

Bilkent University
Econ 101 - Fall 2022
Chapter 3: Equilibrium in an Exchange Economy

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1 Introduction

In the previous chapter, we analyzed the choice made by a consumer in isolation (i.e., there are no other consumers). Now, we will put two consumers together and see what happens.

I should note that this is a chapter that most introductory economics textbooks do not cover. (Make no mistake, however; what you are about to see contains some of the most canonical ideas of economics, and you *will* encounter them in the future.) So you will have to rely on the notes we posted here.

“Why are we covering this?” you might ask. Here are some answers:

1. We want you to get a sense of *equilibrium* as early as possible.
2. In the consumer theory section, we took the prices as given. The model analyzed in this section will give you a sense of how the prices are determined, and the role they play in equilibrium.
3. This seems like a good time to draw a distinction between *positive* and *normative* analysis – which we will do.
4. You will need this in Econ 102. (Always a great reason.)

Anyway, without further ado, here is a description of an exchange economy.

2 An Exchange Economy

In this section, I will describe an **exchange economy**. The reason why there is an emphasis on *exchange* is: there is no *production* taking place in this economy. Each consumer in this economy owns some goods, and they just exchange them. Where do these goods come from? At this stage, just imagine they fall from sky. People have some goods to begin with, and they may be willing to exchange them. For the sake of a motivation, you may imagine a bazaar where every agent has her own stand. Some agents bring their old CDs, some bring their old DVDs, and they set up their own stands. Each agent can go to another stand and may exchange CDs for another agent's DVDs.

To keep the matters simple, we will analyze a simplified model where there are only two agents and two goods. It is good to keep in mind that the conclusions we derive here will generalize to cases with more than two agents and more than two goods. But that general case becomes too intractable and is definitely beyond the scope of undergraduate economics.

So, there are two agents and two goods. Each agent comes to market with a bundle. Another way to say this is: *each agent is endowed with a bundle*.

For convenience, let me refer to agent 1 as **Robinson** and agent 2 as **Friday**. Similarly, let me refer to good 1 as **Apple** and good 2 as **Banana**. This will help us remember the storyline of our exchange economy:

Suppose there are only two agents on an island: Robinson and Friday. The island is divided into two: Robinson's side and Friday's side. There are only two types of trees on the island: Apple trees and Banana trees. Every agent owns the trees (and hence apples and bananas on these trees) on their side of the island.

Suppose Robinson is endowed with e_A^R apples and e_B^R bananas at the beginning. That is, Robinson has an initial endowment, which is a bundle $e^R = (e_A^R, e_B^R)$. Similarly, Friday's endowment is $e^F = (e_A^F, e_B^F)$.

The total number of apples on the island is e_A , i.e., $e_A = e_A^R + e_A^F$. Similarly, the total number of bananas on the island is e_B , i.e., $e_B = e_B^R + e_B^F$.

Here is the catch, though: Robinson and Friday **do not** have to consume their endowments. Instead, they can go on to exchange some apples and bananas. That is, their consumption bundles do not have to be the same as their endowments. After all, we are in an exchange economy!

You may ask: how do we represent consumption bundles? The answer is: the same way we represented consumption bundles in consumer theory. A consumption bundle for Robinson is $q^R = (q_A^R, q_B^R)$, where q_A^R is the quantity of apples Robinson consumes, and q_B^R is the quantity of bananas Robinson consumes. Just as in the consumer theory, Robinson has preferences over q^R 's. Let these preferences be denoted by \mathcal{R}^R . Throughout this section, we will make the assumptions that preferences are well-defined (complete, transitive, monotonic), preferences are smooth and satisfy diminishing marginal rate of substitution.

Similarly, Friday's consumption bundle specifies the quantity of apples Friday consumes and the quantity of bananas Friday consumes, i.e., $q^F = (q_A^F, q_B^F)$. Friday has preferences over q^F 's, denoted by \mathcal{R}^F , and the preferences satisfy the usual assumptions.

An **allocation** q is a pair of consumption bundles, which specifies the bundle Robinson gets and the bundle Friday gets, i.e., $q = (q^R, q^F) = ((q_A^R, q_B^R), (q_A^F, q_B^F))$. Keep in mind that for this model, an allocation is represented by four numbers. An allocation q is **feasible** if it distributes all the apples and bananas in the island between Robinson and Friday. In other words, an allocation q is feasible if it satisfies the following equalities:

$$\begin{aligned}q_A^R + q_A^F &= e_A^R + e_A^F &= e_A \\q_B^R + q_B^F &= e_B^R + e_B^F &= e_B\end{aligned}$$

Here are some broad level questions we will try to answer:

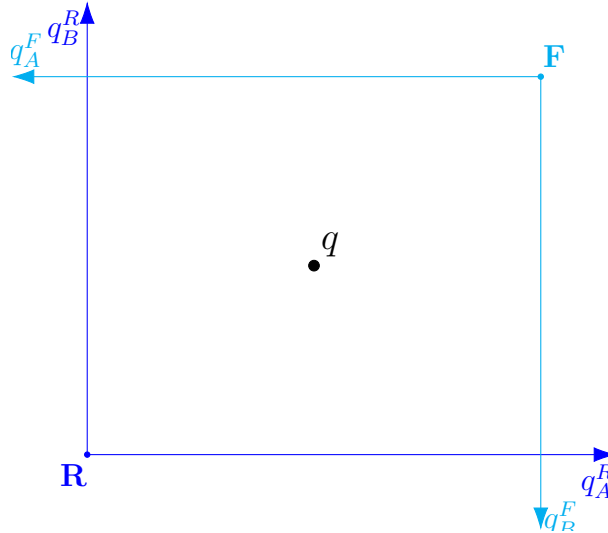
- (i) What are some *desirable* allocations in this economy? That is, are there desirable consumption bundles for Robinson and Friday, possibly different from their endowments?

- (ii) How can we achieve that desirable allocation? Is there a *mechanism* that achieves this allocation without requiring too much information?

Before we move on to these questions, let us discuss a very neat way to illustrate this economy.

2.1 The Edgeworth Box

Thanks to Irish philosopher and economist Francis Y. Edgeworth (1845-1926), we have a very intuitive to visualize/graphically represent this exchange economy. The **Edgeworth Box** (sometimes referred to as Edgeworth-Bowley Box), depicted below, is a visualization of an exchange economy where there are only two agents and two goods.

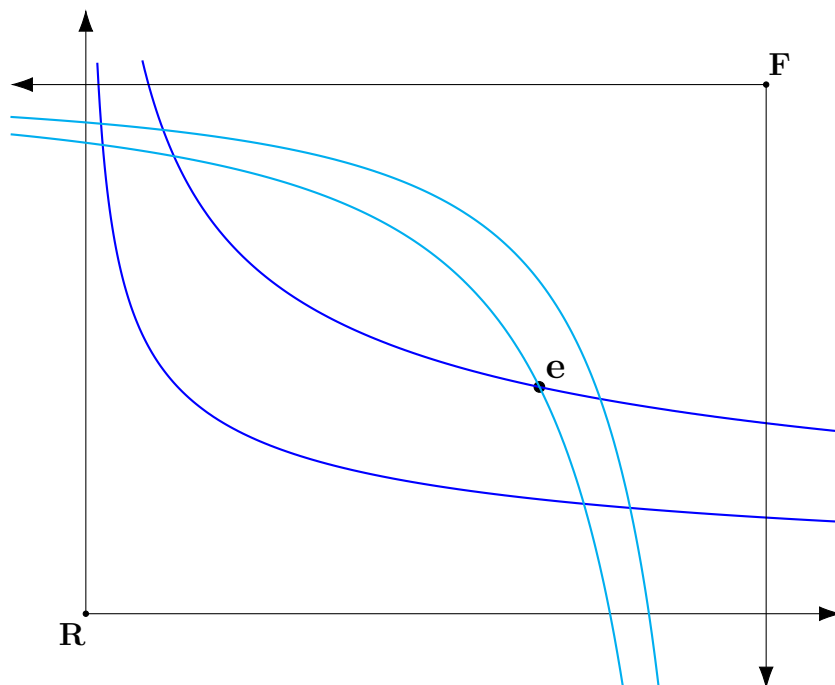


The Edgeworth Box is a rectangle with a width e_A and height e_B . Take any feasible allocation in this exchange economy. It can be represented as a point on the Edgeworth Box. Moreover, any point in this Edgeworth Box corresponds to a feasible allocation.

The figure looks like the figures we had in the previous chapter, where we covered consumer theory. Take any point in this box q . If you take point R as the origin, the x -axis represents Robinson's apple consumption (q_A^R) and the y -axis represents Robinson's banana consumption (q_B^R). The reason why there is a **box** is that Robinson's allocation has to be feasible: $q_A^R \leq e_A$ and $q_B^R \leq e_B$. But as long as these two inequalities are satisfied, such an allocation is feasible. Therefore, the Edgeworth Box is a box with width e_A and height e_B . Any point in this box is a feasible allocation for Robinson.

But... Here is the beauty of Edgeworth Box. If you rotate this figure by 180-degrees, you will see that the exact same reasoning applies to Friday! Do me a favor and rotate the figure upside down. Take point F as the origin. Now, the x -axis is $e_A - q_A^R$, which is equal to q_A^F , because q is a feasible allocation! Moreover, the y -axis is $e_B - q_B^R$, which is equal to q_B^F . Therefore, just a single point in this box is sufficient to represent four different numbers in an allocation $q = ((q_A^R, q_B^R), (q_A^F, q_B^F))$.

Two more things. **First**, note that the endowment $e = (e^R, e^F) = ((e_A^R, e_B^R), (e_A^F, e_B^F))$ is also a feasible allocation. Therefore, we can represent the endowment e as a point in the Edgeworth Box. **Second**, just as we drew the indifference curves of a consumer in consumer theory, we can draw the indifference curves of Robinson and Friday in this graph. We just need to be a little careful: when drawing Robinson's indifference curves, we take R as the origin (so that Robinson's higher indifference curves are in the northeastern direction). When drawing Friday's indifference curves, we rotate the figure and take F as the origin (so that Friday's higher indifference curves are in the southwestern direction). In the figure below, we illustrate two indifference curves of each agent, including the ones passing through the endowment point e . Robinson's indifference curves are depicted with dark blue, and Friday's indifference curves are depicted with light blue.



3 “Desirable” Allocations

Now we are ready to discuss the question: what are the desirable allocations in this Edgeworth Box? A **HUGE WARNING** that I have just asked a **normative** question. Yes, for the first time in this class, we have stepped into normative analysis. We are about to say something about what *should happen*, instead of what *happens*. To be honest, this is making me a little uneasy. After all, I do not feel comfortable making statements about what should happen between Robinson and Friday. Luckily for us, Wilfredo Pareto (1848-1923) felt less uncomfortable years ago, and invented a criterion to assess *desirability* of an allocation. We will use his criterion of assessment, which is called **Pareto Efficiency**.

Weirdly, I find it a better order when we first describe Pareto **inefficiency** before describing Pareto efficiency. So let’s start with that. In words, an allocation is **Pareto inefficient** if it is possible to make an agent strictly better off without making another agent worse off. Formally,

Definition 1. An allocation $q = (q^R, q^F)$ is **Pareto inefficient** if there exists another allocation $\tilde{q} = (\tilde{q}^R, \tilde{q}^F)$ such that:

- (i) $\tilde{q}^R \mathcal{R}^R q^R$ and $\tilde{q}^F \mathcal{R}^F q^F$,
- (ii) either $\tilde{q}^R \mathcal{P}^R q^R$, or $\tilde{q}^F \mathcal{P}^F q^F$, or both.

Intuitively, a Pareto inefficient allocation is so undesirable that there is an obvious improvement: we can reallocate the bundles so that one agent will be happier, and the other agent would not object to the reallocation. From a normative point of view, it is easy to see why such an allocation is undesirable. Simply put, the economy has not achieved *efficiency*: an agent can be made happier at no cost (i.e., without hurting the other agent).

To see a visualization, consider the point e in the picture above. We can move to an allocation in the “lens” between the indifference curves that pass through e . Such an allocation would make both agents better off, and at least one of them strictly better off (because they are both moving to “higher” indifference curves). Therefore, if the economy is stuck at point e , that would be really undesirable. An implication is, at point e , there is room for *mutually beneficial exchange* between Robinson and Friday.

Moving on to Pareto efficiency, you can guess where I am going. In words, an allocation is **Pareto efficient**

if it is not Pareto inefficient. That is, it must be impossible to make an agent better off without making the other one worse off. Formally,

Definition 2. An allocation $q = (q^R, q^F)$ is **Pareto efficient** if there does not exist another allocation $\tilde{q} = (\tilde{q}^R, \tilde{q}^F)$ such that:

- (i) $\tilde{q}^R \mathcal{R}^R q^R$ and $\tilde{q}^F \mathcal{R}^F q^F$,
- (ii) either $\tilde{q}^R \mathcal{P}^R q^R$, or $\tilde{q}^F \mathcal{P}^F q^F$, or both.

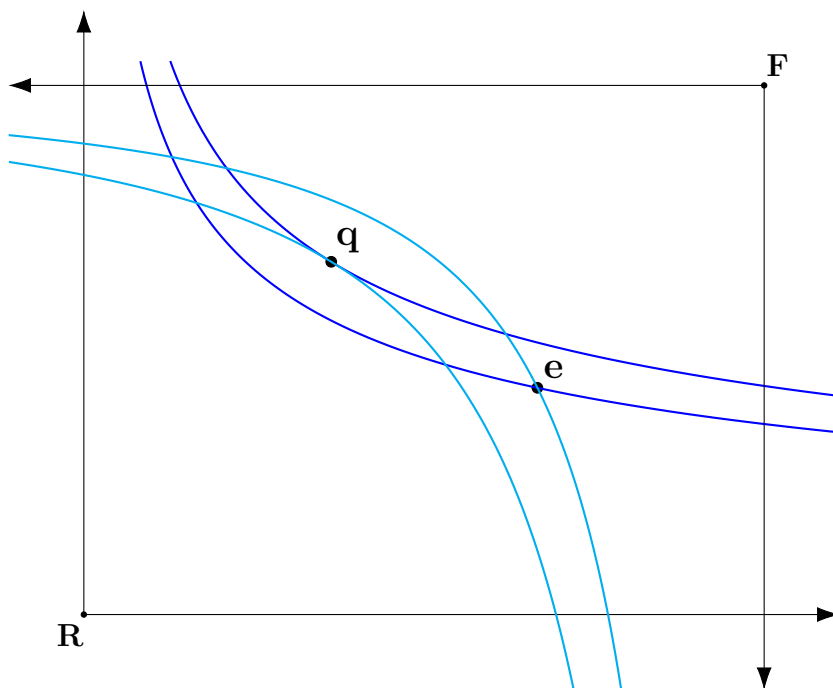
Once we are in a Pareto efficient allocation, if we want to make one agent happier by reallocating the goods, the other agent would object to such a reallocation. This means we have reached the point where the economy achieved some efficiency: there is no more “free happiness” one agent can attain without the other agent being hurt.

Let’s move on to the graphical interpretation. If an allocation q is Pareto efficient, then there should be no “lens” between the indifference curves that pass through q . Therefore: an allocation q is Pareto efficient if the indifference curves of Robinson and Friday passing through q are tangent to each other. In other words, if a feasible allocation $q = (q^R, q^F)$ is Pareto efficient, then the following equality must hold:

$$MRS_{B,A}^R(q^R) = MRS_{B,A}^F(q^F) \tag{1}$$

In economic terms, at a Pareto efficient allocation, Robinson’s valuation of apples in terms of bananas is equal to Friday’s valuation of apples in terms of bananas. Why? Suppose, for the sake of the mental exercise, that they are different; for instance, suppose Robinson’s valuation of apples is higher than Friday’s. This means, at the margin, Robinson likes apples more than Friday does. Conversely, at the margin, Friday likes bananas more than Robinson does. But then, Robinson and Friday would find a mutually beneficial exchange: Robinson would give up some bananas and receive some apples from Friday in return. This would make both agents happier, which means the allocation is Pareto inefficient.

If you are in need of a visualization, see the next figure, where q is a Pareto efficient allocation and e is a Pareto inefficient allocation (as before).



It is time for the next exercise. Using the observations we made, one can find all the Pareto efficient allocations (by checking all the points where the indifference curves passing through are tangent). We can give a name to it:

Definition 3. *The set of all Pareto efficient in an Edgeworth Box is called the **Pareto set** or the **contract curve**.*

The notion of a “contract curve” captures the idea that once these agents find a Pareto efficient allocation, they will be able to sign self-enforcing contracts that allow them to stay there. After all, in a Pareto efficient allocation, if one agent wants to “break” the contract and offer a new one, the other agent will object to it.

I would like to conclude this section by making a few remarks about Pareto efficiency.

- Do not forget that it contains a normative judgment (i.e., a judgment about what *should* happen)! Even as I write this document, my mind sometimes slips and I use phrases such as “an agent is happier...” etc. These are dangerous sentences, and we should not be talking about “happiness” unless we know what we mean by it.¹ We evaluate Pareto efficiency based on the agents’ preferences. But remember what I told when we started covering consumer theory: these preferences contain anything (social concerns, disregard for others, future considerations...) So they may not equal “happiness”, or one person’s happiness may contain the other person’s unhappiness. In general, it is a good idea to be careful not to equate preferences with happiness. This is the primary reason why we should stick with positive analysis whenever possible. But life is life: sometimes, we find ourselves in a position to conduct normative analysis. In such cases, it is a good idea to at least remind ourselves about the perils of conducting normative analysis.
- Pareto efficiency, as it is clear by its name, is an *efficiency* concept. It says very little about *fairness*. What I mean by this is: an allocation can be Pareto efficient, but very unfair. Consider, for instance, the point F (the upper right corner of the Edgeworth Box). That allocation is Pareto efficient: if you wanted to make Friday happier by moving to another point in the Edgeworth Box, Robinson would object to it. Note, however, that this is a *terribly unfair* allocation. Robinson has everything, and Friday has nothing! That is a reason why you may find that allocation “undesirable”, even though it is Pareto efficient.

What I want to arrive at is: just because an allocation is Pareto efficient does not mean that we, as the society, would find it “desirable”. There may be other grounds to find it “undesirable” as a society. This is the reason why many people find Pareto efficiency a *too permissive* concept: it allows for *too many* allocations to be efficient, including some which are very unfair.

The flip side of the coin is that: when an allocation is Pareto inefficient, there are usually grounds to find it “undesirable”. I mean, obviously, there are ways to improve upon such allocations. So most people think of Pareto efficiency as a *necessary condition* for an allocation being “desirable”. That is, for an allocation to be “desirable”, it must be at least Pareto efficient. But Pareto efficiency is not a *sufficient condition*: even when an allocation is Pareto efficient, it may still be “undesirable”.

Subject to the caveats discussed above, we have finally defined what the “desirable” allocations are. The next question is: is there a mechanism to guarantee that the economy will end up in an allocation in the contract curve? This is the question we will explore in the next section.

4 Allocation Mechanisms

Up to this point, we discussed *what should* happen, and have not discussed *how it should* happen. That is, we have not introduced a mechanism to find a Pareto efficient allocation. Such a mechanism is called an **allocation mechanism** (for obvious reasons), and you can think of many allocation mechanisms! An obvious possibility is having a *central planning* mechanism. Suppose there is a social planner who can seize and reallocate the goods. Both Robinson and Friday report their endowments and their preferences to the social planner. The social planner draws the Edgeworth Box based on the endowments, draws the contract curve based on their preferences, chooses an allocation on the contract curve and reallocates. This is certainly a way of achieving Pareto efficiency. Note that this mechanism does not use prices or such: it is purely a central planning mechanism. It should be noted, however, that it uses a lot of *information*: it requires the

¹We do not, in fact, know what we mean by it.

central planner to know Robinson and Friday's endowments and preferences. Endowments may be verifiable (even though that would be a difficult and time-consuming process), but preferences are just impossible to know. Therefore, the central planner must ask Robinson and Friday their preferences, and it should rely on the agents being truthful. But if the agents, for some reason, realize that they can lie to the central planner and receive a better allocation, there is nothing stopping them from doing so.² Therefore, the informational requirements of a central planning mechanism makes it a fragile allocation mechanism.

You can consider other allocation mechanisms as well. For instance, you can just rely on Robinson and Friday's ability to sort it out between each other. That is, Robinson and Friday can bargain over possible allocations and maybe reach a desirable allocation through bilateral negotiations and exchange. This would be called a *bargaining mechanism*, but guess what: it also suffers from informational issues (what if Robinson knew about Friday's preferences but Friday didn't know Robinson's preferences?), also we have to ensure that the bargaining is fair (i.e., Robinson cannot bully Friday into accepting an offer etc.)... Informational issues again.³

This brings us to our next question: is there an allocation mechanism that does not rely heavily on the information conveyed by agents? The answer to this question is the market mechanism.

4.1 The Market Mechanism

Here is another obvious possibility: why don't open a **market** for apples and bananas, and allow Robinson and Friday to buy/sell as many apples and bananas as they want? Opening a market means that there is a *market price* for apples and bananas, and the agents will be able to buy or sell at these prices. Let's denote the market price of apples with p_A , and the market price of bananas with p_B .

4.1.1 Competitive Markets

Let me make an observation we will revisit multiple times this semester. In the market I consider, Robinson and Friday **take the prices as given** and act accordingly. That is, the agents in this exchange economy are **price-takers**.

A market where every agent acts as a price-taker is frequently referred to as a **competitive market**. The underlying story here is that there are many agents, so that no single agent is powerful enough to affect the price. As you probably realize, there is a bit of cognitive dissonance here: in our exchange economy, there are only two agents – this can hardly be called an economy with “many agents”. Nevertheless, I should once again remind you that all the results we discuss here will extend to multiple agents. It is true that we are analyzing a model with only two agents, but that's for analytical convenience.

So, how does an agent act in a competitive market? Just like the agent we covered in the previous chapter: the agent consumes the *optimal bundle*. That is, the agent consumes the most preferred bundle among the feasible bundles. As a reminder, the feasible bundles are those affordable under the prices p_A , p_B and income I .

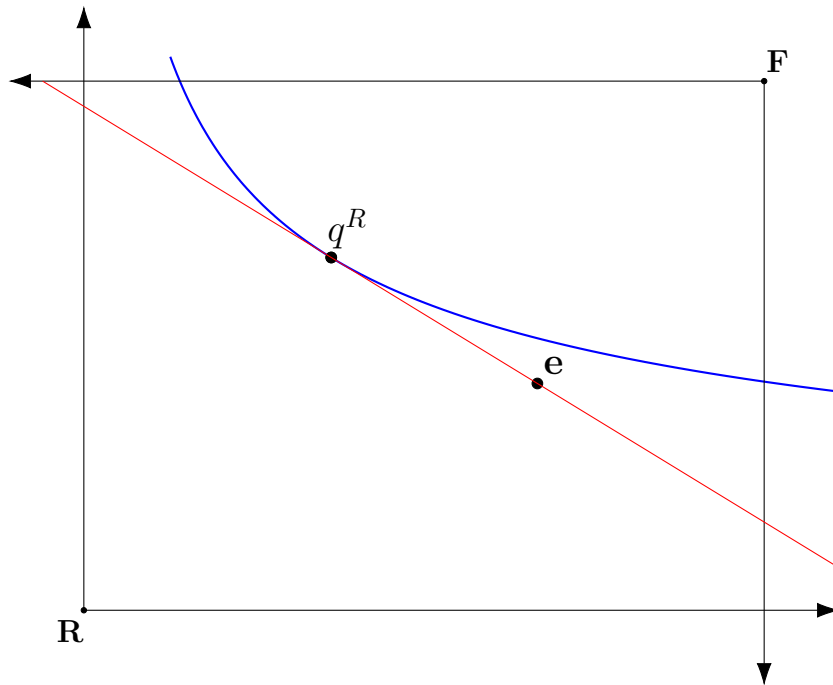
But... What is the income of an agent? Quite simply, the income of an agent is *the monetary worth of her endowment*. This corresponds to the following scenario: an agent can sell all her endowment, receive some money in return, and then spend that money on the optimal bundle. Therefore, when prices are p_A and p_B , Robinson's income is:

²The search for mechanisms that ensure the agents do not lie to the central planner is a long-lasting pursuit. There is a field of microeconomic theory, called *mechanism design*, which specializes on this question. In 2007, a trio of economists consisting of Leonid Hurwicz, Eric Maskin and Roger Myerson won the Economics Nobel Prize for their contributions to this field. I will be posting the press release for the 2007 Nobel Prize, and a 1973 paper by Hurwicz, to Moodle. The press release should be reasonably accessible, but the paper is not an easy read. Also, note that the 1973 paper should be read in context: this is the height of the Cold War and people have been asking the question “Is it possible to have a central planning mechanism that achieves desirable allocations?”. Within the context, this is a heavily political question. You can see the hints of Hurwicz's personal leanings, informed by growing up in Soviet Russia. Anyway... If you find mechanism design interesting, you should check out Econ 448 (Economics of Information) offered in our department.

³If you find bargaining interesting, you should check out Econ 444 (Bargaining Theory and Experiments in Economics) offered in our department.

$$I^R = p_A \cdot e_A^R + p_B \cdot e_B^R$$

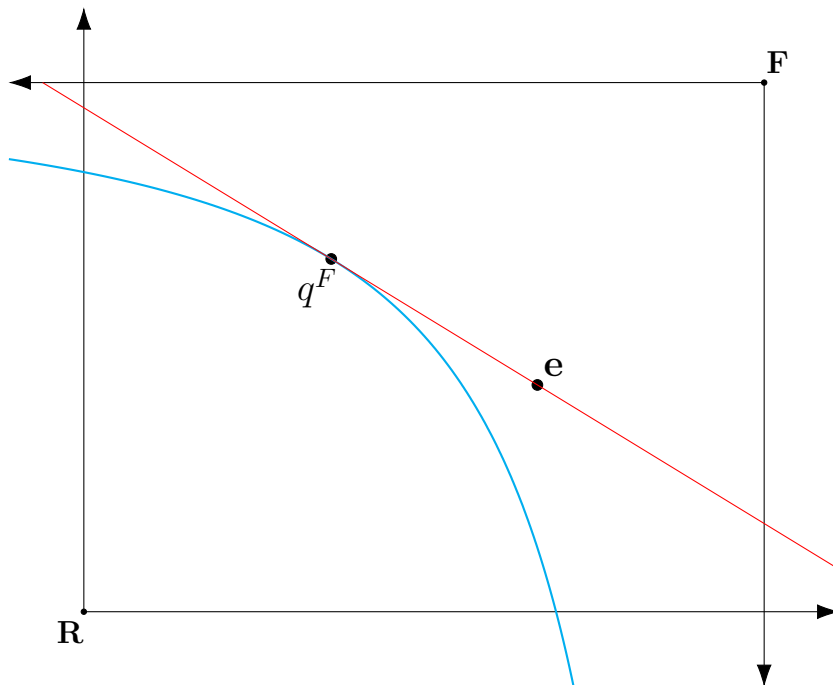
Under this income and prices, Robinson chooses the optimal bundle. Therefore, Robinson has a budget line with the (absolute value of) slope $\frac{p_A}{p_B}$. Moreover, the budget line passes through Robinson's endowment (because Robinson's endowment, as a bundle, has a price equal to Robinson's income). We simply repeat the exercise in the consumer theory chapter: we draw Robinson's budget line and find the optimal bundle under this budget line. Below is a representative figure showing Robinson's optimal bundle $q^R = (q_A^R, q_B^R)$. In this figure, the red line is Robinson's budget line (it passes through e and its slope has an absolute value of $\frac{p_A}{p_B}$).



Things are not really different for Friday. We just need to rotate the figure upside down and repeat the same analysis. When prices are p_A and p_B , Friday's income is:

$$I^F = p_A \cdot e_A^F + p_B \cdot e_B^F$$

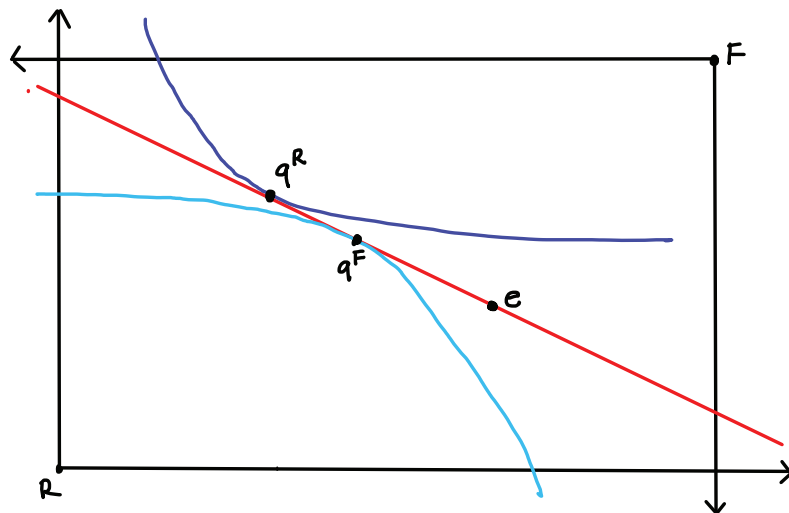
Once again, Friday has a budget line with the (absolute value of) slope $\frac{p_A}{p_B}$. If you rotate the figure upside down, that line still has a slope of $\frac{p_A}{p_B}$. The budget line again passes through Friday's endowment. Then, we can draw Friday's budget line: it passes through e and its slope has an absolute value of $\frac{p_A}{p_B}$. Below is a representative figure showing Friday's budget line in red and his optimal bundle q^F .



So far so good. We figured out how each agent behaves in a competitive market. As you recall Chapter 1, an equilibrium requires that *each agent is optimizing*, so we are almost there. We need to take care of one final thing before we define what the equilibrium is.

4.1.2 Market Clearing

There is one final question we need to answer before we close the loop: how are the prices p_A and p_B determined? To make sense of this question, let's see the following figure. (I'm sorry I had to draw it by hand :(((



In this figure, given prices p_A and p_B , Robinson and Friday are both optimizing. Robinson is consuming his optimal bundle $q^R = (q_A^R, q_B^R)$ and Friday is consuming his optimal bundle $q^F = (q_A^F, q_B^F)$. Still, my claim is

that this cannot be an equilibrium. Why? Because in this figure, there is too much consumption of bananas:

$$q_B^R + q_B^F > e_B$$

So much so that the total consumption of bananas exceeds the total quantity of bananas in the island. Moreover, there is too little consumption of apples:

$$q_A^R + q_A^F < e_A$$

Clearly, this combination of optimal bundles is not sustainable – there are simply not enough bananas around, and some apples are just not consumed at all. An economist would say that there is **excess demand** for bananas and **excess supply** of apples. In this case, it is reasonable to imagine that the prices will adjust to reflect the excess demand and supply. That is, the price of apples will drop and the price of bananas will rise. This will result in $\frac{p_A}{p_B}$ decreasing, i.e., the budget line getting flatter. Because apples are cheaper, both agents will increase their consumption of apples and reduce their consumption of bananas. At the end, the prices will settle at a point where there is no excess supply or demand. This is the situation where the **markets clear**:

$$\begin{aligned} q_A^R + q_A^F &= e_A, & \text{(the market for apples clears), and,} \\ q_B^R + q_B^F &= e_B, & \text{(the market for bananas clears).} \end{aligned}$$

This is the role prices play in a competitive market: **in equilibrium, prices clear the markets**. You may still ask “but who sets these prices?”, which is a very reasonable question. However, at this point, we will be a little nontransparent about it. If you desire, you may imagine a social planner playing the role of the mediator between Robinson and Friday. The mediator gives hypothetical prices to Robinson and Friday, asks about their optimal bundles, adjusts the prices if there is excess demand/supply and asks again... Continuing until the markets clear. There are two things I want to emphasize: (i) the mediator in this scenario requires *much less* information than the social planner who needs to know the preferences. Here, the mediator only needs to know the excess demand and supply! (ii) Regardless of there is a mediator or not, the situation where the markets do not clear is unsustainable. Somehow, the prices will have to change – that situation cannot be an equilibrium. We just imagine there is a mediator to describe this adjustment process.

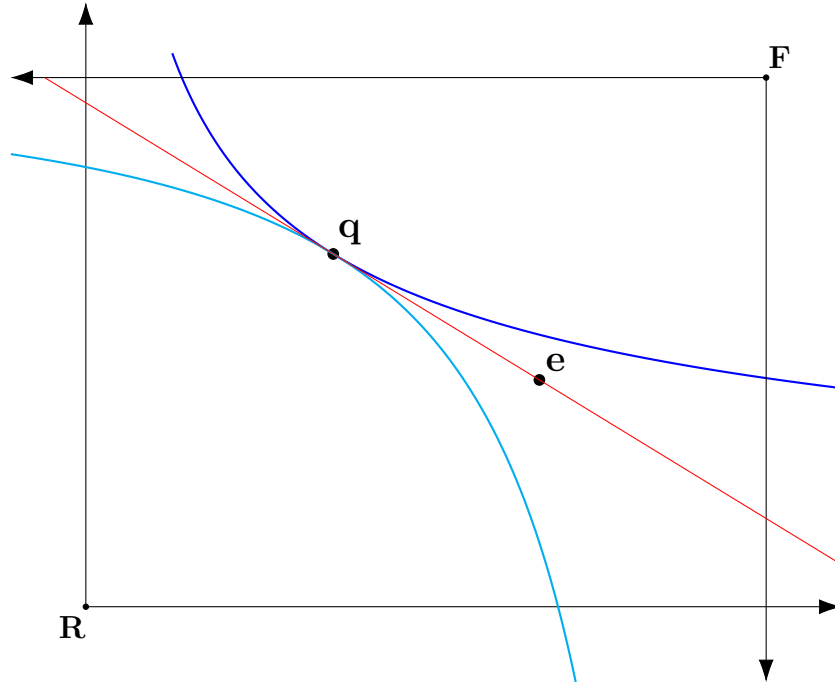
4.1.3 Competitive Equilibrium

All this work, and we are finally ready to define the notion of an **equilibrium** in the competitive market of an exchange economy. Simply put, the equilibrium is the situation where every agent optimizes given the prices, and the prices clear the markets. This is called the **competitive equilibrium**, or **market equilibrium**, or **Walrasian equilibrium** (named after economist Leon Walras, 1834-1910).

Definition 4. A **competitive equilibrium** of an exchange economy is an allocation $q = (q^R, q^F)$ and prices (p_A, p_B) such that the following conditions hold:

1. $q^R = (q_A^R, q_B^R)$ is the optimal bundle for Robinson given prices p_A, p_B , and income $I^R = p_A \cdot e_A^R + p_B \cdot e_B^R$.
2. $q^F = (q_A^F, q_B^F)$ is the optimal bundle for Friday given prices p_A, p_B , and income $I^F = p_A \cdot e_A^F + p_B \cdot e_B^F$.
3. Markets clear: $q_A^R + q_A^F = e_A$ and $q_B^R + q_B^F = e_B$.

Given our discussions so far, you may have imagined how the competitive equilibrium will look graphically. Below is an illustration of the competitive equilibrium in the Edgeworth Box.



5 Equilibrium Allocation is “Desirable”

Look at the figure illustrating Pareto efficient allocation: at a Pareto efficient allocation, the indifference curves of both agents are tangent to each other. Now, look at the competitive equilibrium allocation. The indifference curves are tangent to each other!

This is no surprise. We know that the indifference curves of each agent is tangent to the budget line, so they satisfy:

$$MRS_{B,A}^R(q^R) = \frac{p_A}{p_B} \text{ and } MRS_{B,A}^F(q^F) = \frac{p_A}{p_B}.$$

But then:

$$MRS_{B,A}^R(q^R) = MRS_{B,A}^F(q^F).$$

This is identical to condition (1), which is the condition for Pareto efficiency. Therefore, competitive equilibrium allocation is Pareto efficient!!!

This observation is obviously true for the indifference curves we drew here, but it is also fairly generalizable. It holds under very weak conditions —conditions we will not talk about in this class. What you need to remember is that this result is known as the **first fundamental theorem of welfare**, or **first welfare theorem**:

Theorem 1 (First Welfare Theorem). *Any competitive equilibrium allocation is Pareto efficient.*

Let us take a step back and remember what we have done. We started with a description of an environment with two agents, who have preferences over their allocations. We specified a “desirability” criterion for allocations, and argued that finding a desirable allocation may not be that easy. Then, we invented a mechanism that guarantees a desirable allocation! Note that the mechanism does not require coordination between the agents or anything: each agent can be quite self-interested, but from a societal point of view, we achieve a desirable allocation. This is the fundamental reason why a lot of economists love *market* as an allocation mechanism: it uses very little information, and it guarantees a desirable allocation.

A couple of notes before we leave you with your contemplation:

- First Welfare Theorem is a very strong result, and it has fundamentally shaped a lot of people's thinking. (You may argue that any political discussion about a "market economy", one way or another, is a discussion about First Welfare Theorem.) Due to this reason, it is a good idea to remind ourselves that the model contains a bunch of heroic assumptions: each agent has well-defined preferences, they face the same prices, agents act as price-takers, markets exist for all the relevant goods... Relaxing any of these assumptions may break the result.
- I know I am repeating myself, but this is a point that cannot be overemphasized: Pareto efficiency is an *efficiency* criterion, not a *fairness* criterion. Therefore, first welfare theorem says that a market mechanism achieves efficiency, but the resulting allocation may be really unfair.
- You may be wondering of there is a *Second Welfare Theorem*, and the answer is yes. Informally, it says that "Any Pareto efficient allocation can be achieved with a reallocation of endowments and a market mechanism." But we will leave this to Econ 203.

We will give you some numerical exercises to play with in the recitations and problem sets.

So much for the detour, but we have learned some useful ideas about *efficiency*, *competitive markets*, *market clearing*, and *equilibrium*. They will reappear later this semester. For now, we will go back to the one agent model.