Proximity and the Formation of Diverse Social Networks: Theory and Evidence *

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Abstract

We propose a friendship formation model that distinguishes the role of similarity and physical proximity. This is a learning-based theory of friendship where individuals spend time exploring the value of a friendship. The model predicts that friendship patterns exhibit homophily: similarity increases the likelihood of friendship. Higher proximity also increases the likelihood of friendship, and this effect is more pronounced for dissimilar individuals: proximity fosters diversity. To verify the predictions, we use an experiment at selective boarding schools in Peru. While social networks exhibit homophily along multiple dimensions, proximity fosters more diverse friendships. This evidence stands in contrast to the predictions offered by a preference-based theory of homophily.

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1 Introduction

Friendships and social networks affect individuals' well-being, social and labor market outcomes (Caldwell and Harmon, 2019; Lleras-Muney et al., 2020; Zimmerman, 2019). A common feature of social networks in educational environments is *homophily*: students prefer to have friends of their same race (Currarini et al., 2009; Marmaros and Sacerdote, 2006) or similar academic achievement (Carrell et al., 2013). Furthermore, students prefer that their indirect connections (friends of their friends) also share their characteristics (Mele, 2020). This evidence paints a precarious picture for school integration policies. While such policies can place students from different races or economic backgrounds in the same school, they may fail to foster more diverse friendships and study networks.

An important instrument to foster social interactions is within-school policies. School authorities have control over the allocation to classrooms, desks, and dormitories, thereby influencing students' physical proximity. Proximity is a central factor in the formation of friendships (Marmaros and Sacerdote, 2006) and social preferences (Rao, 2019). The literature has provided separate evidence on homophily and the effect of proximity; yet, whether homophilic preferences persist when students are close to each other is unknown. Understanding the impact of proximity on students with different similarity levels is crucial for the success of school integration policies and a central piece to identify the underlying cause of homophily in social networks.

This paper proposes a learning-based approach to friendship formation and provides an empirical test to identify whether homophily is driven by learning. Theoretically, we introduce a model where homophily results from students learning the value of friendships *faster* when they interact with similar peers. Unlike other existing models on link formation in social networks (Currarini et al., 2009; Mele, 2017), homophily is not a result of students deriving *higher* utility from these friendships. We show that under a learning-based model, the impact of proximity is higher for dissimilar peers. We test this prediction by leveraging an experiment at selective boarding schools in Peru and find evidence that learning, rather than preferences, is a driver of homophily. Taken together, our theoretical and empirical results imply that within-school policies can break patterns of homophily and lead to more diverse social networks.

Our theoretical model is an application of the theory of exponential bandits (Keller et al., 2005). In the model, a student decides whether to maintain interactions with a peer, where the value of maintaining the interaction are initially uncertain and is revealed over time. In the optimal strategy, there is an *exploration phase*: a student engages with a peer for a period where she "tests the waters". If she receives a signal that the interaction is valuable, she maintains the interactions; otherwise, she stops engaging with that peer.

The crucial assumption of the model is that when a student engages with a peer from a similar background, learning about the interaction's value occurs faster. Then, for a given proximity, students end up being friends with similar peers more often than with dissimilar peers: friendship patterns exhibit homophily. When the cost of interaction is lower due to physical proximity, the overall rate of friendship is higher: proximity fosters friendships. Moreover, the effect of physical proximity is higher for dissimilar peers: proximity fosters diversity. Intuitively, this is because proximity and similarity are substitutes in this model. When two peers have similar characteristics, the value of interaction is already explored even when the cost is high. In this case, a lower cost due to proximity has little impact on the outcome. This stands in contrast to a model of preference-based homophily (Section 2.3), under which proximity has a higher impact on similar peers.

We then verify the theoretical predictions with an experiment at selective boarding schools in Peru. These are public schools designed for the most talented low-income students in the country. The details of the experiment are in Zárate (2021) and consist of randomizing students to (i) more socially central vs. less socially central peers and to (ii) higher-achieving vs. lower-achieving peers. The assignment generates random variation in the physical proximity between a pair of students in the allocation to dormitories. We leverage this variation to explore the effects of proximity on social interactions.

We have access to three rounds of social networks: one before and two after the intervention. In all surveys, students could nominate all their friends, study partners, and peers with whom they engage in social activities such as playing sports or games. The surveys' take-up rate was above 95% in all schools, and there were no restrictions on the number of peers students could list for each question. These features allow us to observe the entire social networks in the schools.

Our empirical analysis starts by exploring homophily in three dimensions: baseline academic achievement, baseline centrality level,[1] and poverty status. Our findings show that social networks exhibit homophily along these three characteristics. On average, poor students have 0.66 fewer non-poor friends (p-value < 0.01) than non-poor students. Likewise, compared to higher-achieving students, lower-achieving students have, on average, 1.05 more interactions with other lower-achieving students and 1.26 fewer interactions with higher-achieving students. The results by other social skills variables (measured by centrality at baseline) also exhibit homophily. Compared to high central students, less central students have around 1.07 fewer interactions (p-value < 0.01) with more central peers.

[1] We use eigenvector centrality that measures a student's influence within their social network. High values of this measure indicate that a student is connected to other individuals who also have high values of eigenvector centrality.

Next, we study the effect of proximity on social interactions. As part of the experiment, students' names were organized on a list that school administrators used to determine the assignment to dormitories. Conditional on students' characteristics, the distance on the list is random and it is a strong predictor of the physical distance between two students in the dormitories. We show that distance on the list is unrelated to social interactions before the intervention, as expected from the random variation in proximity generated by the experiment. The effect of distance on social interactions after the intervention is high and statistically significant. Two adjacent students on the list are 16.6 percentage points more likely to become friends. This effect is 23.6 percentage points higher for first-year students, which is intuitive as they did not know each other before the experiment.

Finally, we explore heterogeneous effects of proximity by student characteristics. Our empirical findings are consistent with the theoretical predictions of the learning-based model. The effect of proximity on interactions is higher for students with different achievement levels and poverty status.

Related literature This paper contributes to three branches of literature.

First, we contribute to the literature on social networks in schools. While there has been extensive work estimating homophily by type in the formation of direct and indirect friendships (Currarini et al., 2010; Mele, 2020; Paula et al., 2018), less is known about the effects of mixing students of different types within the school. To our knowledge, this is the first paper that combines physical distance between each pair of students and social network surveys. Furthermore, we show that even though students prefer friends similar to them, these differences are less prevalent when students are physically close to each other. This finding is an essential factor to consider when designing policies for school integration.

Second, we contribute to the literature on how exposure to peers of a different type changes attitudes towards that group. When two students have different poverty status and academic achievement levels, the effect of exposure is stronger than when they have similar characteristics. These results add to the evidence of contact on attitudes and preferences such as race (Boisjoly et al., 2006), poverty (Rao, 2019), caste (Lowe, 2021), and religion (Mousa, 2020).

On the theoretical front, we propose a novel model of friendship formation. As in Currarini et al. (2009), our model also predicts homophily. In our model, faster learning (rather than preferences) is the driver of homophily. The distinction between learning-based and preference-based homophily is crucial for our analysis. Kets and Sandroni (2019) also propose an information-driven theory, where homophily alleviates the strategic uncertainty because similar individuals receive correlated impulses. In comparison, our model makes predictions on the effects of proximity on friendship patterns. Peski (2008) and Baccara and Yariv (2013) propose models of homophily where group formation is a critical part of the argument; our model considers the isolated instance of a student choosing whether to interact with a peer.

2 The Model

We model the friendship process as a game of *experimentation* with exponential bandits. The model is based on Keller et al. (2005) and our treatment is closest to Bardhi et al. (2020). For other models of experimentation, see Keller and Rady (2010), Keller and Rady (2015), and Strulovici (2010). The model allows us to decompose the effects of proximity (a *design* variable) and similarity (students' *exogenous* characteristics) on friendship patterns, and understand their interaction.

The model conveys a *learning-based* theory of homophily. In the model, there is no discrimination based on the *value* of a friendship: on average, a student derives the same value from interacting with a peer with a similar background as she does with peers with different backgrounds. The only feature that distinguishes the peers with similar backgrounds is that they learn the interaction's value *faster* compared to peers with different backgrounds. This represents the fact that individuals from the similar backgrounds have an ease in communication, perhaps because they share some common knowledge (Mayhew et al., 1995) or because they share the same mental models of the world (Carley and Palmquist, 1992).

The model builds on the following idea. A student i's personal benefit from interacting with a peer j is initially unknown to i. Observing whether i benefits from j takes time and is a costly activity for i. As i spends time with j, a conclusive signal about i's benefit from j arrives in random times. The arrival of a signal captures the moment where i "clicks" with j and realizes the interaction will be worth maintaining. The arrival rate of the signal depends on the observable characteristics of i and j: if they have matching characteristics, the signal arrives faster. In the optimal solution, there is an exploration phase where i waits for the signal to arrive. If the signal does not arrive within the phase, i stops interacting with j. In equilibrium, i will have a higher probability of naming peers with matching characteristics as friends after a sufficiently long time horizon. In other words, the friendship dynamics exhibit *homophilic patterns.*

The model also helps us interpret the effect of proximity on friendship patterns. In the model, an increase in proximity reduces the cost of maintaining interactions. This is because when i is within close proximity of j, it is physically less costly to maintain interactions with j. We show that a reduction in the cost increases the probability that students name each other as friends. That is, *proximity fosters friendships.* Finally, the proximity has a *heterogeneous effect:* if i has different characteristics with j and matching characteristics with ℓ, i benefits from increased proximity with j more compared to the benefits from increased proximity with ℓ. That is, *proximity fosters diversity.*

Before moving on to the formal description, it is worth pointing out that this is a model where i learns her personal benefit from interacting with j, and individually decides whether to continue her interaction with j. Whether i names j as a friend only depends on i, and this is independent from whether j names i as a friend. In other words, this is a model of *one-sided* friendship formation. We present such a model because empirical analysis uses a directed network. Section 2.4 discusses alternative formulations.

2.1 The Friendship Formation Process

There is a population of students denoted by I. There is a finite set of categories $\mathcal{K}$ with $|\mathcal{K}| = K$. Each category $k \in \mathcal{K}$ represents a binary classification based on observable characteristics (poverty levels, academic achievement etc.) For each student $i \in I$, her characteristic in category $k \in \mathcal{K}$ is given by:

$$\kappa_{i,k} \in \{0, 1\}$$

For instance, if k represents poverty status, students with $\kappa_{i,k} = 1$ are those with high poverty, and those with $\kappa_{i,k} = 0$ are those with low poverty. For a student i, let

$$\kappa_i \equiv (\kappa_{i,1}, \kappa_{i,2}, \ldots, \kappa_{i,K})$$

denote the vector of her observable characteristics. Also, for a student $i \in I$ and a category $k \in \mathcal{K}$, let

$$\kappa_{i,-k} \equiv (\kappa_{i,1}, \ldots, \kappa_{i,k-1}, \kappa_{i,k+1}, \ldots, \kappa_{i,K})$$

denote the characteristics of i other than the characteristic in category k.

At the beginning of the game, an individual i is matched with a subset of students $M_i \subseteq I \setminus \{i\}$. The students included in M_i are the students who are in close proximity to i (i's *neighbors*). For two individuals $i, j \in I$, let

$$n_{ij} \equiv \mathbf{1}_{j \in M_i} \quad \in \{0, 1\}$$

denote the variable that captures whether j is a neighbor of i.

After i observes M_i, she starts the following process of maintaining interactions with every student $j \in I \setminus \{i\}$. The time is continuous, denoted by $t \in [0, \infty)$, and the discount rate is $r > 0$. At any t, i chooses $a_{ijt} \in \{0, 1\}$. Here, $a_{ijt} = 1$ if i maintains the interactions with j and $a_{ijt} = 0$ if not. Maintaining the interactions imposes a flow cost of $c(n_{ij}) > 0$ on i. We impose the following structure on the cost.

Assumption 1 (Proximity decreases cost of interactions)**.** *Maintaining interactions with j is less costly when j is a neighbor of i, i.e.,*

$$c(1) < c(0)$$

For i, maintaining an interaction with j may be *valuable* or not. When an interaction with j is valuable for i, i receives a flow payoff of $v > 0$. Otherwise, i receives a payoff of zero: a nonvaluable interaction does not yield any payoff. This is modeled through an i.i.d. random variable $\theta_{ij} \in \{0, 1\}$. The realization of θ_{ij} is initially unknown to i, who has a prior $p_0 \equiv \Pr(\theta_{ij} = 1)$. i receives a flow payoff of $\theta_{ij} \cdot v$ from maintaining interactions with j, which is realized at the end of the time horizon. We assume:

$$p_0 \cdot v \geq c(0)$$

That is, initially, each interaction is valuable in expectation regardless of the match. Note that this implies $v \geq c(0)$, i.e., once an interaction is revealed to be valuable, it is worth maintaining interaction even when j is not a neighbor.

i can learn the value of interaction with j via a signal, which arrives at exponentially distributed random times with parameter $\lambda(\kappa_i, \kappa_j)$. In particular, the probability that a signal arrives within the time interval $[t, t + dt)$ is

$$a_{ijt} \cdot \theta_{ij} \cdot \lambda(\kappa_i, \kappa_j) \cdot dt$$

This implies:

- If i does not maintain interactions with j ($a_{ijt} = 0$) she does not receive any signals, and,
- If i's interaction with j is not valuable ($\theta_{ij} = 0$) it does not generate any signals. This formulation is critical for the evolution of beliefs: because only the valuable interactions generate signals, i is *certain* that $\theta_{ij} = 1$ upon the arrival of the first signal. Arrival of the first signal is a *breakthrough* as in Keller et al. (2005).
- The arrival rate of signal depends on the characteristics of i and j, (κ_i, κ_j). We impose the following structure on the arrival rate.

Assumption 2 (Similarity speeds up learning)**.** *If i and j have matching characteristics, i learns the value of maintaining interactions faster, i.e.,*

$$\lambda\left(\kappa_{i,-k}, \kappa_{j,-k}, \kappa_{i,k}, \kappa_{j,k} = \kappa_{i,k}\right) > \lambda\left(\kappa_{i,-k}, \kappa_{j,-k}, \kappa_{i,k}, \kappa_{j,k} \neq \kappa_{i,k}\right)$$
$$\textit{for all } k \in \{1, \ldots, K\}, \kappa_{i,-k}, \kappa_{j,-k}.$$

All in all, i's expected discounted payoff from the action profile $\{a_{ijt}\}_{t\geq 0}$ is:

$$\mathbb{E}\left[\int_0^\infty re^{-rt} a_{ijt} \cdot (\theta_{ij} \cdot v - c(n_{ij}))\, dt\right]$$

where the expectation is taken over θ_{ij}, the stochastic arrival of signals and $\{a_{ijt}\}_{t\geq t}$. We will consider the strategy $\{a^*_{ijt}\}_{t\geq 0}$ that maximizes the total expected payoff of student i at each history h_t. Here, h_t involves the action profile up to period t, $\{a_{ij\tilde{t}}\}_{\tilde{t}<t}$, and the arrival of signals.

2.2 Analysis

The following proposition characterizes the optimal decision rule of i for each $j \in I \backslash \{i\}$. It is a corollary of Proposition 3.1 of Keller et al. (2005) and Lemma A.1 of Bardhi et al. (2020). For the sake of conciseness, we are omitting the proof here.

Proposition 1. In the optimal solution, $a^*_{ijt} = 1$ if and only if the belief $p_t = \Pr(\theta_{ij} = 1|h_t)$ is above

$$p^*_{ij} \equiv \frac{r \cdot c(n_{ij})}{(r + \lambda(\kappa_i, \kappa_j)) \cdot v - \lambda(\kappa_i, \kappa_j) \cdot c(n_{ij})}$$

A couple of notes about the equilibrium dynamics is in order. First, consider the posterior belief at time t, $p_t = \Pr(\theta_{ij} = 1|h_t)$. Due to the *breakthrough* nature of signal, in the optimal solution the posterior decreases as the individuals maintain the interaction and the signal does not arrive:

$$\frac{dp_t}{dt} = -a_{ijt} \cdot \lambda(\kappa_i, \kappa_j) \cdot p_t \cdot (1 - p_t) \tag{1}$$

As soon as the signal arrives, the posterior beliefs jump to $p_t = 1$.

Second, the cutoff belief and the rate of change of beliefs imply an "exploration phase" where i *experiments* by waiting for a signal to arrive. If the signal arrives within the exploration phase, i maintains interactions with j from then on. If it does not, i stops her interactions with j onwards. The length of exploration phase is:

$$\begin{aligned} t^*_{ij} &= \frac{1}{\lambda(\kappa_i, \kappa_j)} \log\left(\frac{p_0}{1 - p_0} \frac{1 - p^*_{ij}}{p^*_{ij}}\right) \\ &= \frac{1}{\lambda(\kappa_i, \kappa_j)} \log\left(\frac{p_0}{1 - p_0} \frac{r + \lambda(\kappa_i, \kappa_j)}{r} \frac{v - c(n_{ij})}{c(n_{ij})}\right) \end{aligned}$$

It is immediately apparent that the exploration phase is longer when $c(n_{ij})$ is lower. Combined with Assumption 1, this implies that neighbors tend to explore more.

$p_0 \geq \frac{c(0)}{v}$ ensures that $t^*_{ij} > 0$, i.e., there is an exploration phase for each pair. Let:

$$\bar{t} \equiv \max_{i,j} t^*_{ij}$$

denote the longest exploration phase within the population.

Our outcome of interest is the ex ante probability of i maintaining interactions with j sufficiently far ahead in time. That is:

$$\gamma_{ij} \equiv \Pr\left(a^*_{ijt} = 1 | \kappa_i, \kappa_j, n_{ij}, t \geq \bar{t}\right)$$

Given Proposition 1 and Equation (1), this is equal to:

$$\gamma_{ij} = p_0 \cdot \left(1 - e^{-\lambda(\kappa_i, \kappa_j) \cdot t^*_{ij}}\right) \tag{2}$$

$$= p_0 \cdot \left(1 - \frac{1 - p_0}{p_0} \frac{r}{r + \lambda(\kappa_i, \kappa_j)} \frac{c(n_{ij})}{v - c(n_{ij})}\right) \tag{3}$$

It is important to emphasize that *learning* about θ_{ij} is the essential part of this story. If the value of θ_{ij} was revealed to i at $t = 0$, γ_{ij} would be equal to p_0 as long as $v \geq c(0)$. The $-(1-p_0)\frac{r}{r+\lambda(\kappa_i,\kappa_j)}\frac{c(n_{ij})}{v-c(n_{ij})}$ is the term that corresponds to interactions that are not maintained due to the initial lack of information about its value. Not surprisingly, as $\lambda(\kappa_i, \kappa_j) \to \infty$ (i.e., as the information is revealed immediately) or as $c(n_{ij}) \to 0$ (maintaining interaction is not costly), this term goes to zero.

We are now ready to present our first empirically testable result.

Result 1 (Proximity Fosters Friendships)**.** Consider three students i, j, j' such that $\kappa_j = \kappa_{j'}$, $n_{ij} = 1$ and $n_{ij'} = 0$. Then,

$$\gamma_{ij} > \gamma_{ij'}$$

Result 1 implies that, controlling for student characteristics, one would expect a positive impact of proximity on the likelihood of being friends.

Our second result is about the effect of matching characteristics on friendship patterns.

Result 2 (Homophily)**.** Consider three students i, j, ℓ and a category k such that $n_{ij} = n_{i\ell}$, $\kappa_{j,-k} = \kappa_{\ell,-k}$, $\kappa_{j,k} = \kappa_{i,k}$ and $\kappa_{\ell,k} \neq \kappa_{i,k}$. Then,

$$\gamma_{ij} > \gamma_{i\ell}$$

Result 2 implies that, controlling for proximity and other dimensions of characteristics, one would expect a positive impact of matching characteristics along one dimension on the likelihood of being friends. That is, the friendship patterns exhibit *homophily*.

Consider a designer who wants to foster friendships in a population. By Result 1, one would expect neighbors to be more likely to be friends compared to non-neighbors. It is, however, unclear whether this effect will be stronger for similar students or dissimilar students. The designer would prefer placing dissimilar students as neighbors if and only if the latter is true. Our next result shows that the marginal effect of proximity is stronger for dissimilar students.

Result 3 (Proximity Fosters Diversity)**.** Consider five students i, j, j', ℓ, ℓ' and a characteristic k such that:

- $\kappa_{j,-k} = \kappa_{j',-k} = \kappa_{\ell,-k} = \kappa_{\ell',-k}$,
- $\kappa_{j,k} = \kappa_{j',k} = \kappa_{i,k}$, $\kappa_{\ell,k} = \kappa_{\ell',k} \neq \kappa_{i,k}$,
- $n_{ij} = n_{i\ell} = 1$, $n_{ij'} = n_{i\ell'} = 0$.

Then,

$$\gamma_{ij} - \gamma_{ij'} < \gamma_{i\ell} - \gamma_{i\ell'}$$

Result 3 implies that, controlling for other dimensions of characteristics, the interaction of proximity and matching characteristics along one dimension has a negative effect of on the likelihood of being friends. There are heterogeneous effects of proximity, and in particular, the effect is more pronounced on students with different characteristics. Consequently, the designer would target dissimilar students and assign them as neighbors.

Intuitively, Result 3 is driven by the substitutability of *proximity* and *similarity* for fostering friendships. In equilibrium for a given c, students with higher λ explore more and learn about the value of interaction better. Since they already have better information about θ, lowering c leaves less room for learning for such pairs. The easiest way to see this is considering the extreme case where $\lambda = \infty$ for similar students, i.e., similar students immediately learn about the value of interaction. In this case, a lower value of c would have no effect on the likelihood of friendship for similar students, because all the learning takes place regardless of c.

2.3 Preference-Based Homophily

Our current formulation yields homophily based on different rates of learning about the payoff from an interaction. We call this *learning-based homophily*. One way to restore Result 1 without having heterogeneity in learning is to add heterogeneity in preferences. To this end, let us define an alternative formulation of the model where $\lambda > 0$ is independent of (κ_i, κ_j). We now introduce heterogeneity in v by assuming that the flow payoff of interactions is $\theta_{ij} \cdot \tilde{v}_{ij}$. Here, $\tilde{v}_{ij} \in \{0, v\}$ is a random variable drawn according to:

$$\tilde{v}_{ij} = \begin{cases} v, & \text{w.p. } \mu(\kappa_i, \kappa_j) \\ 0, & \text{w.p. } 1 - \mu(\kappa_i, \kappa_j) \end{cases}$$

where we impose the following structure on the distribution of values.

Assumption 3 (Similarity begets higher values). *If i and j have matching characteristics, i has a higher probability of having $\tilde{v}_{ij} = v$, i.e.,*

$$\mu\left(\kappa_{i,-k}, \kappa_{j,-k}, \kappa_{i,k}, \kappa_{j,k} = \kappa_{i,k}\right) > \mu\left(\kappa_{i,-k}, \kappa_{j,-k}, \kappa_{i,k}, \kappa_{j,k} \neq \kappa_{i,k}\right)$$
$$\textit{for all } k \in \{1, \ldots, K\}, \kappa_{i,-k}, \kappa_{j,-k}.$$

The timing is now modified as follows. First, i learns the realization of $\tilde{v}_{ij}$. Then, i chooses $\{a_{ijt}\}_{t \geq 0}$ to learn about θ_{ij}.

Following the same steps as above shows that in this formulation,

$$\gamma_{ij} = \mu(\kappa_i, \kappa_j) \cdot p_0 \cdot (1 - \frac{1 - p_0}{p_0} \frac{r}{r + \lambda} \frac{c(n_{ij})}{v - c(n_{ij})})$$

Clearly, under Assumption 3, an analogous version of Result 2 holds. That is, friendship patterns also exhibit homophily in this formulation, because there is a higher fraction of peers with $\tilde{v} = v$ among similar students, and thus there is a higher fraction of interactions that are worth exploring. We call this *preference-based homophily* as it is driven by the heterogeneity of payoffs from an interaction.

Under preference-based homophily, the opposite of Result 3 holds: the effect of proximity is more pronounced on students with similar characteristics. This is because, under preference-based homophily, *proximity* and *similarity* are complements. For a student to benefit from lower c, she must already be finding the interaction worth exploring.

The most general formulation of this model includes both learning-based and preference-based homophily by introducing heterogeneity in both λ and $\tilde{v}$. There, heterogeneity in $\tilde{v}$ generates homophily due to the variation in the *extensive margin*: some interactions are not explored at all, and such interactions are more likely to be amongst dissimilar students. In contrast, heterogeneity in λ generates homophily due to variation in the *intensive margin*: out of explored interactions, similar students explore them longer and learn their value precisely. The relative strength of extensive versus intensive margin determines the direction of heterogeneous effect of proximity. Our empirical results suggest that learning-based homophily is the prevalent case.

2.4 Some Comments on Modeling Assumptions

We finish our theoretical treatment by providing some remarks on our modeling choices.

In the model, the flow payoff from maintaining interactions is realized at the end of the horizon. An alternative is assuming that payoffs arrive in lump-sum fashion in exponentially distributed random times with arrival rate $\lambda(\kappa_i, \kappa_j)$. In this formulation, arrival of the first payoff would play the role of the perfectly conclusive signal. Whereas this alternative would yield the same predictions, it would also imply that the payoffs arrive faster when the students have matching characteristics. Our formulation, however, isolates the *learning* channel from the *preferences* channel.

The model relies on perfectly conclusive good news. The polar opposite of this model would include perfectly conclusive bad news ("breakdowns" as in Keller and Rady (2015)). Perfectly conclusive bad news would overturn Result 2: when learning is faster, interactions are more likely to break down after a certain time period.

Another assumption of the model is that the realization of θ_{ij} does not change over time. Realistically, the value of an interaction may change over time, yet we abstract away from this possibility. One justification for this choice is that the time frame we have for our empirical application is not too long to allow for a reversal of values.

Finally, as discussed before, this is a model of *one-sided* friendship formation. An alternative setup would have a common θ_{ij} for both players, i and j choosing a_{ijt} and a_{jit} respectively, and having the signals flow with the rate of $a_{ijt} \cdot a_{jit} \cdot \theta_{ij} \cdot \lambda(\kappa_i, \kappa_j)$. That model would contain a number of equilibria due to the strict complementarity of actions, but the "most cooperative" equilibrium would exhibit the patterns discussed here. We opt for a model of one-sided formation to motivate our empirical analysis which uses a directed network.

3 Experimental Setting

The setting of the experiment is the COAR Network of selective boarding schools in Peru, a network of public schools directly managed by the Peruvian government. These schools are designed for the most talented students in the country. Only students in public schools are eligible to apply to the COAR Network. The schools operate for the last three years of high school. Our cohorts of analysis are enrolled at the COAR Network between 2015 and 2017.

The COAR Network meets the standards of elite Latin American private high schools, where students have access to all the required inputs for high-quality education. COAR schools are boarding schools and are deliberately located close to the capital city of each region to reduce the daily transportation costs for both families and the government. Upon admission, students receive school materials, uniforms, and a laptop for school use. All of the schools have a high-quality infrastructure, including a library and scientific laboratories. The government covers all the necessary operating expenses, including laundry service and food.

Students spend the entire academic year in the schools. The schools operate Monday through Friday from approximately 7:30 a.m. to 3:45 p.m. and Saturday from 7:30 a.m. to 12:45 p.m. Outside of school hours, students can study in the dormitories and spend time with their classmates. Students have the opportunity to visit their families on weekends as long as a family member can pick them up from school. Because many students come from a different region than the school's region, some stay at the school on weekends, which offer new opportunities to interact with their peers.

Applicants are eligible for admission to COAR if they are ranked in the top ten of their public school cohort in the previous academic year. The admission process consists of two rounds. In the first round, applicants take a written test covering reading comprehension and mathematics. The highest-scoring applicants move on to second round where psychologists rate them based on two activities: a one-on-one interview, and observation of their peer interactions while they complete a set of tasks. Admission decisions are determined by a composite score of three tests, the region of origin, and the applicants' school preferences.

We collected three rounds of network surveys to observe how students form friendships and study groups, with the first round of surveys collected in December of 2016. The survey included questions on preferred roommates, friendships, study partnerships, and other social activities such as playing sports or games. During the 2017 school year, we partnered with the Ministry of Education to conduct an experiment to investigate the effect of the allocation to dormitories on students' learning and social outcomes. This experiment generates random variation in the physical proximity of students. We leverage this variation to test our predictions in Section 2.2. After the experiment, we collected two other rounds of social network surveys that we pooled together for the empirical analysis. Specifically, two students formed a social link if, in either of the two rounds surveys, one of them named the other as their friend, study partner, or someone with whom they engage in social activities such as sports and board games.

3.1 Descriptive Statistics

Even though the schools are designed for students enrolled in public schools, there is important variation in demographic characteristics of students enrolled in the COAR Network. Social workers perform a survey upon entrance of students in which they classify them either as poor or non-poor. We restrict our sample to the schools for which this information is available. Table 1 presents summary statistics of students in our sample. Approximately 43% of the students enrolled are male, around 25% come from a rural household, and 40% are classified as poor. Lower-achieving and less socially central students are more likely to come from a poor and rural household than higher-achieving and more socially central students.

The experiment, described in more detail in Zárate (2021) and Appendix B, estimates peer effects on social and academic skills by randomizing students to a particular peer type. First, we classify students into lower- or higher-achieving types. In particular, for all the school-by-grade-by-gender cells, students with an admission score above (below) the cell-specific median are classified as higher-achieving (lower-achieving). Second, we classify students as either less socially central or more socially central, based on the *eigenvector centrality*[2] of the social network at baseline. Students with a centrality above (below) the cell-specific median are classified as more (less) socially central. Because the social network surveys were not available for students who enroll in 2017, these students are not classified as less or more socially central. Hence, the analysis on social centrality focuses on the 2015 and 2016 cohorts. Table 1 presents the distribution of student types. By construction, 50% of the students are higher-achieving and 50% are more socially central.

Table 1 also presents summary statistics of the number of connections students have

[2] Eigenvector centrality measures a student's influence within his or her social network. High values indicate that a student is connected to many other individuals who also have high values.

after the experiment. We consider two students to be connected if either of them names the other as a friend, a study partner, or someone with whom they play sports or engage in other social activities. On average, a student has 10.5 friends, 6.6 study partners, and 8.2 social activities partners. These account for a total of 14.15 connections. Columns 2 and 3 show that poor and non-poor students have a similar number of links. Columns 4 and 5 show that lower-achieving students have approximately 0.5 fewer connections than higher-achieving students. Although not surprising, the most striking comparison is in columns 6 and 7: less-central students have, on average, 3.5 fewer connections than the more-central students.

3.2 Diagnosing Homophily

In light of Result 2, we explore whether social networks exhibit homophily by poverty, academic achievement, and centrality. We estimate the following equation:

$$y_{ic} = \alpha + \beta_1 \text{poor}_{ic} + \beta_2 \text{lower-achieving}_{ic} + \beta_3 \text{less-central}_{ic} + \gamma_c + \delta y_{ic_{t-1}} + \varepsilon_{ic}, \qquad (4)$$

where y_{ic} is the number of connections of individual i in cell c, given by the combination of school, cohort, and gender. Our outcomes include the total number of connections a student has after the intervention, as well as the number of connections with students of different types. The variables poor_{ic}, $\text{lower-achieving}_{ic}$, and less-central_{ic} are dummy variables that take the value of one when individual i is poor, lower-achieving, and less central, respectively. We control for the number of connections at baseline $y_{ic_{t-1}}$, and include cell fixed effects. Finally, ε_{ic} is an error term.

Table 2 reports the estimates of Equation (4). The first column reports the estimates on the total number of connections. The results show that poor students have on average 0.5 fewer connections than non-poor students. While lower-achieving students have on average 0.22 fewer links, this difference is statistically indistinguishable from zero. Less central students have 1.15 fewer links than more central students, and this difference is statistically different from zero.

In general, students have more connections with those similar to them. Columns 2 and 3 show that while poor and non-poor students have a similar number of connections with other poor students, there is a difference of 0.66 links (p-value< 0.001) with non-poor students. Columns 4 and 5 show that compared to higher-achieving students, lower-achieving students have around 1.05 more links with other lower-achieving students and around 1.25 fewer links with higher-achieving students. Finally, the results in column 6 and 7 show that compared to the more-central students, less-central students form around 0.23 more links (p-value$= 0.035$) with other less-central peers and 1.07 fewer links (p-value < 0.001) with more-central peers. Overall, this evidence shows that social networks exhibit homophily in academic achievement, poverty, and social centrality.

4 The Role of Proximity

4.1 Random Variation in Proximity

The experiment generates random variation in the physical proximity of two students in the allocation to dormitories. In particular, students' names were randomly sorted on a list, and the schools follow the list to allocate students to specific beds in the dormitories. The position of a student on the list depends on the type of the student (the combination of their academic achievement and social centrality) and the type of peer that they are randomly assigned. The order on the list predicts the physical distance between two students in dormitories. The details of this design are explained in Appendix B and in Zárate (2021).

We verified compliance of the experiment by comparing the lists we sent to the schools with the final allocation to dormitories. We corroborate whether school authorities followed the protocols by estimating the following equation on the likelihood that two students are neighbors in dormitories:

$$y_{ij} = \alpha + \gamma l_{ij}^d + \omega' X_{ij} + \phi_i + \phi_j + \theta_{g_i,g_j} + \theta_{\tau_i,\tau_j} + \varepsilon_{ij}, \tag{5}$$

where y_{ij} is a dummy variable equal to one when students i and j are direct neighbors in the allocation to dormitories.[3] The variable l_{ij}^d is a dummy variable equal to one when student j is in the neighborhood of size d of student i according to the list that schools followed to assign students to dormitories. That is:

$$l_{ij}^d = \begin{cases} 1 & \text{if } distance(i,j) \leq d, \\ 0 & \text{otherwise.} \end{cases}$$

We control for individual fixed effects ϕ_i, ϕ_j, and also for the combination of gender fixed effects, θ_{g_i,g_j}[4], as students are more likely to befriend peers of their same gender. Likewise, as part of the experimental design, students were more likely to be allocated to peers of the same type, so we also control for the combination of type fixed effects θ_{τ_i,τ_j}. Finally, ε_{ij} is an error term. We cluster the standard errors at the network (school-by-grade) level.

Panel A of Figure 1 presents the estimates for $d \in \{1, \ldots, 9\}$. The distance on the list predicts that students become neighbors in dormitories, which confirms that schools followed the list to assign students in the dormitories. Panel B presents the interaction of the neighborhood's size dummies with being a first-year. There is no difference in

[3] As there is variation in the size of dormitories, we define neighbors as follows: (i) for schools with dorm size less or equal to five, neighbors are roommates; (ii) for schools with larger dormitories, neighbors are students in the same bunk bed or in the adjacent bunk beds.

[4] These fixed effects correspond to whether students i and j are both female, both male, or one is a female and the other one a male.

how the distance on the list impacts the likelihood of being neighbors for first-years versus the other cohorts.

We also estimate Equation (5) on the likelihood that individuals i and j form of social interactions before and after the intervention. We define y_{ij} as a dummy variable equal to one when i and j form a social connection. We report the results for $d = 1$ in Table 3. Panel A of Table 3 shows that being adjacent on the list does not predict social interactions before the intervention. This is consistent with the location on the list being random and therefore independent of baseline connections. In contrast, Panel B of Table 3 shows that being adjacent on the list increases the likelihood of friendships by 12.8 percentage points, study partnerships by 8.4 percentage points, and social partnerships by 11.1 percentage points after the intervention. All of these estimates are statistically different from zero at the 99% level. These impacts account for a total increase of 16.6 percentage points (p-value < 0.01) on any of these social interactions. These results are consistent with the predictions of Result 1.

We also include an interaction term between adjacency and a dummy variable for first-year students. We include this interaction as first-years did not know each other before the intervention and the lists were also for the allocation to classrooms. Overall, the effects are more pronounced for first-year students, consistent with higher proximity impacts when students have not formed previous social links and they share the same classroom. For example, column 4 shows that the impact of adjacency on social interactions is higher by 23.6 percentage points (p-value < 0.01) for first-years. As seen in Panel B of Figure 1, these differences are not driven by higher compliance of the lists in dormitory assignments for first-years.

4.2 Heterogeneity under Learning-Based Homophily

We now explore whether there are heterogeneous effects of proximity by students' characteristics. In particular, we consider whether students from different poverty statuses, achievement levels, and centrality at baseline form more or less social connections with their neighbors than students that share these characteristics. Result 3 predicts that under learning-based homophily, the effect of proximity is higher for neighbors of different characteristics. As discussed in Section 2.3, we would expect the opposite conclusion under preference-based homophily.

To empirically verify Result 3, we estimate the following equation:

$$\begin{aligned} y_{ij} = {} & \alpha + \beta_p D_p + \beta_a D_a + \beta_s D_s + \\ & \gamma l_{ij}^d + \delta_p l_{ij}^d \times D_p + \delta_a l_{ij}^d \times D_a + \delta_s l_{ij}^d \times D_s + \\ & \omega' X_{ij} + \phi_i + \phi_j + \theta_{g_i,g_j} + \theta_{\tau_i,\tau_j} + \varepsilon_{ij}, \end{aligned} \tag{6}$$

where D_p, D_a, and D_s are dummy variables equal to one when students have a different poverty status, achievement level, and centrality at baseline, respectively. The remaining variables and parameters are as in Equation (5). The parameters $\beta \in \{\beta_p, \beta_a, \beta_s\}$ capture whether social networks exhibit homophily. Specifically, when $\beta < 0$ students of different types are less likely to form social interactions.

Our parameters of interest are $\delta \in \{\delta_p, \delta_a, \delta_s\}$, which capture whether the effect of proximity is weaker or stronger when students are of a different type. Learning-based homophily predicts that $\delta > 0$, whereas preference-based homophily predicts $\delta < 0$.

Panel C of Table 3 reports the estimates of Equation (6). First, consistent with the results in Table 2, the estimates show that social networks exhibit homophily. Column 4 shows that students with a different poverty status, achievement, and centrality are 0.8, 1.9, and 1.1 percentage points less likely to form social interactions. These estimates are all statistically significant at the 99% level and reduce the likelihood of social interactions between 7% and 17% of the sample mean.

The estimates in Panel C of Table 3 reject a theory of preference-based homophily. The results on academic achievement and poverty are consistent with learning-based homophily. The impact of being adjacent on the list is higher by 2.7 percentage points (p-value 0.072) for students from a different poverty status. Likewise, the impact of being adjacent on the list is higher for students from different academic achievement levels (3.3 percentage points, p-value 0.018). The point estimate for students with a different centrality is also positive, although not statistically significantly different from zero.

4.3 Larger Neighborhoods

In our estimates of Equation (6) so far, we reported the impact of being adjacent on the list. That is, $l_{ij}^d = 1$ if and only if i and j are immediate neighbors of each other in the list. In this section, we consider larger *neighborhoods* and consider i and j to be *neighbors* if they are within a distance $d \in \{1, \ldots, 9\}$ of each other on the list. We present the estimates of Equation (6) with values of d larger than one.

Panel A of Figure 2 shows that for larger neighborhoods, proximity still has a positive and highly significant effect on social connections. As expected, there is also a decreasing pattern of how distance on the list affects social connections. While being adjacent on the list (a neighborhood of size 1) increases social connections by 14 percentage points, being in a neighborhood of size 9 increases the likelihood of interaction by 9 percentage points.

Panels C to E of Figure 2 report heterogeneous effects of distance by students' characteristics. Similar to the case of $d = 1$, the results are consistent with a model of learning-based homophily. Students with a different poverty status are more likely to form social connections. In fact, this effect is greater (around 4 percentage points) and statistically significant at the 95% level when considering larger neighborhoods.

The heterogeneous effect by different academic achievement levels is slightly lower but still statistically significant for larger neighborhoods. Although estimates are positive for students with different social centrality at baseline, these impacts are statistically indistinguishable from zero.

5 Conclusion

This paper introduces a learning-based model of friendship formation that can explain patterns of homophily in social networks. In contrast to preference-based homophily, the model predicts that physical proximity and similarity are substitutes when students decide how to form social connections. Hence, the effect of proximity would be larger for dissimilar students.

We corroborate the model by leveraging an experiment at selective boarding schools in Peru that generates random variation on the physical proximity between students. We interpret this variation as changes in the cost of forming social connections. The empirical results show that social networks exhibit homophily along poverty, academic achievement, and baseline social centrality. However, the effect of proximity on social interactions is larger for students with different poverty statuses and academic achievement levels.

Considering the effects of within-school policy variation is also vital to understand the consequences of school integration policies. Even though social networks in the COAR Network exhibit homophily, policies that foster diversity by mixing students of different types in dormitories can enhance the structure of more diverse social networks.

Table 1: Summary Statistics

Variable	All students	By poverty		By Academic Achievement		By Social Centrality	
		Poor students	Non-poor students	Lower-achieving	Higher-achieving	Less central	More central
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Demographics							
Male (%)	0.43	0.44	0.43	0.43	0.43	0.41	0.40
Rural (%)	0.25	0.43	0.14	0.29	0.21	0.29	0.23
Poor (%)	0.39	1.00	0.00	0.44	0.35	0.44	0.38
Network variables							
Connections	14.15	14.20	14.12	13.93	14.37	11.51	14.97
	(6.85)	(6.23)	(7.22)	(6.90)	(6.78)	(5.55)	(5.99)
Connections with poor students	5.56	6.93	4.66	5.64	5.47	4.88	6.11
	(4.11)	(4.41)	(3.63)	(4.16)	(4.05)	(3.81)	(4.48)
Connections with lower-achieving students	7.03	7.27	6.88	7.53	6.53	5.76	7.34
	(4.25)	(3.98)	(4.41)	(4.57)	(3.84)	(3.31)	(3.57)
Connections with less central students	9.58	9.43	9.68	9.50	9.66	5.76	5.96
	(7.23)	(6.73)	(7.53)	(7.21)	(7.25)	(3.24)	(3.34)
N	5,467	2,153	3,314	2,732	2,735	1,696	1,687

Notes: This table reports summary statistics for demographic characteristics and the total number of connections students have with their peers. The demographic characteristics include whether the student is a male, whether they come from a rural household, or from a household classified as poor. Column 1 reports the statistics for all students enrolled at the COAR Network in 2017. Columns 2 and 3 classifies students by poverty status, columns 4 and 5 by academic achievement, and columns 6 and 7 by baseline social centrality. The standard deviations are in parenthesis.

Table 2: Estimates of Students' Characteristics on Number and Type of Connections

	Number of connections with:						
	All students	Poor students	Non-poor students	Lower-achieving	Higher-achieving	Less central	More central
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Poor	-0.527**	0.138	-0.660***	-0.050	-0.469***	-0.464**	-0.038
	(0.168)	(0.085)	(0.118)	(0.113)	(0.106)	(0.143)	(0.075)
Lower-achieving	-0.220	0.167**	-0.381***	1.051***	-1.265***	-0.071	-0.131*
	(0.154)	(0.077)	(0.108)	(0.101)	(0.097)	(0.131)	(0.071)
Less central	-1.153***	-0.276**	-0.681***	-0.451***	-0.594***	0.228**	-1.074***
	(0.247)	(0.098)	(0.168)	(0.121)	(0.137)	(0.101)	(0.163)
Mean	14.15	5.56	8.59	7.03	7.12	9.58	4.56
N	5,467	5,467	5,467	5,467	5,467	5,467	5,467

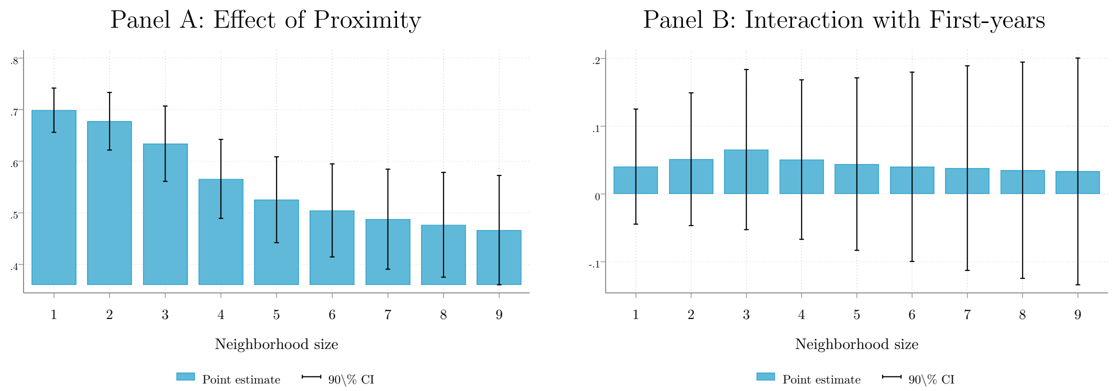
Notes: This table reports estimates of the impact of students' characteristics on the number and type of connections. Specifically, the estimates show how poor, lower-achieving, and less socially central students have different connections and links with specific types of students. Standard errors are clustered at the peer-group-type-by-student-type level; *** p-value<0.01, ** p-value<0.05, * p-value<0.1.

Table 3: The Impact of Proximity on Social Interactions

Dependent variable:	Friendships (1)	Study groups (2)	Social activities (3)	Any connection (4)
	Panel A: Connections at Baseline			
Adjacent	0.004	0.002	0.010*	0.007
	(0.005)	(0.005)	(0.005)	(0.006)
Mean	0.04	0.02	0.03	0.06
N	213,799	213,799	213,799	213,799
	Panel B: Connections at Endline			
Adjacent	0.128***	0.084***	0.111***	0.166***
	(0.013)	(0.009)	(0.010)	(0.014)
Adjacent × first years	0.210***	0.188***	0.195***	0.236***
	(0.038)	(0.030)	(0.030)	(0.039)
Mean	0.08	0.05	0.07	0.11
N	348,079	348,079	348,079	348,079
	Panel C: Heterogeneous Effects			
Adjacent	0.113***	0.071***	0.101***	0.143***
	(0.016)	(0.012)	(0.014)	(0.018)
Adjacent × first years	0.207***	0.185***	0.196***	0.237***
	(0.038)	(0.031)	(0.030)	(0.039)
Different poverty status	-0.007***	-0.003***	-0.007***	-0.008***
	(0.001)	(0.001)	(0.001)	(0.002)
Different achievement level	-0.016***	-0.011***	-0.011***	-0.019***
	(0.002)	(0.002)	(0.002)	(0.003)
Different social centrality	-0.011***	-0.007***	-0.011***	-0.011***
	(0.002)	(0.001)	(0.002)	(0.002)
Adjacent × different poverty status	0.015	0.012	0.012	0.027*
	(0.014)	(0.011)	(0.015)	(0.015)
Adjacent × different achievement level	0.032**	0.030**	0.014	0.033**
	(0.013)	(0.014)	(0.013)	(0.014)
Adjacent × different social centrality	-0.002	-0.007	0.006	0.008
	(0.016)	(0.013)	(0.016)	(0.016)
Mean	0.08	0.05	0.07	0.11
N	348,079	348,079	348,079	348,079

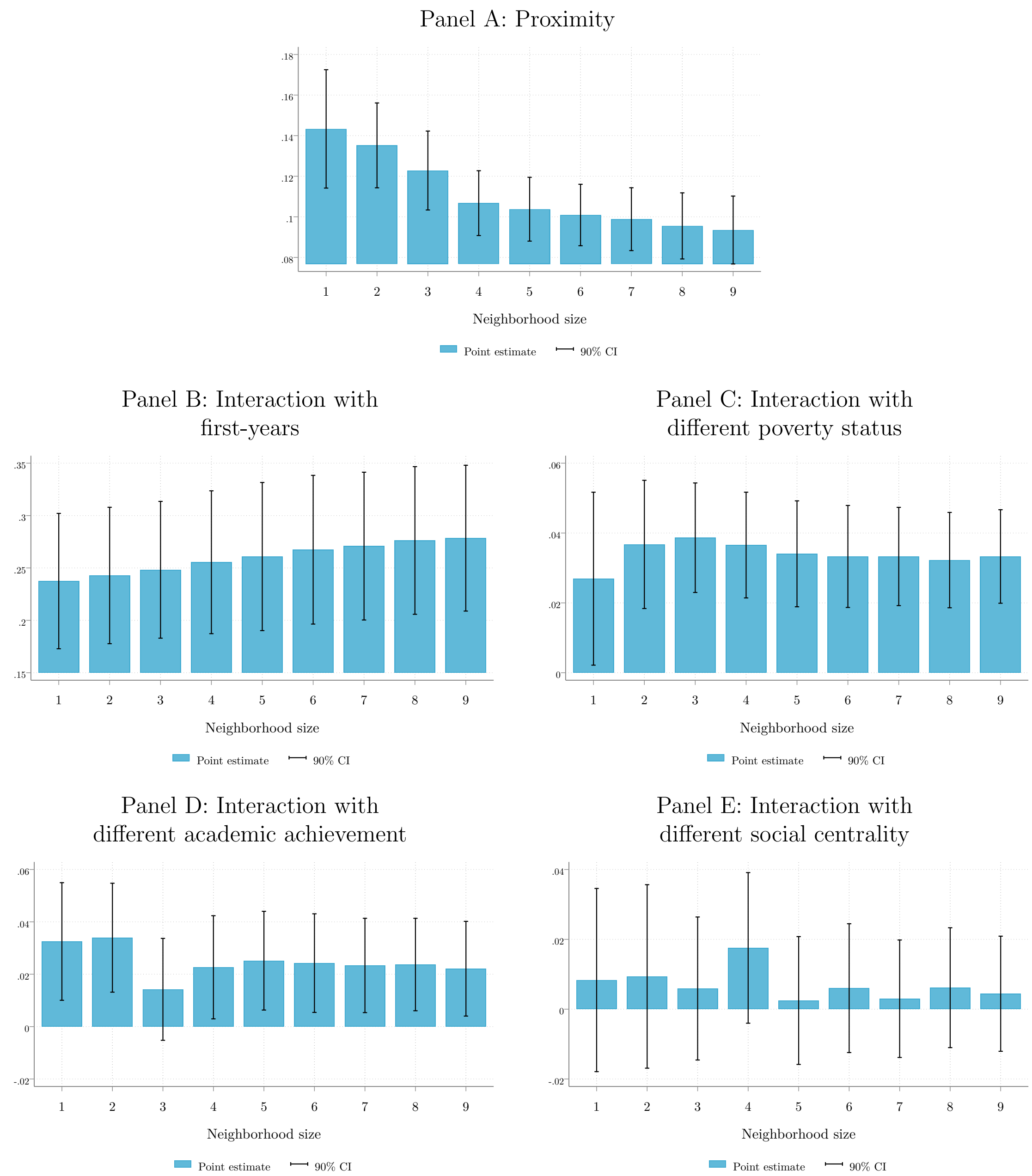
Notes: This table reports the impact of being adjacent on the list on social interactions at the dyad level. Panel A presents the estimates on connections before the intervention, Panel B presents the estimates on connections after the intervention, and Panel C heterogeneous effects of being neighbors by students' covariates. All estimations control for school-by-grade-by-gender and individual fixed effects. Standard errors are clustered at the network (school-by-grade) level; *** p-value < 0.01, ** p-value < 0.05, * p-value < 0.1.

Figure 1: Impact of Proximity on Being Neighbors in Dormitories



Notes: This figure presents estimates of Equation (5) for different values of distance on the list, d. Panel A shows the direct effect of being at a distance less or equal to d on the list, and Panel B presents the differentiated effect for first-year students.

FIGURE 2: Heterogeneous Effects of Proximity on Social Interactions for Other Values of Distance



Notes: This figure presents estimates of Equation (6) at different values of distance on the list, d. Panel A shows the direct effect of being at a distance less or equal to d on the list, and Panel B presents the differentiated effect for first-year students. Panels C, D, and E report the interaction with whether students have a different poverty status, academic achievement, and social centrality, respectively.

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Appendix

A Proofs

Proof of Result 1. Since $\kappa_j = \kappa_{j'}$, $\lambda(\kappa_i, \kappa_j) = \lambda(\kappa_i, \kappa_{j'})$. Moreover, $n_{ij} = 1$, $n_{ij'} = 0$ and Assumption 1 implies that $c(n_{ij}) < c(n_{ij'})$. The result then follows from Equation (3). □

Proof of Result 2. Since $n_{ij} = n_{i\ell}$, $c(n_{ij}) = c(n_{i\ell})$. Moreover, $\kappa_{j,-k} = \kappa_{\ell,-k}$, $\kappa_{j,k} = \kappa_{i,k}$, $\kappa_{\ell,k} \neq \kappa_{i,k}$ and Assumption 2 implies that $\lambda(\kappa_i, \kappa_j) > \lambda(\kappa_i, \kappa_\ell)$. The result then follows from Equation (3). □

Proof of Result 3. Defining:

$$\gamma(c, \lambda) \equiv p_0 \cdot \left(1 - \frac{1-p_0}{p_0} \frac{r}{r+\lambda} \frac{c}{v-c}\right)$$

By Equation (3), $\gamma_{ij} = \gamma(c(n_{ij}), \lambda(\kappa_i, \kappa_j))$ for any $i, j \in I$. Then,

$$\begin{aligned}
\gamma_{ij} - \gamma_{ij'} &= \gamma(c(n_{ij}), \lambda(\kappa_i, \kappa_j)) - \gamma(c(n_{ij'}), \lambda(\kappa_i, \kappa_{j'})) \\
&= \gamma(c(1), \lambda(\kappa_i, \kappa_j)) - \gamma(c(0), \lambda(\kappa_i, \kappa_{j'})) \\
&= \gamma(c(1), \lambda(\kappa_i, \kappa_j)) - \gamma(c(0), \lambda(\kappa_i, \kappa_j)) \\
&= \int_{c(0)}^{c(1)} \frac{\partial}{\partial c} \gamma(c, \lambda(\kappa_i, \kappa_j)) \cdot dc \\
&= -\int_{c(1)}^{c(0)} \frac{\partial}{\partial c} \gamma(c, \lambda(\kappa_i, \kappa_j)) \cdot dc
\end{aligned}$$

and

$$\gamma_{i\ell} - \gamma_{i\ell'} = -\int_{c(1)}^{c(0)} \frac{\partial}{\partial c} \gamma(c, \lambda(\kappa_i, \kappa_\ell)) \cdot dc$$

Therefore,

$$\begin{aligned}
(\gamma_{ij} - \gamma_{ij'}) - (\gamma_{i\ell} - \gamma_{i\ell'}) &= -\left(\int_{c(1)}^{c(0)} \frac{\partial}{\partial c} \gamma(c, \lambda(\kappa_i, \kappa_j)) \cdot dc - \int_{c(1)}^{c(0)} \frac{\partial}{\partial c} \gamma(c, \lambda(\kappa_i, \kappa_\ell)) \cdot dc\right) \\
&= -\int_{c(1)}^{c(0)} \left(\frac{\partial}{\partial c} \gamma(c, \lambda(\kappa_i, \kappa_j)) - \frac{\partial}{\partial c} \gamma(c, \lambda(\kappa_i, \kappa_\ell))\right) dc \\
&= -\int_{c(1)}^{c(0)} \int_{\lambda(\kappa_i, \kappa_\ell)}^{\lambda(\kappa_i, \kappa_j)} \frac{\partial^2}{\partial c \partial \lambda} \gamma(c, \lambda) \cdot d\lambda dc
\end{aligned}$$

Since $\frac{\partial^2}{\partial \kappa \partial \lambda} \gamma(\kappa, \lambda) = (1 - p_0) \frac{r}{(r+\lambda)^2} \frac{v}{(v-\kappa)^2} > 0$, the result follows. □

B Experimental Design

The experiment described in detail in Zárate (2021) estimates peer effects in academic achievement and social centrality. In the design, students are classified into types using the median of baseline achievement and centrality and, conditional on student's type, randomizes them to the type of peer. In this appendix, we describe the experimental design and how it generates random variation in students' distance in the allocation to dormitories.

The experimental design guarantees strong variation in academic achievement and social centrality by proceeding in two steps. First, it classifies students into types defined by the quantiles of the distribution of the variable of interest x. Second, students are randomly assigned to the type of peer. Each type of peer corresponds to a treatment arm or a control group. By randomizing students to the type of peer instead of randomizing them to groups, the design ensures that students neighbor peers with different levels of achievement and social centrality. In particular, assigning students to a type of peer is equivalent to allocating students to *peer-group types*, where a peer-group type is the combination of a student's own type and her treatment arm.

To better understand this, consider the simple case of two types of students classified by the median of the distribution of x, and let's denote by τ the set of types, with $\tau = \{high, low\}$. Students can be assigned either to the high-type-peer treatment, in which they are paired with high-type students, or to the control group, in which they are paired with low-type students.

The randomization to the treatment is equivalent to allocating students to peer-group types—that is, combinations of the student's own type and the type of her peer. There are three peer-group types with two types of students and one treatment (high-type peers): two *homogenous* group types, each composed of individuals of a single type, and one *heterogeneous* group type, composed of individuals of both types:

a) Group A: a group composed of the high type only.

b) Group B: a mixed group, in which half are high-type students and the other half are low-type students.

c) Group C: a group composed of the low type only.

Denote the peer-group type by ρ_i, which is a function of a student's type and her assigned type of peer. Here, $\rho_i = \rho(\tau_i, t_i)$, where τ_i denotes the type of the student, and t_i the assigned peer type. The following matrix shows the composition of peer-group types:

	High	Low
High	Group A	Group B
Low	Group B	Group C

Each row in this matrix represents a type of student, and each column the type of peer. Notice that the diagonal of the matrix shows all combinations of a single type of student. Outside of the diagonal, the matrix is symmetric, as students are matched to peers of the opposite type and therefore jointly have the same peer-group type (Group B). This implies that ρ_i is a symmetric function in τ and t: $\rho(\tau_i, t_i) = \rho(t_i, \tau_i)$.

The experiment uses data from the baseline social-network survey to identify socially central students, and performance on the admissions test to identify higher-achieving students. The randomization in the experiment is analogous to the one-variable example introduced before. However, with two variables of interest, rather than two types of students, there are four types of students. The four types of students are also the peer types that define the treatment groups. These four combinations are the following:

1. Less socially central and lower-achieving peers
2. Less socially central and higher-achieving peers
3. More socially central and lower-achieving peers
4. More socially central and higher-achieving peers

Notice that this design is equivalent to two treatments—more socially central peers and higher-achieving peer—and an interaction between the two.

Instead of the three peer-group types (A, B, and C) in the simplest case, there are ten potential peer-group types.[5] Figure B.1 shows the ten possible combinations of types of peers and student types. Each row corresponds to the student type, each column to the type of peer to whom she was assigned, and each cell to the combination of student type and type of peer–that is, the peer-group type.[6] Each group takes a different cell color in the symmetrical matrix of Figure B.1.

After randomizing students to peer-group types, as described above, the design uses these groups to allocate students to the dormitories of the COAR Network. The structure of dormitories across the COAR Network is heterogeneous. For example, while the school in Lima has dormitories of three to five students, its counterpart in Cusco has four dormitories, with approximately seventy-five students per dormitory.

[5]With four types of students, there are sixteen possible combinations, but six of them are redundant, as $\rho(\tau_i, t_i) = \rho(t_i, \tau_i)$.

[6]Group 1, for example, is composed of only more socially central and higher-achieving students. Group 3 is composed of (i) less socially central and higher-achieving students and (ii) more socially central and lower-achieving students.

FIGURE B.1: Groups of Peers in the Experimental Design

Student Type	Type of Peers: higher-achieving more central	higher-achieving less central	lower-achieving more central	lower-achieving less central
higher-achieving more central	Group 1	Group 2	Group 3	Group 4
higher-achieving less central	Group 2	Group 5	Group 6	Group 7
lower-achieving more central	Group 3	Group 6	Group 8	Group 9
lower-achieving less central	Group 4	Group 7	Group 9	Group 10

Notes: this figure shows the ten peer-group-types in my experimental design. It represents all possible combinations between student type and type of peers. Rows are described by student types, and columns show the types of peers to which they were randomly assigned. The diagonal of the matrix is composed by groups of a single type. The matrix is symmetric by virtue of the fact that students are matched with peers of the assigned type.

To make the peer-group types consistent with the widely varying number of dorm sizes across schools, students' names are sorted on a list based on the ten peer-group types. This list was later used to allocate students to specific beds in the dormitories. The position of each student on the list was determined as follows:

(i) For each school-by-grade-by-gender, the peer-group types were randomly ordered on the list.

(ii) Within each peer-group type, the students' order is also random. The only condition is that in the mixed groups —those with two different types of students—the list alternates between students of different types. This rotation guarantees that the type of the closest neighbors in the dormitories is always the student's treatment.

To understand how the design works in practice, we present an example with twelve students and the two-type design introduced earlier in the paper. Recall that with two types of students, there are three potential peer-group types: Group A (only high types), Group B (the mixed group), and Group C (only low types). In the first step, the peer-group types are randomly ordered on the list, and one of six potential orders is selected.[7] In the following example, we assume that the random order selected is Group A-Group C-Group B:

$$\underbrace{H-H-H-H}_{\text{Group A}}-\underbrace{L-L-L-L}_{\text{Group C}}-\underbrace{H-L-H-L}_{\text{Group B}}$$

[7] The six potential orders are: (i) Group A-Group B-Group C, (ii) Group A-Group C-Group B, (iii) Group B-Group A-Group C, (iv) Group B-Group C-Group A, (v) Group C-Group A-Group B, (vi) Group C-Group B-Group A.

Within each peer-group type, students are randomly ordered, subject to the condition that students in the mixed group alternate. The letter H or L represents the student's type, and those in red are the students assigned to high-type-peer treatment.

By using the lists, the design can adapt the allocation of students to different types of dorms. For example, if students were assigned to six dorms of two students each, the assignment would be as follows:

$$\underbrace{H - H}_{\text{Dorm 1}} - \underbrace{H - H}_{\text{Dorm 2}} - \underbrace{L - L}_{\text{Dorm 3}} - \underbrace{L - L}_{\text{Dorm 4}} - \underbrace{H - L}_{\text{Dorm 5}} - \underbrace{H - L}_{\text{Dorm 6}},$$

where each student would end up with exactly one roommate whose type would always correspond to the assigned treatment. The list also determines the allocation for larger dorms. For example, if the dormitories' size was four students, then the allocation to dorms would perfectly coincide with the peer-group types:

$$\underbrace{H - H - H - H}_{\text{Dorm 1}} - \underbrace{L - L - L - L}_{\text{Dorm 2}} - \underbrace{H - L - H - L}_{\text{Dorm 3}}$$

There might be some cases of noncompliance, when the dorm structure does not coincide with the size of the peer-group types. For example, consider the case in which the dorms house three students. The allocation would be as follows:

$$\underbrace{H - H - H}_{\text{Dorm 1}} - \underbrace{H - L - L}_{\text{Dorm 2}} - \underbrace{L - L - H}_{\text{Dorm 3}} - \underbrace{L - H - L}_{\text{Dorm 4}},$$

For some students, there is noncompliance between the treatment and the type of the neighbor. For example, the last student in Group A ends up with low-type roommates despite being assigned to the high-type-peers treatment. While this noncompliance due to the dorm structure would weaken the first stage, the allocation for most students would still guarantee that the assignment to the treatment could be used as an instrument for peer characteristics in the group.

The list is also flexible enough to adapt to large dorms by placing in the same bunk bed or in adjacent bunk beds, students who are adjacent to each other on the list. Overall, the list's order is directly linked to the physical distance between two students in a dormitory. Students who are adjacent on the list are more likely to be near each other in the dormitories. In small dorms, the assigned peers likely share the same room. In bigger dorms, students and assigned peers are placed either in the same bunk bed or in beds next to each other. Our empirical strategy exploits the random variation on the list as a measure of proximity.