Bilkent University
Econ 101 - Fall 2022
Chapter 4: Towards the Demand Curve

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After the detour with two agents, we now go back to the one-agent model. Now that we know how the consumer chooses her optimal bundle, we have the machinery to study how the optimal bundle changes with the parameters of the model (i.e., income of the consumer and prices of goods.) This is the fun stuff!

1 What If the Income Changes?

Let's start with an simple case. Consider an increase in a consumer's income (i.e., I goes up.) What happens?

Mathematically, the optimization problem changes because the *constraint set* changes. But that's fine – we did our analysis using a generic set of parameters, so the analysis still applies. In general, the optimal bundle $q^* = (q_1^*, q_2^*)$ satisfies

$$p_1 q_1^* + p_2 q_2^* = I$$

Moreover, if $q_1^* > 0$, $q_2^* > 0$, and preference is smooth, then

$$\text{MRS}_{2,1}(q^*) = \frac{p_1}{p_2}$$

So far so good. Just solve this problem with a higher I. Geometrically, it corresponds to shifting the budget line higher, finding a new indifference curve tangent to it, and marking the point of tangency as the optimal bundle.

Let's do this graphically. Notation:

- Fix the prices at p_1 and p_2.
- Initial income: I^i.
- Optimal bundle under initial income: $q^i = (q_1^i, q_2^i)$.
- Final income: I^f.
- Optimal bundle under final income: $q^f = (q_1^f, q_2^f)$.

If $I^f > I^i$, it may look like Figure 1. The red line is the budget line under I^i. The dark red line is the budget line under I^f. Note that the two budget lines are parallel, because their slopes are the same: they are $-\frac{p_1}{p_2}$, which we keep fixed for this exercise. The dark red line is higher than the red line, because $I^f > I^i$.

The first thing that you should realize is that the consumer is *at least as happy as before* when she consumes q^f rather than q^i. There are two ways in which you can verify this.

1. The consumer is *richer* under income I^f compared to income I^i. This is because $I^f > I^i$, but you can verify this by looking at Figure 1. The budget set under I^f is *larger* than the budget set under

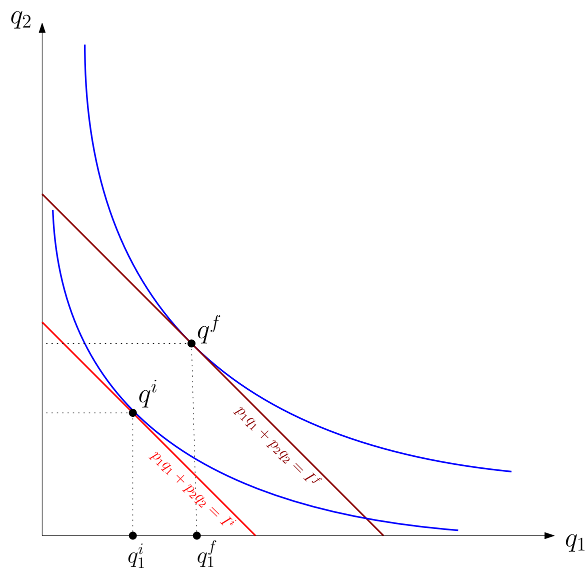

Figure 1: Optimal bundles under I^i and I^f.

I^i. This means that any feasible bundle under I^i is also feasible under I^f. Therefore, any bundle that the consumer can afford initially, she can also afford now. This means that the consumer **cannot be** worse off! In the worst case, she can consume the same bundle, q^i. This implies that q^f must be at least as good as q^i, i.e.

$$q^f \ \mathcal{R} \ q^i$$

2. Just eyeballing Figure 1, you can see that q^f is on a *higher* indifference curve than q^i. This is not surprising: because the consumer is richer, she cannot find her in a lower indifference curve. A higher indifference curve means that

$$q^f \ \mathcal{P} \ q^i$$

You may be tempted to say "But isn't there a third way in which we can verify $q^f \ \mathcal{P} \ q^i$? Monotonicity?" My answer is: yes for Figure 1, but not in general. Because one may have: $q_1^f < q_1^i$, but $q_2^f > q_2^i$. In such a case, monotonicity would not imply a preference between q^i and q^f. For instance, you may have a case like Figure 2. You can verify, using bullet points 1 and 2 above, that consumer is at least as happy as before when she consumes q^f rather than q^i. It is not due to monotonicity, though!

This begs the question: what is the exact difference between Figure 1 and Figure 2? Here is the answer.

- If the indifference curves are as in Figure 1, the consumer consumes more of good 1 when she has higher income. We call goods like these **normal goods**.

  Definition 1. *Good i is a* **normal good** *if the consumer's consumption of good i increases with the consumer's income.*

  Examples of normal goods: goods that you consume more as you get richer. Cars, iPhones, sweaters, herbal teas, dishwashers...

- If the indifference curves are as in Figure 2, the consumer consumes *less* of good 1 when she has higher income. We call goods like these **inferior goods**.

  Definition 2. *Good i is an* **inferior good** *if the consumer's consumption of good i decreases with the consumer's income.*

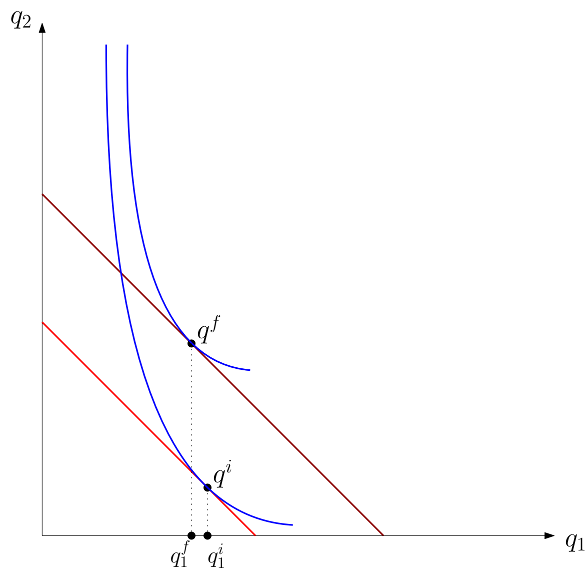

Figure 2: Optimal bundles under I^i and I^f, when good 1 is an inferior good.

Examples of inferior goods: goods that you consume less as you get richer. Public transportation, rice, bulgur, instant noodle, instant coffee...

This classification of goods into two categories will be useful later.

You may have two questions at this point.

1. What if $I^f < I^i$? Just switch the labels of I^i and I^f. The budget line shifts inwards, and the consumer becomes worse off. If good 1 is a normal good, the consumer's consumption of good 1 decreases as the income decreases. If good 1 is an inferior good, the consumer's consumption of good 1 increases as the income decreases.
2. What if good 2 is an inferior good? Once again, just switch the labels of goods. If good 2 is a normal good, the consumer's consumption of good 2 increases as the income increases. If good 2 is an inferior good, the consumer's consumption of good 2 decreases as the income increases.

Below, I summarize what we have discussed so far. The relationship between the consumption of good i under optimal bundle (q_i^*) and income I is as follows.

	as $I \uparrow$...	as $I \downarrow$...
if i is a normal good, q_i^*...	$\uparrow$	$\downarrow$
if i is an inferior good, q_i^*...	$\downarrow$	$\uparrow$

2 What If the Price of a Good Changes?

Now, let's move on to a slightly more complicated case. Consider an increase in the price of good 1 (i.e., p_1 goes up.) Notation:

- Fix the income at I and the price of good 2 at p_2.
- Initial price of good 1: p_1^i.
- Optimal bundle under initial price of good 1: $q^i = (q_1^i, q_2^i)$.
- Final price of good 1: p_1^f.
- Optimal bundle under final price of good 2: $q^f = (q_1^f, q_2^f)$.

We can conduct a graphical analysis. If $p_1^f > p_1^i$, it may look like Figure 3. The red line is the budget line under p_1^i. The dark red line is the budget line under p_1^f. Note that the two budget lines are **not** parallel. The slope of the red line is $-\frac{p_1^i}{p_2}$, and the slope of the dark red line is $-\frac{p_1^f}{p_2}$. The budget set under the final price is smaller than the budget set under the initial price, because $p_1^f > p_1^i$.

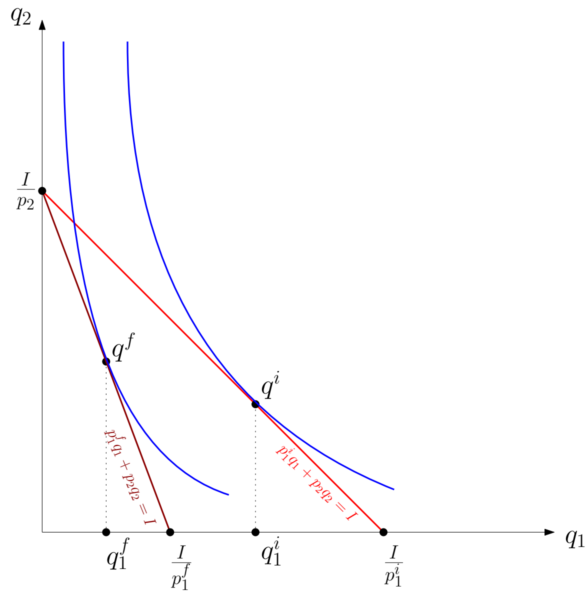

Figure 3: Optimal bundles under prices p_1^i and p_1^f.

Now, my claim is that the consumer is *at most as happy as before* when she consumes q^f rather than q^i. There are two ways in which you can verify this.

1. The consumer is *effectively poorer* under price p_1^f compared to price p_1^i. You can verify this by looking at Figure 3. The budget set under p_1^f is *smaller* than the budget set under p_1^i. This means that some feasible bundles under p_1^i are not feasible under p_1^f any more. The *purchasing power* of the consumer has decreased, even though she has the same income as before!

2. Just eyeballing Figure 1, you can see that q^f is on a *lower* indifference curve than q^i. This is because the consumer is effectively poorer.

What I am trying to say is: there is an **income effect** hidden in this graph. In Figure 3, the consumer reduces her consumption of good 1 from q_i^i to q_i^f due to two reasons.

1. Due to the **income effect**, the consumer is poorer. If good 1 is a normal good, the consumer reduces her consumption of good 1.

2. The relative price of good 1 in terms of good 2 is higher! Even if the consumer was not effectively poorer, she would choose to consume less of good 1 and more of good 2. Why?

   Mathematically: Good 2 is now relatively cheaper, so that the consumer can reduce her consumption of good 1 a little bit and consume a lot of good 2 instead. Recall that the optimal bundle requires marginal rate of substitution of good 2 for good 1 to be equal to the price ratio. If price ratio is higher, the marginal rate of substitution is higher. But if the preferences satisfy diminishing marginal rate of substitution, this is achieved only when the quantity of good 1 is lower and the quantity of good 2 is higher.

   Economically: The trade-off between good 1 and good 2 has changed. Now, in order to consume the same amount of good 1, the consumer needs to give up more of good 2. That is, the **cost** of good 1 in terms of good 2 is higher. Because of this, the consumer is less willing to consume good 1.

In any case, the consumer would *substitute* some of good 1 with good 2. This would happen *even if the consumer was not effectively poorer*. The consumer just finds it optimal to reduce her consumption of good 1 and increase her consumption of good 2. This is called the **substitution effect**.

So, in Figure 3, $q_1^f < q_1^i$ due to two effects. We want to decompose these two effects: how much is the reduction in quantity of good 1 due to the consumer being poorer, and how much of it is due to good 1 being more expensive relative to good 2?

Recall what I said just above: the substitution effect would work towards the reduction in the quantity of good 1 "even if the consumer was not effectively poorer". This is the key: how can we think of a consumer who is not effectively poorer when the price of good 1 changes? The idea is: we will, hypothetically, **compensate** the consumer for the price change. That is, we will imagine we increase the consumer's income up to the point where, under the new prices, she is exactly as happy as she was before under the old prices.

More formally, we will find a level of income I^c such that the following holds. Suppose, under the prices p_1^f and p_2, if the consumer's income was I^c, her optimal bundle would be $q^c = (q_1^c, q_2^c)$. We want this optimal bundle to satisfy:

$$q^c \; \mathcal{I} \; q^i$$

This construction makes sure that the consumer is exactly as happy as before, even though she is consuming a different bundle. After all, she is indifferent! This means that she is **compensated** for the increase in the price of good 1.

We will call I^c the **compensated income**, and q^c the **compensated demand**. The naming choice should be obvious by now.

Graphically, what we are doing is shifting the dark red line in Figure 3 until it is tangent to the indifference curve that contains q^i. The tangency point is q^c.

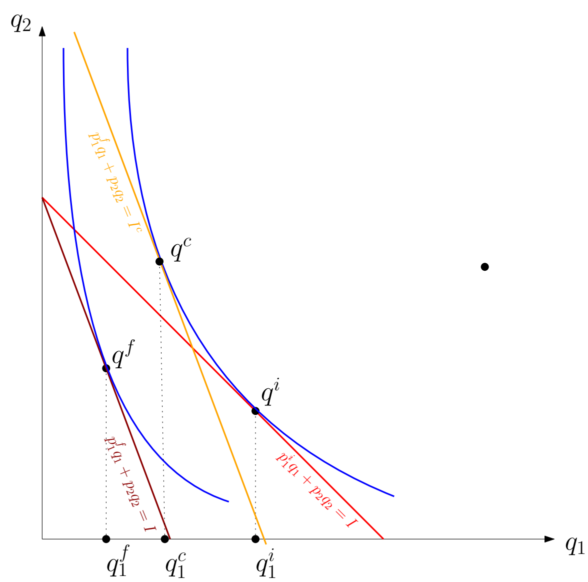

Figure 4: Compensated demand for good 1 (q_1^c) and compensated income I^c.

In Figure 4, the orange line is the budget line under compensated income I^c. If the consumer's income was I^c instead of I, she would consume q^c and be exactly as happy as if she was consuming q^i. Therefore,

- The move from q^i to q^c is due to the change in relative prices. It isolates the consumer's unhappiness due to being effectively poorer! She is as happy as before, she is just finding it optimal to consume less of good 1 and more of good 2 because good 1 is relatively more expensive.

- The move from q^c to q^f is due to the consumer being poorer. There is no effect of relative price, the consumer just changes her consumption because she is poorer.

Now, the move $q^c \rightarrow q^f$ should be familiar to you: this is the **income effect**. The comparison of q_1^c and q_1^f is the same as before. If good 1 is a normal good, $q_1^f < q_1^c$. If good 1 is an inferior good, $q_1^f > q_1^c$.

But what about the relationship between q_1^i and q_1^c? That is, what is the direction of **substitution effect**? My claim is that, as long as diminishing marginal rate of substitution is satisfied, we must have $q_i^c < q_1^i$. Why?

- **Intuitively**, the move from q_1^i captures the effect of good 1 being relatively more expensive in terms of good 2. When something is more expensive, you consume less of it!
- **Mathematically**, q^i in Figure 4 satisfies:

$$MRS_{2,1}(q^i) = \frac{p_1^i}{p_2}$$

  and, by construction, q^c satisfies:

$$MRS_{2,1}(q^c) = \frac{p_1^f}{p_2}$$

  But since $p_1^f > p_1^i$, $\frac{p_1^f}{p_2} > \frac{p_1^i}{p_2}$. Therefore,

$$MRS_{2,1}(q^c) > MRS_{2,1}(q^i)$$

  But recall that q^c and q^i are on the same indifference curve by construction! Since the preferences satisfy diminishing MRS, $MRS_{2,1}(q^c) > MRS_{2,1}(q^i)$ is satisfied only when q^c is to the northwest of q^i. Then, we must have $q_1^c < q_1^i$.

This is what I am saying: as long as the diminishing marginal rate of substitution is satisfied, for any good i:

	as $p_i \uparrow$...	as $p_i \downarrow$...
the substitution effect is such that q_i^*...	$\downarrow$	$\uparrow$

But recall that the total effect is a combination of substitution effect and income effect. For a normal good, let's put them together.

as $p_1 \uparrow$...	substitution effect	income effect ($I \downarrow$)	total effect
if i is a normal good, q_i^*...	$\downarrow$	$\downarrow$	$\downarrow$

Both effects work in the same direction! The total effect is a decrease in the quantity of good 1 consumed. Graphically, this looks like Figure 4. In Figure 5 below, I demonstrate the two effects. Recall that $q_1^i \rightarrow q_1^c$ is the substitution effect, and $q_1^c \rightarrow q_1^f$ is the income effect. As long as diminishing MRS is satisfied, $q_1^c < q_1^i$. As long as good 1 is a normal good, $q_1^f < q_1^c$.

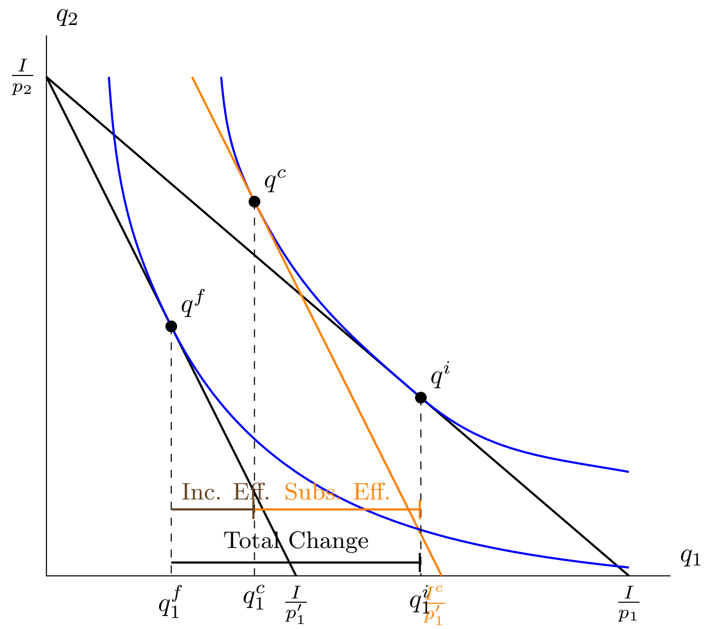

Figure 5: Effect of an increase in the price of good 1, when good 1 is a normal good.

What is good 1 is still a normal good, but it price decreases? You can just imitate the same analysis. All the effects will be reversed.

as $p_1 \downarrow$...

	substitution effect	income effect ($I \uparrow$)	total effect
if i is a normal good...	$q_i^* \uparrow$	$q_i^* \uparrow$	$q_i^* \uparrow$

See Figure 6 below.

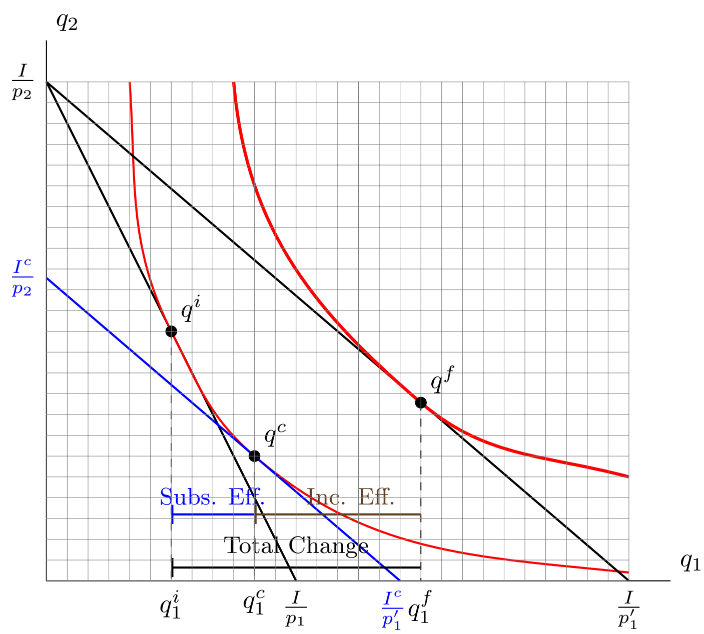

Figure 6: Effect of a decrease in the price of good 1, when good 1 is a normal good.

The next question is: what if good 1 is an inferior good and the price of good 1 increases? The substitution effect would still work in the direction of reducing the consumption of good 1. But now, income effect pulls in the opposite direction.

	as $p_1 \uparrow$...		
	substitution effect	income effect ($I \downarrow$)	total effect
if i is an inferior good, q_i^*...	$\downarrow$	$\uparrow$	?

Hm, this looks like a tricky case. If the income effect dominates, the consumer consumes more of good 1 when it is more expensive! What is happening? The consumer is so poor as a result of the price change that she moves away from higher quality consumption options and starts consuming good 1 even more.

We economists call such good **Giffen goods**, named after Robert Giffen. In a letter written to his friend Alfred Marshall, Giffen suggested the following phenomenon: in the late 19th century, as the price of bread increased, very poor individuals in Britain consumed more bread! Here is a quote from Wikipedia:

> As Mr. Giffen has pointed out, a rise in the price of bread makes so large a drain on the resources of the poorer labouring families [...] that they are forced to curtail their consumption of meat and the more expensive farinaceous foods: and, bread being still the cheapest food which they can get and will take, they consume more, and not less of it.
>
> *-Alfred Marshall, 1895*

Formally,

Definition 3. *Good i is a* **Giffen good** *if the consumer's consumption of good i increases as the price of good i increases.*

Note that for a good to be a Giffen good, it has to be in inferior good: the income effect should pull towards an increase in the quantity consumed. But being an inferior good is not enough in itself! The good has to be **so inferior** that the income effect must dominate the substitution effect! Graphically, it looks like Figure 7.

I would claim that having a Giffen good is a mathematical possibility, but economically it is so unlikely that we can just assume it away. A good such that as it becomes more expensive, you buy more of it! I don't

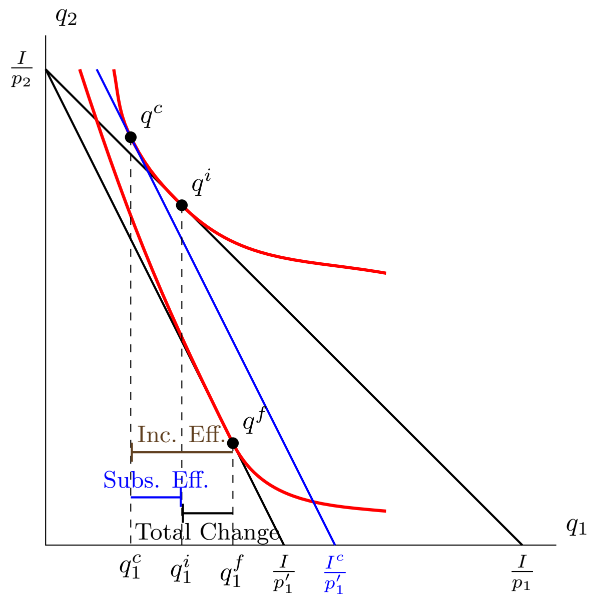

Figure 7: Effect of an increase in the price of good 1, when good 1 is a Giffen good.

find this possibility very compelling. Giffen's observation about the bread in late 19th century Britain is controversial: we are not sure it empirically holds. Some claim that potatoes during the great Irish famine may be considered a Giffen good. Well, maybe, but even if that's true, that is a very particular time and location in history. There is a 2008 paper your textbook discusses, which I will post to Moodle. It argues that in very poor parts of China, rice is a Giffen good. This paper is basically the only empirical evidence we know about the existence of a Giffen good. But in virtually any economic scenario we consider, the likelihood of having a Giffen good is so small that we can just discard that possibility. From now on, we will assume that a good is not a Giffen good. It may still be an inferior good, but even then we will assume that the income effect does not dominate the substitution effect. Those goods are sometimes called **ordinary goods**.

Definition 4. *Good i is an* **ordinary good** *if the consumer's consumption of good i decreases as the price of good i increases.*

From now on, let's agree that a good is not a Giffen good.

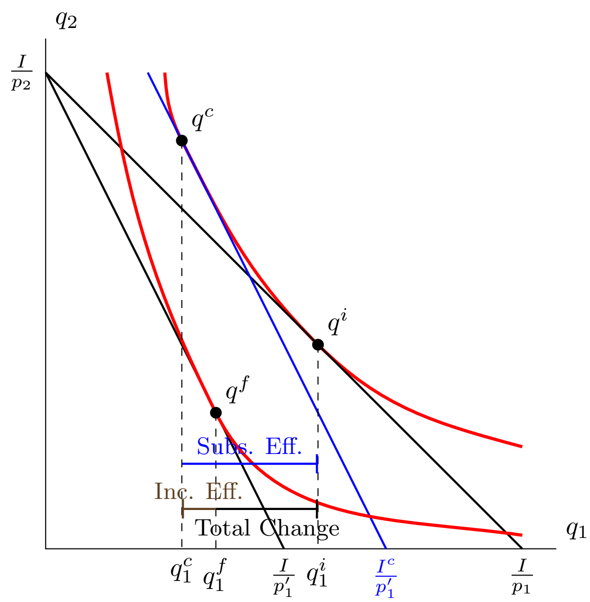

Figure 8: Effect of an increase in the price of good 1, when good 1 is an inferior but not a Giffen good.