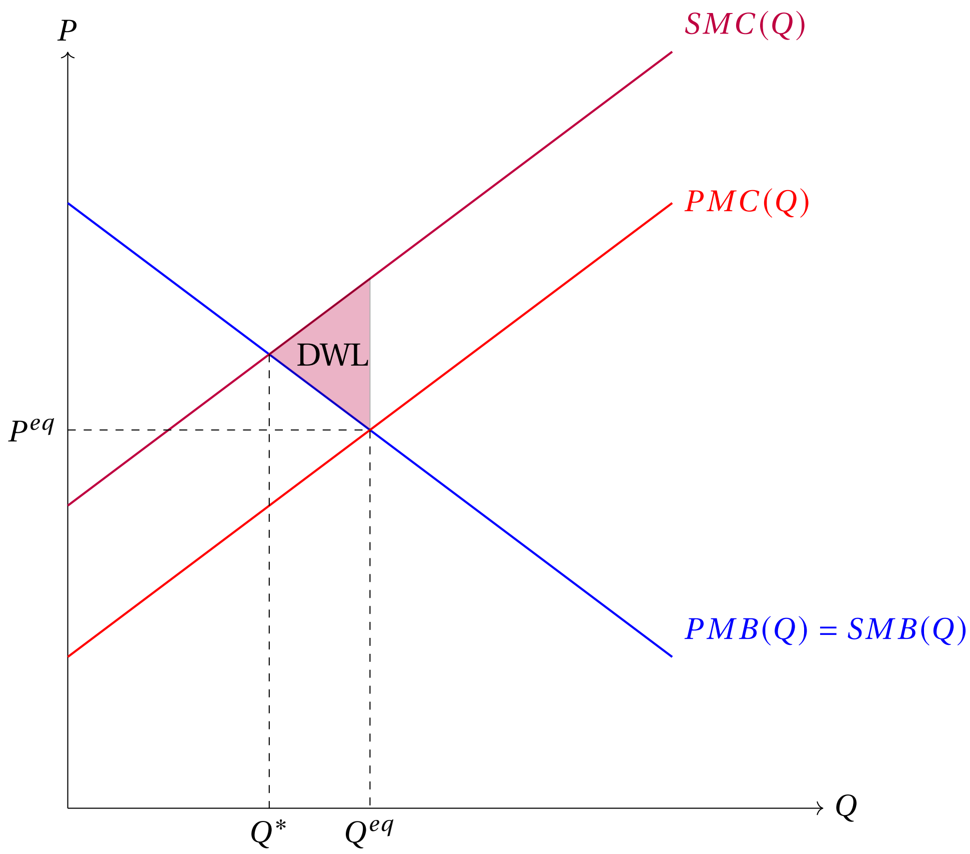


Figure 275.1: Overproduction and deadweight loss when there are negative externalities due to production.

Quantity Regulations: Quotas

The idea is simple: the government can simply ban the trade of the good beyond Q^*.
Example: quotas that limit carbon emissions.

Price Regulations: Pigouvian Taxes

Rather than regulating the quantities, the government can also intervene by working through the prices. The key idea here is: whenever there is a negative externality, there is overproduction: the government would like to reduce the activity. As we already know, a good way to reduce the activity is *taxing it*. Thus, the government could get rid of the deadweight loss by taxing the harmful activity. This revelation occurred to British economist Arthur Pigou (1877-1959), and hence these type of taxes designed for correcting externalities are called **Pigouvian taxes**. They take the form of sales taxes.[1] You may ask: but should the sales tax be imposed on the consumer or the producer? Honestly, if you ask this question I will be very disappointed. From Section 10.3, you should know that it does not matter. For practical purposes, we will imagine that when the negative externality is due to consumption, the taxes are imposed on the consumer (and when the negative externality is due to production, the taxes are imposed on the producer). But it does not matter, we could have taxed the opposite side and we would have identical results.

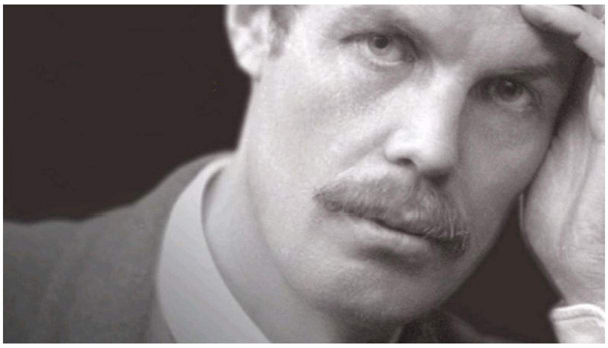


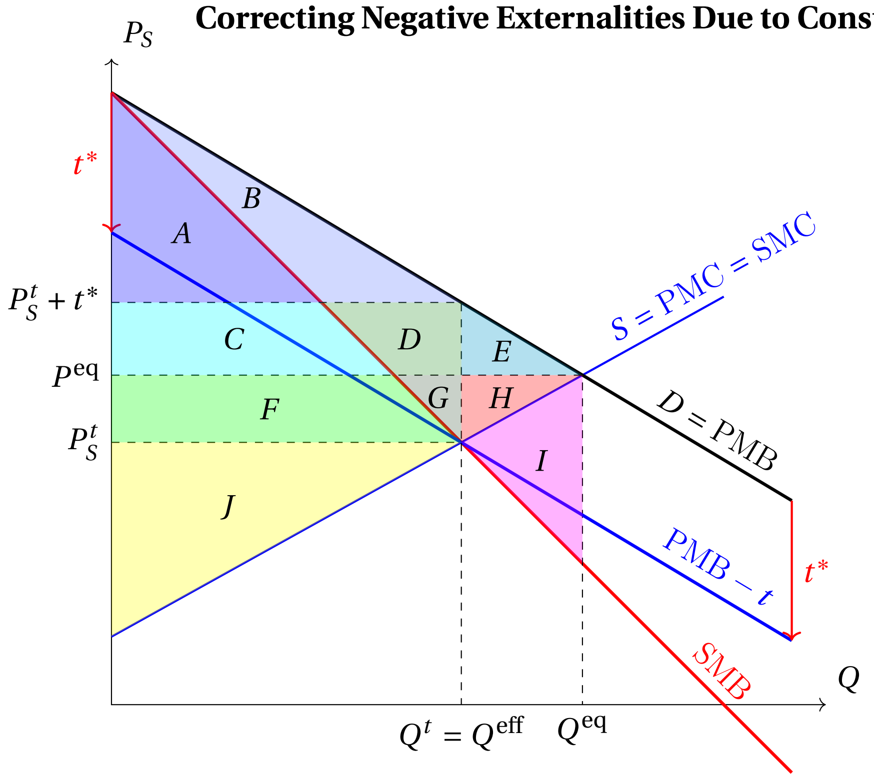
Figure 276.1: Arthur Pigou after seeing you smoking all day, when you think that Phillip Morris pays the sales tax and not you.

Examples: many sales taxes are indeed designed to correct negative externalities. Taxes on tobacco and alcohol are prime examples. Another example is the 0.25 TL you have to pay for every plastic bag you get from Migros.

Why do Pigouvian taxes work? Intuitively, suppose an agent imposes a negative externality of E TL on others by engaging in an activity. If the government requires that the agent pays a tax of E TL whenever the agent engages in that activity, the government makes sure that the agent now *internalizes* the externality. This logic also reveals that corrective taxes work because they are *punishments* on an activity: they make you reconsider whether the activity is worth doing, i.e., whether your private marginal benefit minus the tax is high enough. They do not have to be monetary punishments, they can also be physical punishments. **Example:** think of the designated smoking areas in Bilkent University. It imposes a cost on smoking by making you walk for 5 minutes whenever you want to smoke. Now, you have to consider whether you value the cigarette enough every time you go out for a smoke. This is a form of corrective tax on smoking.

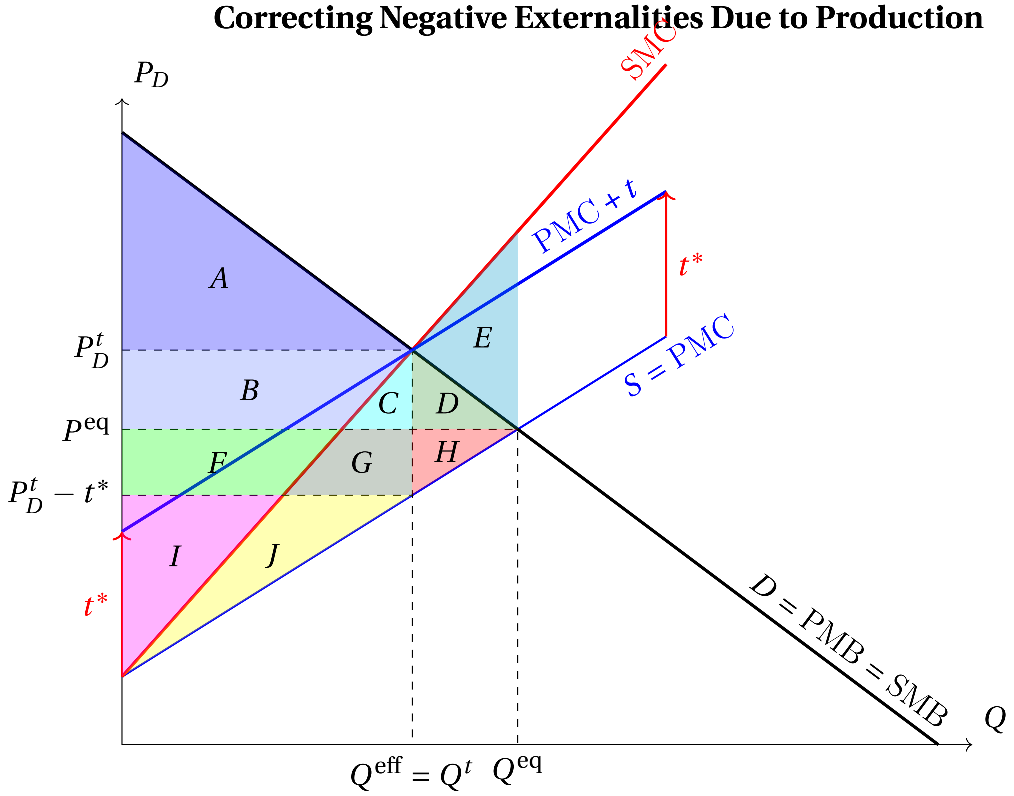
[1]Generally speaking, there are two economically solid justifications for taxation. The first one is *redistribution* – transferring resources from the rich to the poor is a welfare improving activity. Income taxes are a good way to achieve this objective. The second one is *correcting externalities*, which is what we cover here. Sales taxes are a good way to achieve this objective. If you are interested in the intricacies of designing tax systems, you should take a course on public finance.

Correcting Negative Externalities Due to Consumption



	Without Taxation	With Taxation
Price consumers face	P^{eq}	$P_D^t = P_S^t + t^*$
Price producers face	P^{eq}	P_S^t
Quantity traded	Q^{eq}	$Q^t = Q^{eff}$
CS	$A+B+C+D+E$	$A+B$
PS	$F+G+H+J$	J
Cost on "others" due to externality	$B+D+E+G+H+I$	$B+D+G$
Tax Collected	0	$C+D+F+G$
Economic Surplus = CS + PS − Cost on others + Tax collected	$A+C+F+J-I$	$A+C+F+J$
Maximum Economic Surplus	$A+C+F+J$	$A+C+F+J$
DWL	I	0

Correcting Negative Externalities Due to Production



	Without Taxation	With Taxation
Price consumers face	P^{eq}	P_D^t
Price producers face	P^{eq}	$P_S^t = P_D^t - t^*$
Quantity traded	Q^{eq}	$Q^{\text{eff}} = Q^t$
CS	$A + B + C + D$	A
PS	$F + G + H + I + J$	$I + J$
Cost on "others" due to externality	$J + G + C + H + D + E$	$J + G + C$
Tax Collected	0	$B + C + F + G$
Economic Surplus = CS + PS − Cost on others + Tax collected	$A + B + F + I - E$	$A + B + F + I$
Maximum Economic Surplus	$A + B + F + I$	$A + B + F + I$
DWL	E	0

12.3 Positive Externalities: Underproduction

If there are positive externalities, $Q^{eq} < Q^*$: the market *underproduces*. Intuitively, this is because the market participants do not internalize the positive impact on others and end up underdoing an activity, compared to the socially desirable level.

12.3.1 Positive Externalities due to Consumption

When there are positive externalities due to consumption, $SMB(Q) > PMB(Q)$. That is, the social marginal benefit curve is above the private marginal benefit (demand) curve. The result is $Q^{eq} < Q^*$ and a deadweight loss, as seen in Figure 279.1.

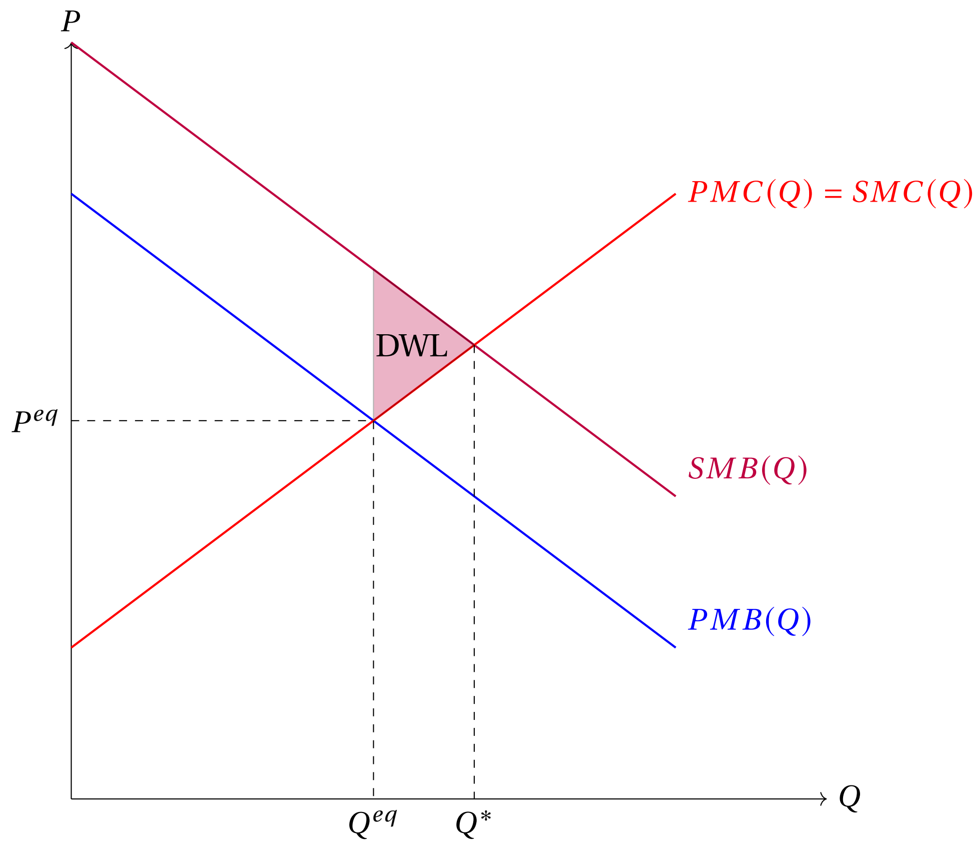


Figure 279.1: Underproduction and deadweight loss when there are positive externalities due to consumption.

Basically, left to market forces, people end up receiving too little education.

12.3.2 Positive Externalities due to Production

When there are positive externalities due to production, $SMC(Q) < PMC(Q)$. That is, the social marginal cost curve is below the private marginal cost (supply) curve. Once again, this results in $Q^{eq} < Q^*$ and a deadweight loss, as seen in Figure 280.1.

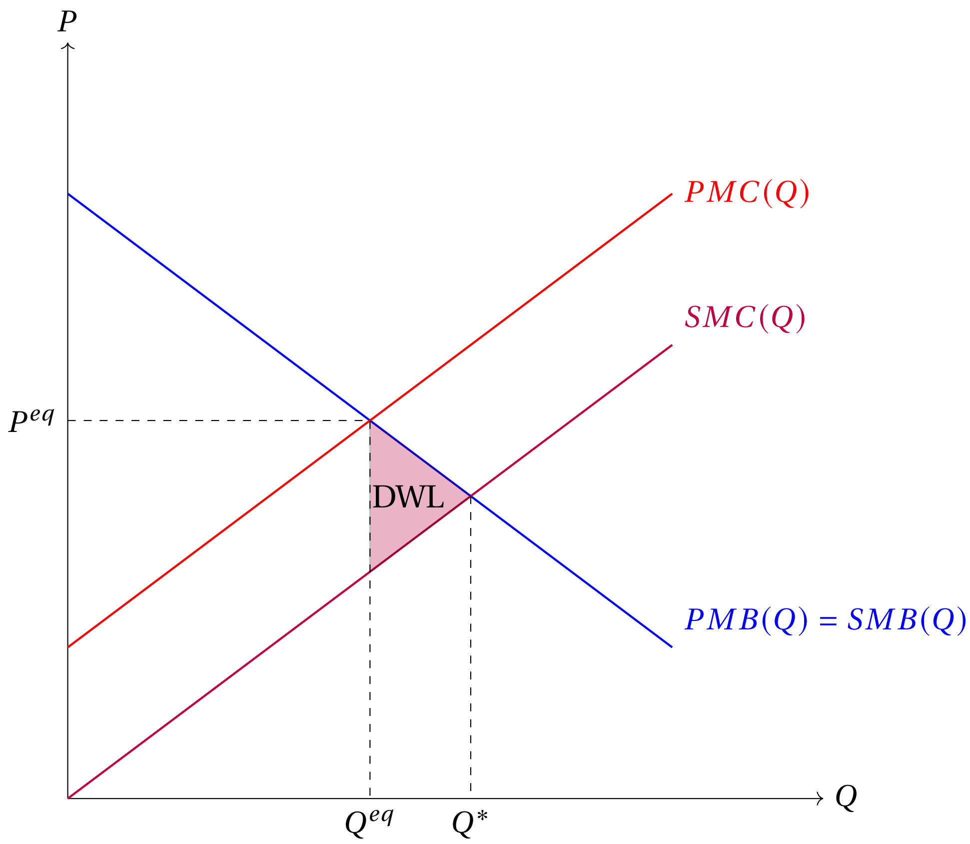


Figure 280.1: Underproduction and deadweight loss when there are positive externalities due to production.

12.3.3 Government Responses to Positive Externalities

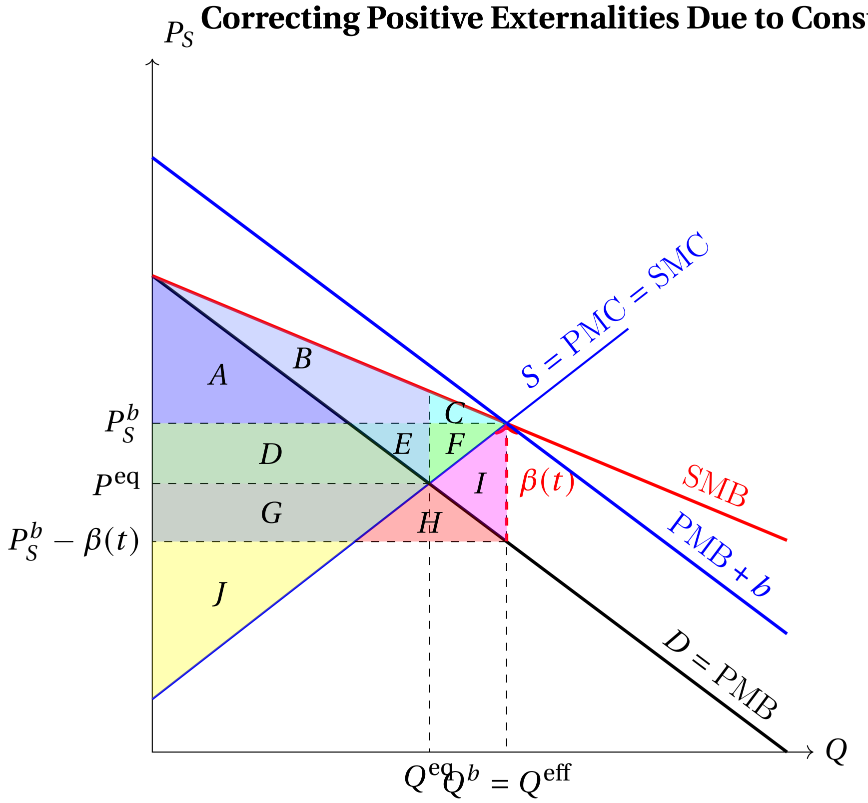
Quantity Regulations

The government may require that people consume at least Q^* units. **Example:** Minimum education requirements.

Price Regulations: Pigouvian Subsidies

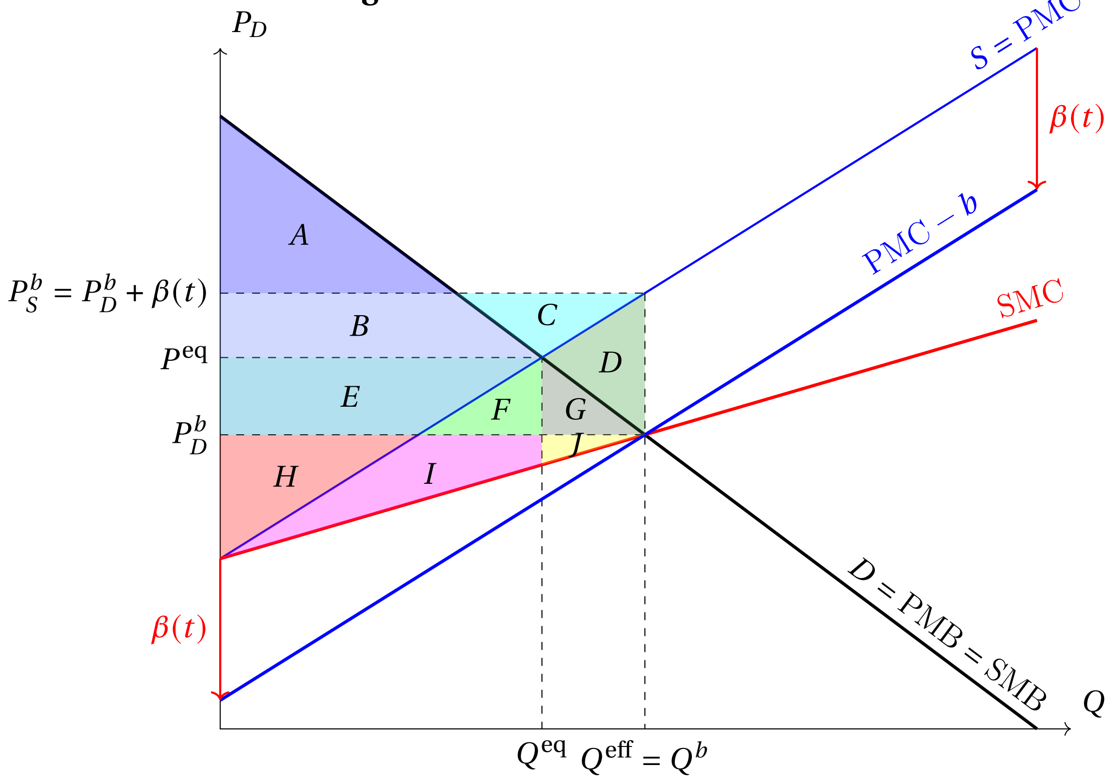
Just like the Pigouvian tax in the case of negative externalities, a Pigouvian subsidy works by making sure that agents internalize the positive externality they impose on others. Intuitively, if you are doing a good thing, government should encourage you.

Correcting Positive Externalities Due to Consumption



	Without Subsidy	With Subsidy
Price consumers face	P^{eq}	$P_D^b = P_S^b - \beta(t)$
Price producers face	P^{eq}	P_S^b
Quantity traded	Q^{eq}	$Q^b = Q^{\text{eff}}$
CS	$A+D$	$A+D+G+H$
PS	$G+J$	$D+E+F+G+J$
Benefit of "others" due to externality	$B+E$	$B+C+E+F+I$
Subsidy distributed by government	0	$D+E+F+G+H+I$
Economic surplus = CS + PS + Benefit of others − Subsidy distributed	$A+D+G+J+B+E$	$A+B+C+D+E+F+G+J$
Maximum economic surplus	$A+B+C+D+E+F+G+J$	$A+B+C+D+E+F+G+J$
DWL	$C+F$	0

Correcting Positive Externalities Due to Production



	Without Subsidy	With Subsidy
Price consumers face	P^{eq}	P_D^b
Price producers face	P^{eq}	$P_S^b = P_D^b + b$
Quantity traded	Q^{eq}	$Q^{\text{eff}} = Q^b$
CS	$A + B$	$A + B + E + F + G$
PS	$E + H$	$B + C + E + H$
Benefit of “others” due to externality	$F + I$	$D + F + G + I + J$
Subsidy distributed	0	$B + C + D + E + F + G$
Economic surplus = CS + PS + Benefit of others − Subsidy Distributed	$A + B + E + F + H + I$	$A + B + E + F + G + H + I + J$
Maximum economic surplus	$A + B + E + F + G + H + I + J$	$A + B + E + F + G + H + I + J$
DWL	$G + J$	0

12.4 Coase Theorem: A Market-Based Solution to Externalities

The whole discussion about government responses to externalities brings another idea to mind. Suppose there is an externality. That means: there is a bystander in the society who is willing to pay the agent to engage/not engage in an activity. Why not let these two (the agent and the bystander) sort it out between themselves?

Example 284.1 I happen to have a 1.5 year-old daughter who *really* needs to get a good sleep through the night. While we all enjoy Ramadan, the sultan of 11 months presents a challenge on us: the friendly neighborhood drummer would like to pass by the street at 3am with the explicit purpose of waking us all up. This is clearly a case of negative externality. Rather than expecting the state to tax the drummer, why don't I just go to the drummer and make a payment, asking him to please not pass through our street?

A "market-based" solution to externalities considers the root of the issue to be a "missing markets" problem. There is no market that allows us to trade the rights to play a drum in the streets at 3am. Because we cannot trade drumming rights, we cannot reach an agreement. Solution: open a market and let people trade drumming rights.

So far so good, but there are two issues that need to be addressed:

a. How can we be sure that the agents will reach a "desirable" outcome, once we allow them to trade drumming rights? Here, let a "desirable" outcome be defined as an efficient outcome. So, how do we know the agents will reach an efficient agreement?

b. Who is going to pay who – should I pay the drummer to stop him from drumming, or should the drummer pay me to receive permission to play his drum?

It turns out the two questions can be answered simultaneously: (under some conditions,) the agents will reach an efficient agreement no matter who owns the initial rights.

Example 284.2 Continuing with the example of Ramadan drummer, let:

- $v > 0$ denote the Ramadan drummer's benefit from playing the drum (his profits etc.),
- $c > 0$ denote the negative externality he imposes by playing the drum in my

street (this is a measure of how much I am willing to pay for him to stop playing the drum).

If $v-c \geqslant 0$, the efficient outcome is drumming: the drummer's value is higher than the externality it imposes; so the efficient thing to do from a social perspective is letting him drum. If $v - c < 0$, the efficient thing from a social perspective is no drumming.

- Suppose the drummer owns the right to drum but I am not allowed to make a payment (i.e., the drumming rights are not tradable). Because $v > 0$, he will always choose to play the drum, even when $v-c < 0$. So, we may get the inefficient outcome.

  However, if the drumming rights are tradable, the situation will change.

  - If $v - c \geqslant 0$, it was efficient for him to drum anyway.
  - If $v-c < 0$, we will be able to reach an agreement: I will propose to pay some price p, between v and c, to stop him from drumming. Because $p > v$, the drummer is happy to take the payment and stop playing. Because $p < c$, I am happy to make the payment.

  In either case, efficiency restored!

- Remarkably, the same logic applies if I own the drumming rights.

  Suppose I own the drumming rights but I the drummer is allowed to make a payment to me. Because $c > 0$, I will never allow the drummer to play, even when $v - c \geqslant 0$. So, we may get the inefficient outcome.

  However, if the drumming rights are tradable:

  - If $v - c < 0$, it was efficient for him not to drum anyway.
  - If $v - c \geqslant 0$, we will be able to reach an agreement: he will propose to pay some price p, between c and v, for me to allow him to drum. Because $p < v$, the drummer is happy to make the payment. Because $p > c$, I will accept the payment. (No worries, I will buy my daughter some noise-canceling headphones with that money.)

  In either case, once again, efficiency restored!

We have observed that the agents will reach an efficient agreement. This is true regardless of who has the initial rights. The ownership of rights changes who pays who, but it does not change the outcome.

The initial rights are referred to as **property rights**: technically speaking, these are the exclusive privileges to use an asset in any way (including transferring them to another party in exchange of some payment).

The observation we have just make (agents reach an efficient outcome, no matter who owns the property rights) is of course not novel. It was first observed by British economist **Ronald Coase** (1910-2013), one of the most influential figures in 20th century economics. A professor of mine once mentioned that "Coase is famous for eight different things", one of which is the Coase Theorem.

Theorem 286.1 [Coase Theorem] *If*

1. *property rights are clearly specified, and,*
2. *there are no transaction costs,*

the outcome will be efficient no matter how the rights are allocated.

Figure 286.1: Ronald Coase signing a contract to transfer his 1991 Nobel Prize in Economics to some other party – WHICH HE CAN, BECAUSE HE OWNS THE PROPERTY RIGHTS.

Coase Theorem has been influential in arguing the necessity of two factors in achieving efficient outcomes:

1. **Property rights matter.** In order to achieve efficient outcomes, the property rights (who owns the asset, and who need to make the payment to who) needs to be clearly specified.
2. **Transaction costs matter.** The term "transaction cost" is an umbrella term that contains many things, including:
    - Literal costs of writing the contract and reaching agreement – costs of paper and pen, legal work etc.,

- Imperfect enforcement of contracts (if we are not confident that the agents will obey the contract, we may not reach an agreement even though it is efficient),
- Credit constraints (if the party who values the asset more lacks the resources and unable to borrow, she will not be able to receive it),
- Information asymmetries (if people do not know each other's valuations, they may not be able to reach an efficient agreement),
- ...

12.4.1 Interpreting Coase Theorem (and Econ 101) Correctly

Some people interpret the Coase Theorem as saying:

> "Open a market, and economic agents will trade until they reach the efficient outcome."

Those people end up being religious believers of markets.

This, in my opinion, is NOT the correct interpretation of Coase Theorem. I interpret Coase Theorem as saying:

> "In real life, oftentimes, we fail to reach the efficient outcome. Why might that be? The answer is: either property rights are not clearly defined, or there are transaction costs."

I don't think I am alone in my interpretation. In his famous book "Putting Auction Theory to Work" (p.20), Paul Milgrom (winner of 2020 Nobel Prize in Economics) says:

> "The 'zero transaction cost' assumption on which the Coasian argument is based, however, is not one that Coase ever advocated as a description of reality. Rather, it was advanced as part of a thought experiment to emphasize the importance of understanding actual transaction costs."

So, rather than treating Coase Theorem as The Answer to the Ultimate Question of Life, the Universe, and Everything, you should really treat it as a jumping point to bigger and better things.

Let me leave you with an important remark: your general attitude towards Econ 101 should be similar.

- We did NOT show you a bunch of results that say: markets are amazing, leave it to markets.

- Rather, we showed you a **model** that says: under such and such conditions, markets work like this.

You should take this model and use it to think about why markets in real life do not work like this. Some assumption we made along the way must be violated. Which one?

This is where the fun part begins. This is also where we leave you to the cozy, comfortable arms of Econ 102.

Extra Readings for Chapter 12

For the original article where Cose formulated his famous theorem, see:

> Coase, Ronald H. "The Problem of Social Cost." *The Journal of Law and Economics* (1960).

This should be an interesting experience: Coase does not use any math in the paper, which is unthinkable nowadays (this paper is one of the last few examples of economics research before Paul Samuelson came in and swept everything).

A relevant observation about Coase Theorem is that it states the agent *will* reach an efficient outcome; it does not state *how long* it will take to reach the efficient outcome. To see how long it may take, see:

> Bleakley, Hoyt, and Joseph Ferrie. "Land Openings on the Georgia Frontier and the Coase Theorem in the Short- and Long-Run." *Working Paper* (2013).

The idea of the paper is very interesting: when the United States settled in Georgia, they allocated the land almost randomly to farmers. Coase Theorem says: no biggie, better farmers value the land more anyway and would be willing to pay the initial owners, so the land will end up in the hands of better farmers (efficient outcome). The authors show that: yes, ultimately we end up with efficient allocation, but it takes 150 years to reach there!

Final note: taking Coase Theorem too seriously may lead to a state of mind towards "marketizing" everything. Be wary of this mindset. There are MANY, MANY dangers of relying on markets to handle interactions. Beyond the economic concerns of market failures (which you will study a lot), sociologists and philosophers have developed a wide body of literature that discusses the perils of commercializing interactions. For two fantastic examples, see:

> Satz, Debra. *Why Some Things Should Not Be for Sale: The Moral Limits of Markets.* Oxford University Press, 2010.

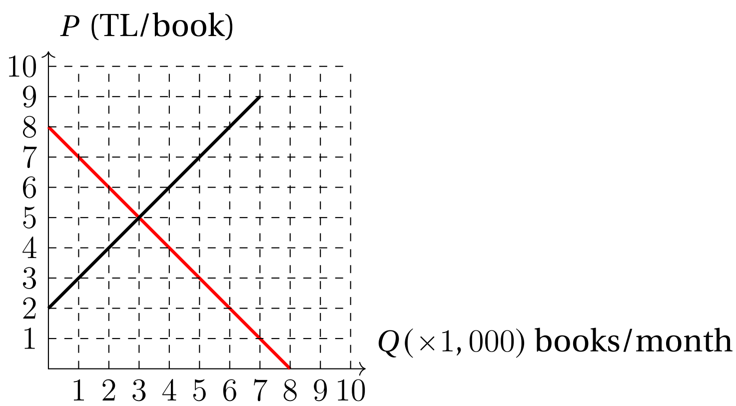
and

> Sandel, Michael J. *What Money Can't Buy: The Moral Limits of Markets.* Macmillan, 2012.

Figure 290.1: Debra Satz and Michael Sandel, waiting patiently to save you from Econ 101 indoctrination.

Exercises for Chapter 12

1) The demand and supply for books is shown in the figure below.



Reading (consuming) books also has a positive externality on the society, which is given as 2 TL/book.

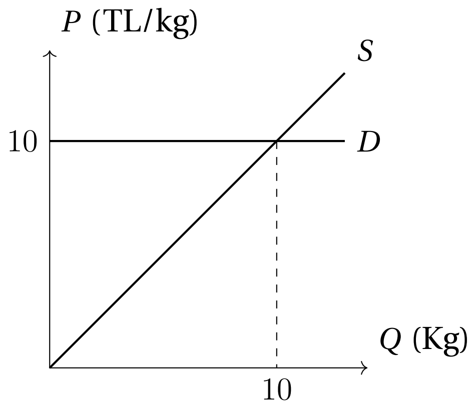
a. What is the free (unregulated) market competitive equilibrium price and quantity? Explain.

b. What is the socially efficient quantity? Explain.

c. What is the dead-weight loss and economic surplus at the free market equilibrium?

d. What kind of policies can be used to move from the free market equilibrium to socially efficient outcome?

2) Consider a market with a perfectly elastic demand, D, and a supply, S, shown in the figure below:



Answer the following questions, assuming that production of the good induces a 2 TL/kg external cost to the society.

a. What is the free-market competitive equilibrium price and quantity?

b. What is the socially efficient quantity?

c. On the picture above add the graph which shows the social marginal cost and shade the area corresponding to dead-weight loss due to externality. Calculate the value of the dead-weight loss.

d. What kind of policies can be used to move from the free market equilibrium to socially efficient outcome?

e. What is the equilibrium price that the consumers face, the equilibrium price that the producers face, the equilibrium quantity under the policy that you have suggested? What is the consumers' surplus the producers' surplus, economic surplus, and the cost to "others" due to externality under the policy you suggested?

3) Consider a market where the market supply is given by the function $P_S = 10 + 1.5Q_S$ and the market demand is given by $P_D = 80 - 2Q_D$.

a. Find the market equilibrium price and quantity.

b. Assume that due to production there is a negative externality such that the marginal cost to society, due to externality, of the Qth unit produced is $1.5Q$.

   (a) Find the social marginal cost function.

   (b) Find the socially efficient quantity and the maximum economic surplus that can be attained taking the externality into account.

   (c) Find the dead-weight loss.

   (d) Can the government, by intervening to the market, reduce the dead-weight loss to zero? If so, how?